



Physics I Lecture06-Circular Motion and Other Applications-I

簡紋濱

國立交通大學 理學院 電子物理系

CONTENTS

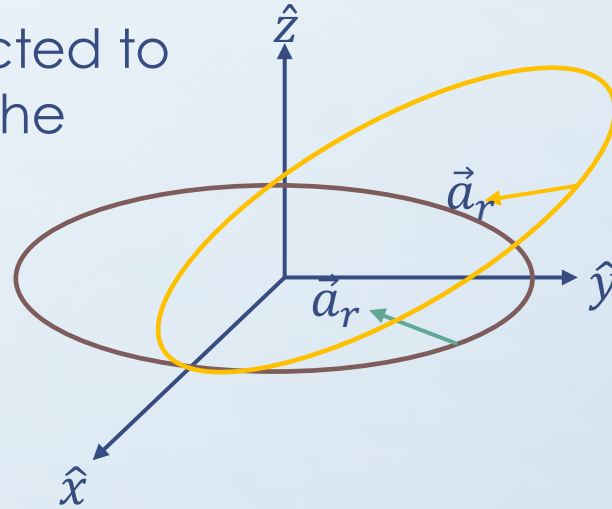
1. Newton's 2nd Law in Uniform Circular Motion
2. Nonuniform Circular Motion
3. Motion in Accelerated Frames
4. Motion in The Presence of Resistive Force
5. Numerical Integration – Euler's Methods

1. NEWTON'S 2ND LAW IN UNIFORM CIRCULAR MOTION

Centripetal Acceleration, Centripetal Force

1. The centripetal force is given at first, then the object can turn its moving direction.
2. The centripetal force is always directed to the center of the trajectory. Find out the plane of the circular trajectory at first.

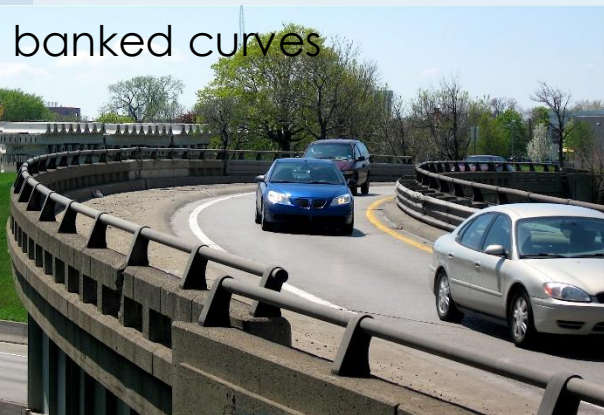
$$\vec{a}_r = -\frac{v^2}{r} \hat{r}$$



$$\vec{F}_{net,centripetal} = \sum_{i=1}^N \vec{F}_i = -m \frac{v^2}{r} \hat{r}$$



<http://www.stephenhodsdenmurray.com/>
2014



1. NEWTON'S 2ND LAW IN UNIFORM CIRCULAR MOTION

Example: A car travels on a circular roadway of radius r . The roadway is flat. The car travels at a high speed v , such that the friction force causing the centripetal acceleration is the maximum possible value. If the same car is now driven on another flat circular roadway of radius $2r$, and the coefficient of friction between the tires and the roadway is the same as on the first roadway, what is the maximum speed of the car such that it does not slide off the roadway?

$$mg\mu_s = F_{\text{centripetal}} = m \frac{v^2}{r}$$

$$r' = 2r \quad v' = ?$$

$$g\mu_k = \frac{v^2}{r} = \frac{v'^2}{r'} \quad v'^2 = \frac{v^2}{r} r' = 2v^2$$

$$v' = \sqrt{2}v$$

1. NEWTON'S 2ND LAW IN UNIFORM CIRCULAR MOTION

Example: An object of mass $m = 0.500$ kg is attached to the end of a cord whose length is $l = 1.50$ m. The object is whirled in a horizontal circle. If the cord can withstand a maximum tension of $F = 50.0$ N, what is the maximum speed the object can have before the cord breaks?

$$F_{centripetal} = F = m \frac{v_{max}^2}{l}$$

$$v_{max} = \sqrt{\frac{Fl}{m}} = \sqrt{\frac{50.0 \times 1.50}{0.500}} = 12.2 \text{ (m/s)}$$

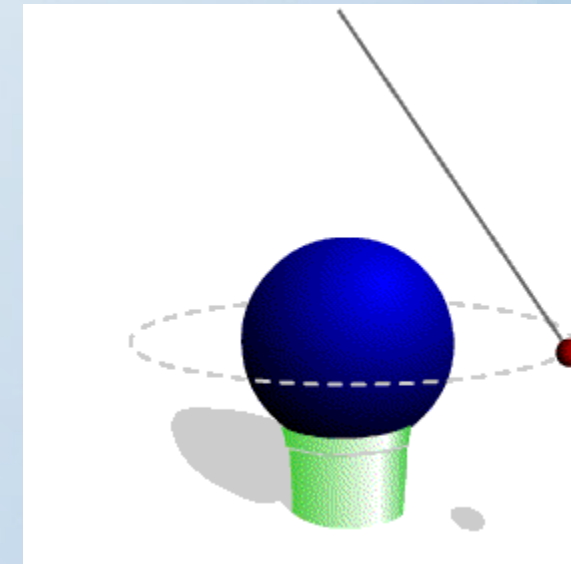
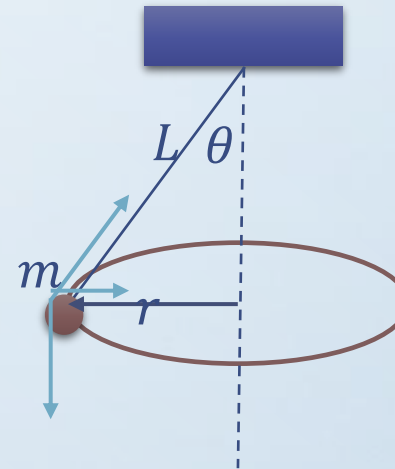
1. NEWTON'S 2ND LAW IN UNIFORM CIRCULAR MOTION

Example: A small object of mass m is suspended from a string of length L . The object revolves in a horizontal circle of radius r with constant speed v . Find (a) the speed of the object, and (b) the period of revolution.

Note that the string tension and the gravitational force give the necessary centripetal force.

$$(a) \quad a_r = g \tan \theta \quad \tan \theta = \frac{r}{\sqrt{L^2 - r^2}}$$
$$a_r = \frac{v^2}{r}$$
$$v = \sqrt{r a_r} = \sqrt{r g \tan \theta} = \sqrt{r^2 g / \sqrt{L^2 - r^2}}$$

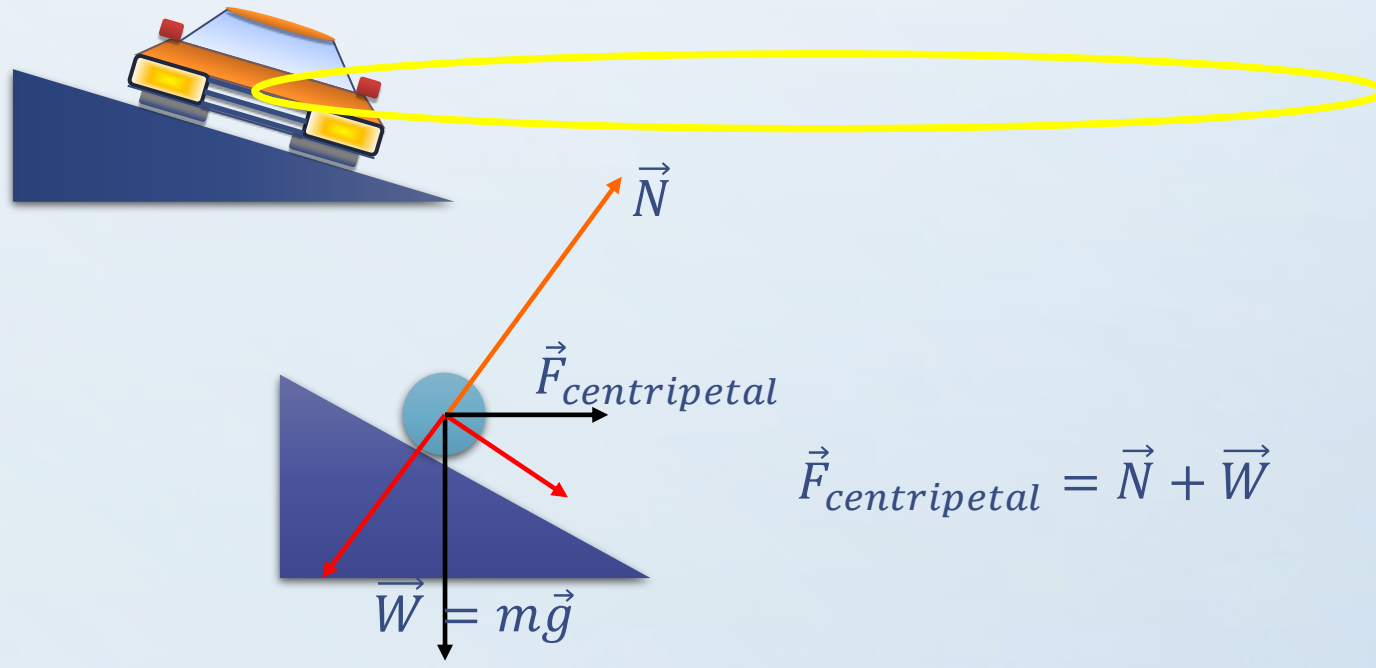
$$(b) \quad T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{r^2 g / \sqrt{L^2 - r^2}}} = 2\pi \sqrt{\frac{\sqrt{L^2 - r^2}}{g}}$$



1. NEWTON'S 2ND LAW IN UNIFORM CIRCULAR MOTION

Example: A curve of radius $r = 30$ m is banked at an angle θ . Find θ for which a car can round the curve at $v = 40$ km/h even if the road is covered with ice that friction is negligible.

Find out the plane of the circular motion at first, then find out the required centripetal force.



1. NEWTON'S 2ND LAW IN UNIFORM CIRCULAR MOTION

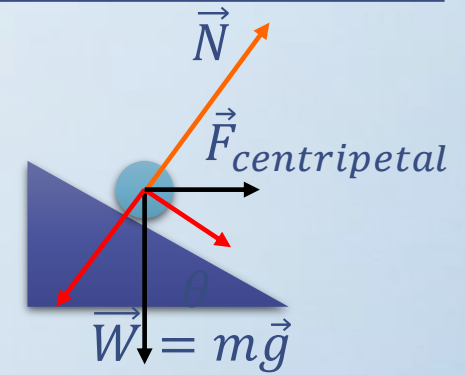
Example: A curve of radius $r = 30.0$ m is banked at an angle θ . Find θ for which a car can round the curve at $v = 40.0$ km/h even if the road is covered with ice that friction is negligible.

$$a_r = g \tan \theta$$

$$r = 30 \text{ m} \quad v = 40 \frac{\text{km}}{\text{h}} = 11.1 \text{ (m/s)}$$

$$g \tan \theta = \frac{v^2}{r} \quad \tan \theta = \frac{v^2}{gr} \cong 0.419$$

$$\theta \cong 22.7^\circ$$



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2. NONUNIFORM CIRCULAR MOTION

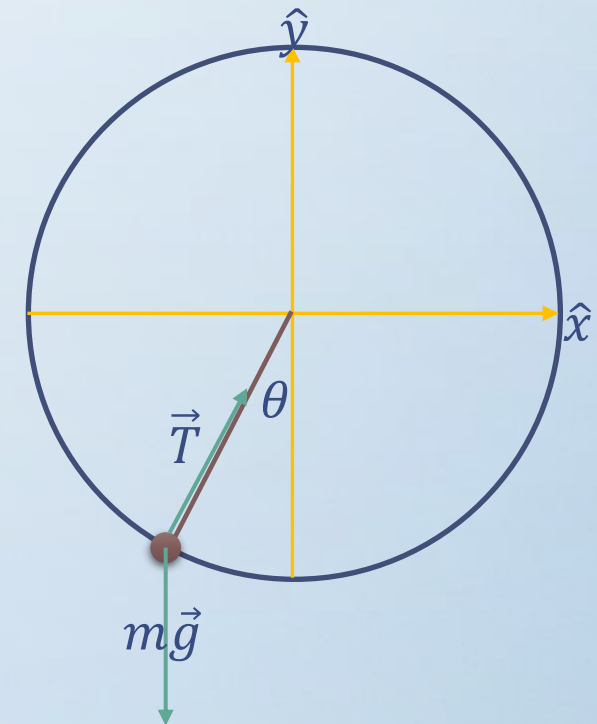
Example: A small sphere of mass m is attached to the end of a cord of length R which rotates under the influence of the gravitational force in a vertical circle about a fixed point O . Let us determine the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle θ with the vertical.

$$F_r = T = m \frac{v(\theta)^2}{R} + mg \cos \theta$$

$$F_t = -mg \sin \theta$$

To solve the problem, you need to specify the condition that, for example, the gravitation totally gives the centripetal force as the sphere is on the top.

$$\theta = \pi, T = 0, \frac{mv(\pi)^2}{R} - mg = 0, v(\pi) = \sqrt{gR}$$



2. NONUNIFORM CIRCULAR MOTION

Example: A small sphere of mass m is attached to the end of a cord of length R which rotates under the influence of the gravitational force in a vertical circle about a fixed point O . Let us determine the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle θ with the vertical.

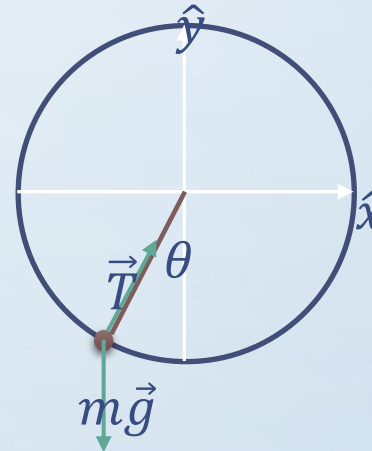
$$v(\pi) = \sqrt{gR} \quad s = R\theta$$

$$F_t = -mg \sin \theta = ma_t = m \frac{d^2 s}{dt^2} = mR \frac{d^2 \theta}{dt^2}$$

$$R \frac{d^2 \theta}{dt^2} = -g \sin \theta \quad R \frac{d^2 \theta}{dt^2} \frac{d\theta}{dt} = -g \sin \theta \frac{d\theta}{dt}$$

$$R\omega \frac{d\omega}{dt} = -g \sin \theta \frac{d\theta}{dt} \quad R \frac{d(\omega^2/2)}{dt} = -g \sin \theta \frac{d\theta}{dt}$$

$$Rd(\omega^2/2) = -g \sin \theta d\theta$$



2. NONUNIFORM CIRCULAR MOTION

Example: A small sphere of mass m is attached to the end of a cord of length R which rotates under the influence of the gravitational force in a vertical circle about a fixed point O . Let us determine the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle θ with the vertical.

$$v(\pi) = R\omega(\pi) = \sqrt{gR} \quad R d(\omega^2/2) = -g \sin \theta d\theta$$

$$R \int_{\omega=\sqrt{g/R}}^{\omega(\theta)} d(\omega^2/2) = -g \int_{\pi}^{\theta} \sin \theta' d\theta'$$

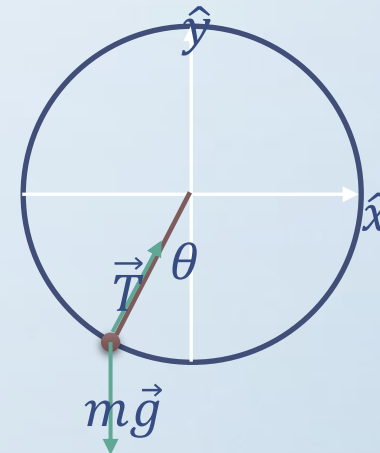
$$R \left(\frac{\omega^2}{2} - \frac{g}{2R} \right) = g(\cos \theta + 1)$$

$$\frac{d\theta}{dt} = \omega = \sqrt{\frac{g}{R} (2 \cos \theta + 3)}$$



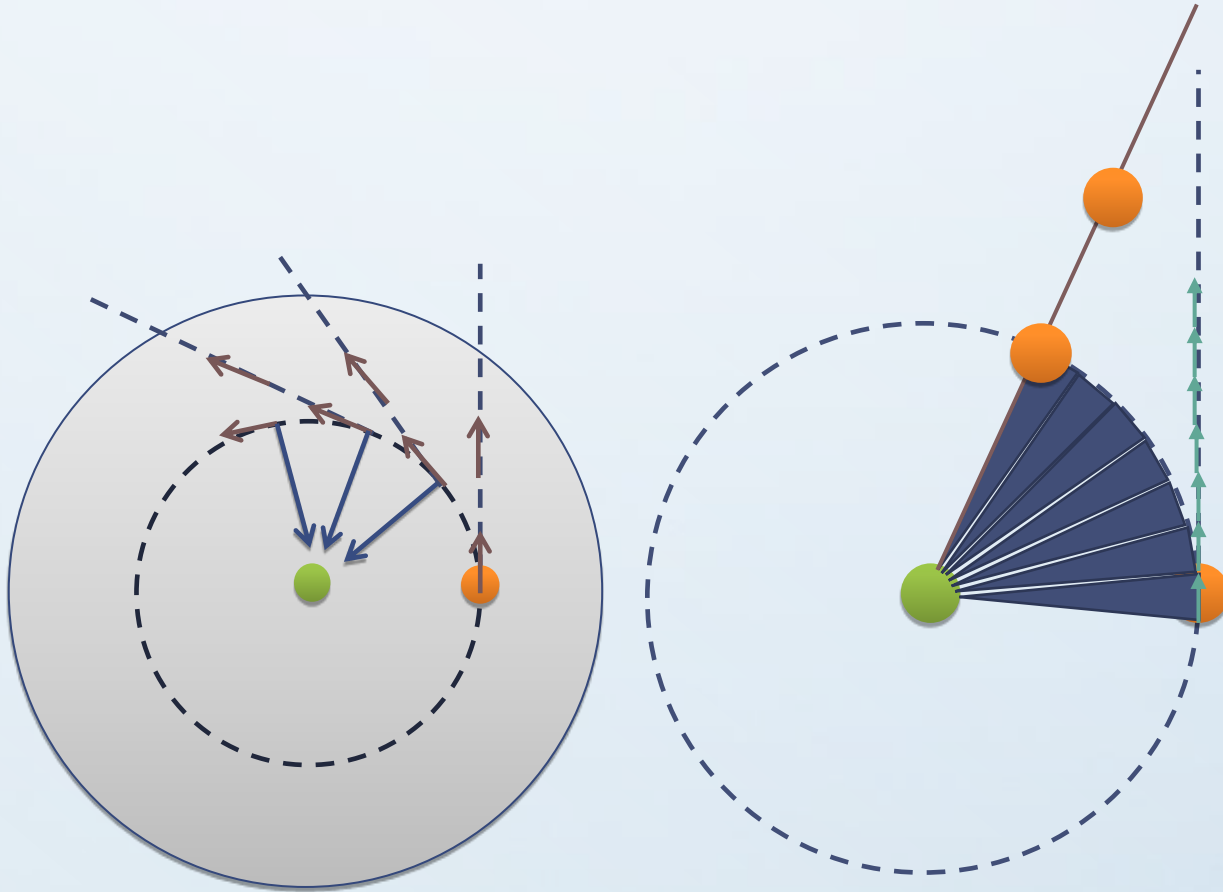
$$\omega(\theta = 0) = \sqrt{5 \frac{g}{R}}$$

$$\omega(\theta = \pi/2) = \sqrt{3 \frac{g}{R}}$$



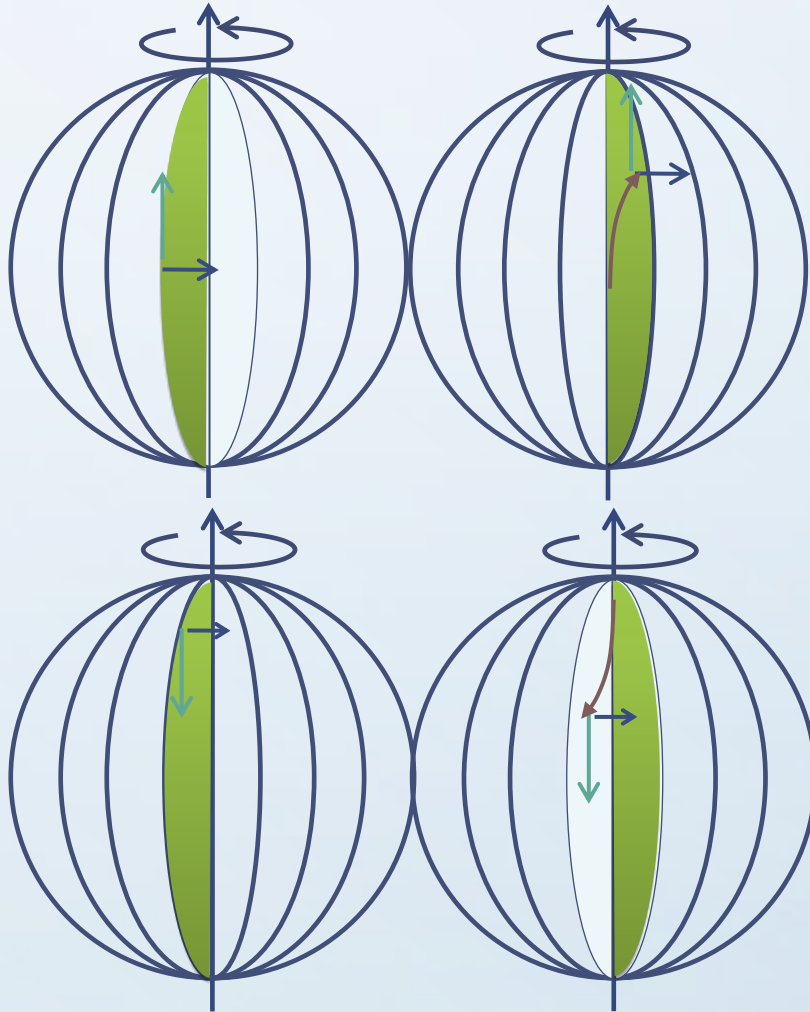
3. MOTION IN ACCELERATED FRAMES

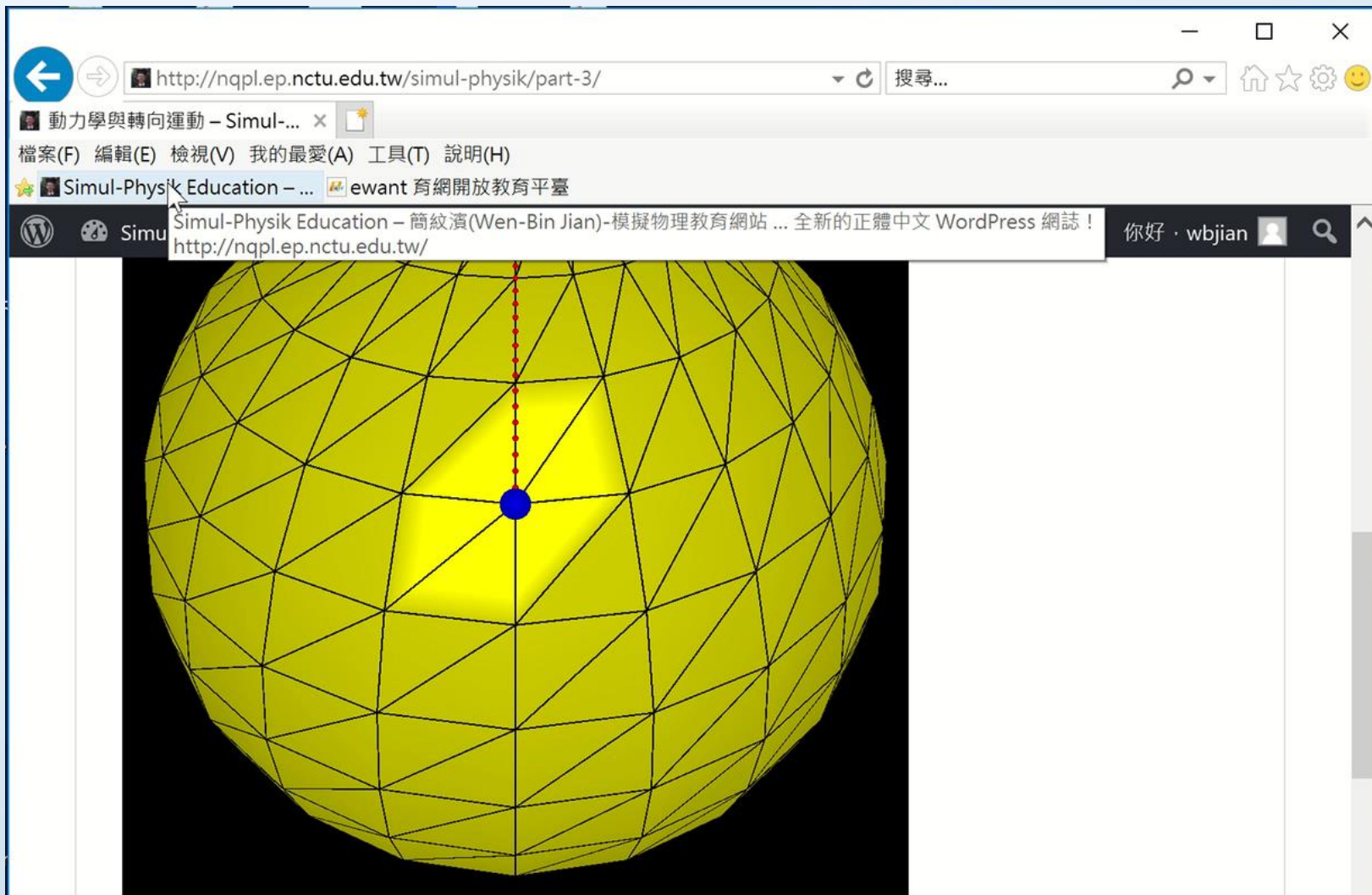
Centripetal or Centrifugal Forces? What's The Mechanism of Spin Dryer?



3. MOTION IN ACCELERATED FRAMES

The Coriolis Force:

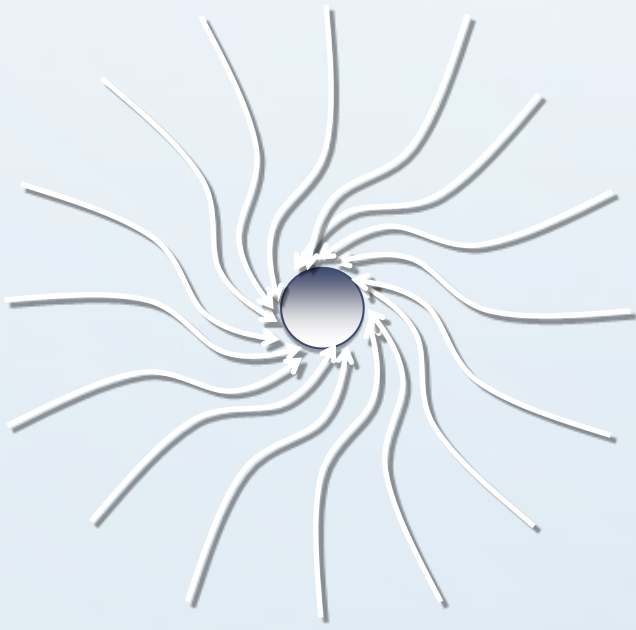




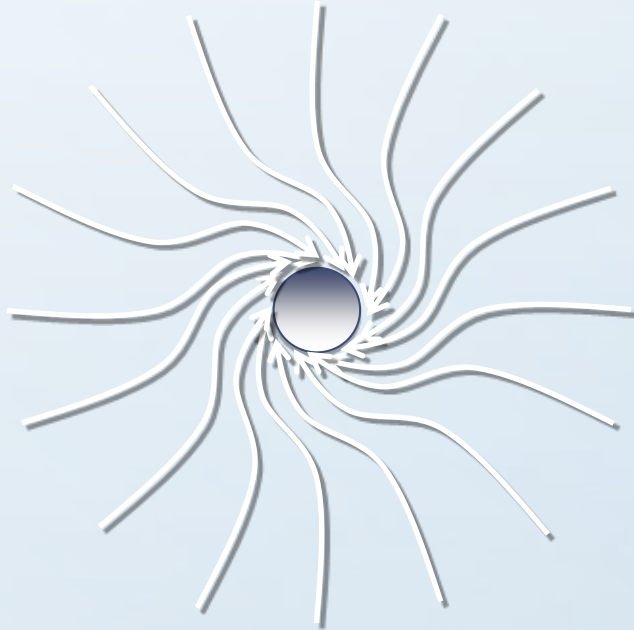
柯氏力(Coriolis force)的產生主要是因為我們以地球為座標系統，而地球是非慣性座標系統，在此座標系統下觀察直線慣性運動，會因座標轉換而產生一個相對的作用力-柯氏力。下圖中黃色球體模擬地球，藍色球體為物體在地球上的起始位置，紅色虛線表示預期的運動軌跡，當按下Start鍵啟動後，綠色的球是球體拋出後的運動，起始速度有地球轉動的角速度加上拋出速度，原本應該是直線運動，但遷移到地心引力的拋物線運動。

3. MOTION IN ACCELERATED FRAMES

Typhoon is counterclockwise in the northern hemisphere of the Earth.



Northern Hemisphere



Southern Hemisphere



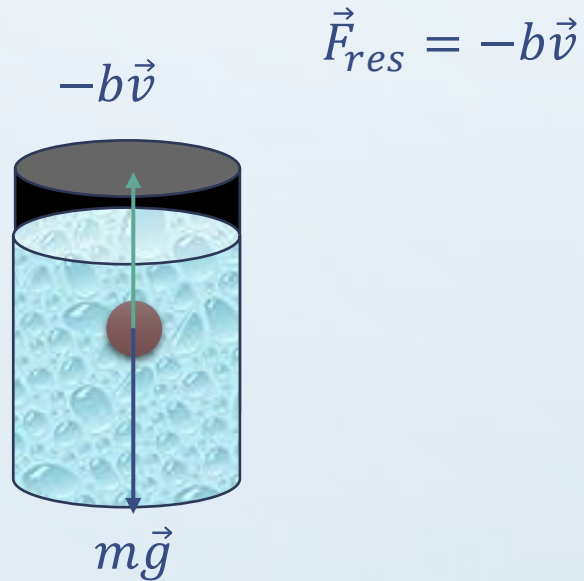
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4. MOTION IN THE PRESENCE OF RESISTIVE FORCE

Two Types of Resistive Force:

1. Objects in Liquid. The resistive force is proportional to the velocity.



2. Objects in Gas. The resistive force is proportional to the square of the speed.

$$\vec{F}_{res} = -\frac{D\rho A}{2}v^2\hat{v}$$

D : drag coefficient (~ 0.6),
 ρ : density of gas,
 A : cross-sectional area



4. MOTION IN THE PRESENCE OF RESISTIVE FORCE

Object in Liquid. Start falling from rest.

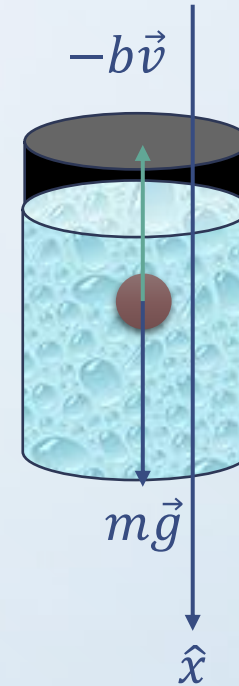
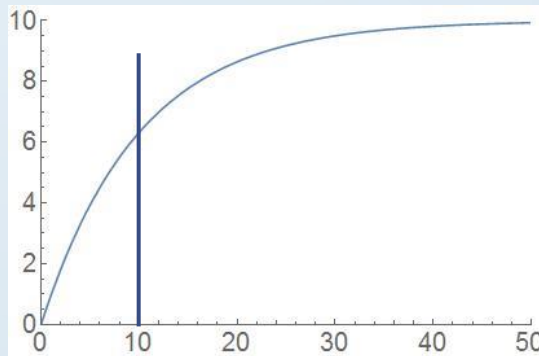
$$F = mg - bv = ma \quad \longrightarrow \quad v_t = \frac{mg}{b}$$

$$m \frac{dv}{dt} = mg - bv$$

$$\int_0^v \frac{m}{mg - bv} dv = \int_0^t dt$$

$$-\frac{m}{b} \left(\ln \left(\frac{mg - bv}{mg} \right) \right) = t \quad \longrightarrow \quad v(t) = \frac{mg}{b} \left(1 - e^{-\frac{bt}{m}} \right)$$

Time Constant: $\tau = m/b$



4. MOTION IN THE PRESENCE OF RESISTIVE FORCE

Example: A Sphere Falling in Oil.

A small sphere of mass 2.00 g is released from rest in a large vessel filled with oil. The sphere approaches a terminal speed of 5.00 cm/s. Determine (a) the time constant τ and (b) the time it takes the sphere to reach 90% of its terminal speed.

$$\begin{aligned} \text{(a)} \quad v_t &= 0.05(m/s) = \frac{mg}{b} \\ b &= \frac{0.002 \times 9.8}{0.05} = 0.392(kg/s) \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \tau &= \frac{m}{b} = 5.1 \times 10^{-3}(s) \\ v(t) &= \frac{mg}{b} \left(1 - e^{-\frac{bt}{m}}\right) \\ t &= -\tau \ln \left(\frac{mg - b \times \frac{0.9mg}{b}}{mg} \right) = 11.7 \times 10^{-3}(s) \end{aligned}$$

4. MOTION IN THE PRESENCE OF RESISTIVE FORCE

For objects dropping in air, terminal speed is:

$$F = mg - \frac{D\rho A}{2}v^2 = ma$$

The condition to reach the terminal speed is $a = 0$.

$$\longrightarrow mg - \frac{D\rho A}{2}v_t^2 = 0 \longrightarrow v_t = \sqrt{\frac{2mg}{D\rho A}}$$

For a human body in the free fall motion in air, the terminal speed is:

$$v_t = \sqrt{\frac{2 \times 60 \times 9.8}{0.6 \times (0.028/0.0224) \times 1}} \cong 39.6 \left(\frac{m}{s}\right) = 143 \left(\frac{km}{h}\right)$$

Object	Mass (kg)	Cross-Section (m ²)	V _t (m/s)
Sky diver	75	0.70	60
Bassball	0.145	4.2x10 ⁻³	43
Golfball	0.046	1.4x10 ⁻³	44
Hailstone	4.8x10 ⁻⁴	7.9x10 ⁻⁵	14
Raindrop	3.4x10 ⁻⁵	1.3x10 ⁻⁵	9.0



4. MOTION IN THE PRESENCE OF RESISTIVE FORCE

Object in Air:

Example: If a falling cat reaches a first terminal speed of 97 km/h while it is tucked in and then stretches out, doubling A , how fast is it falling when it reaches a new terminal speed?

$$v_t = \sqrt{\frac{2mg}{D\rho A}} \quad v_{t1} = 97(\text{km/h})$$
$$\frac{v_{t2}}{v_{t1}} = \sqrt{\frac{A_1}{A_2}} \quad v_{t2} = \frac{97}{\sqrt{2}} = 69(\text{km/h})$$

4. MOTION IN THE PRESENCE OF RESISTIVE FORCE

Object in Air:

Example: A raindrop with radius $R = 1.5$ mm falls from a cloud that is at height $h = 1200$ m above the ground. The drag coefficient D for the drop is 0.60. Assume that the drop is spherical throughout its fall. The density of water ρ_w is 1000 kg/m³, and the density of air ρ_a is 1.2 kg/m³.

$$A = \pi R^2 = \pi(0.0015)^2 = 7.1 \times 10^{-6} (\text{m}^2)$$

$$m = 1000 \times \frac{4\pi(0.0015)^3}{3} = 1.4 \times 10^{-5} (\text{kg})$$

$$v_t = \sqrt{\frac{2mg}{D\rho A}} = \sqrt{\frac{2 \times 1.4 \times 10^{-5} \times 9.8}{0.6 \times 1.2 \times 7.1 \times 10^{-6}}} = 7.3 \left(\frac{\text{m}}{\text{s}} \right) = 26 \left(\frac{\text{km}}{\text{h}} \right)$$

5. NUMERICAL INTEGRATION – EULER'S METHODS

If you cannot solve the exact solutions of $x(t)$, you need to express it numerically. In the real world you may always need the numerical representation of motion.

Example: Consider the initial value problem $\frac{dy}{dx} = 0.1\sqrt{y} + 0.4x^2$, $y(2) = 4$. Use Euler's method to obtain an approximation of $y(2.5)$ using $\Delta x = 0.1$ and $\Delta x = 0.05$.

$\Delta x = 0.1$

	x	y	y'
0	2	4	1.8
1	2.1	4.18	1.9685
2	2.2	4.3768	2.1452
3	2.3	4.5914	2.3303
4	2.4	4.8244	2.5236

$$B3=B2+0.1$$

$$C3=C2+0.1*D2$$

$$D3=0.1*SQRT(C3)+0.4*B3*B3$$

5. NUMERICAL INTEGRATION – EULER'S METHODS

Example: Consider the initial value problem $\frac{dy}{dx} = 0.1\sqrt{y} + 0.4x^2$, $y(2) = 4$. Use Euler's method to obtain an approximation of $y(2.5)$ using $\Delta x = 0.1$ and $\Delta x = 0.05$.

$\Delta x = 0.05$

	x	y	y'
0	2	4	1.8
1	2.05	4.09	1.883237
2	2.1	4.184162	1.968552
3	2.15	4.282589	2.055944
4	2.2	4.385387	2.145413
5	2.25	4.492657	2.236959
6	2.3	4.604505	2.330581
7	2.35	4.721034	2.426279
8	2.4	4.842348	2.524053
9	2.45	4.968551	2.623902
10	2.5	5.099746	2.725826

$$B3=B2+0.05$$

$$C3=C2+0.05*D2$$

$$D3=0.1*\text{SQRT}(C3)+0.4*B3*B3$$

numerical06-1 - Excel

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	A	B	C	D	E	F	G	H	I	J
1	Example 1	Consider the initial value problem $dy/dx=0.1\sqrt{y}+0.4x^2$, $y(2)=4$.								
2		Use Euler's method to obtain an approximation of $y(2.5)$ using								
3		$\Delta x=0.1$ and $\Delta x=0.05$.								
4		step	x	y	dy/dx					
5										
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5. NUMERICAL INTEGRATION – EULER'S METHODS

Example: Compare the numerical results with the integrated function for the constant acceleration of $a = 2.0 \text{ (m/s}^2\text{)}$, $v(0) = 0 \text{ (m/s)}$, $x(0) = 0 \text{ (m)}$.

$a = 2.0 \text{ m/s}^2, \Delta t = 0.1 \text{ s}, v(0) = 0, x(0) = 0 - v(t) = at, x(t) = 1/2 * at^2$						
Step	t	v	x		v(t)	x(t)
0	0.	0	0		0	0
1	0.1	0.2	0		0.2	0.01
2	0.2	0.4	0.02		0.4	0.04
3	0.3	0.6	0.06		0.6	0.09
4	0.4	0.8	0.12		0.8	0.16

EXERCISE

For $t < 0$, an object of mass m experiences no force and moves in the positive direction with a constant speed v_i . Beginning at $t = 0$, when the object passes the position at $x = 0$, it experiences a net resistive force proportional to the square of its speed as $F_r = -mkv^2$, where k is a constant. Please find the speed of the object after $t = 0$.

$$F_{net} = -mkv^2 = ma$$

$$-mkv^2 = m \frac{dv}{dt}$$

Identify two variables: v and t

Use separation of variables

$$-kdt = \frac{dv}{v^2}$$

$$-k \int_0^{t'} dt = \int_{v_i}^{v(t')} \frac{dv}{v^2}$$

EXERCISE

For $t < 0$, an object of mass m experiences no force and moves in the positive direction with a constant speed v_i . Beginning at $t = 0$, when the object passes the position at $x = 0$, it experiences a net resistive force proportional to the square of its speed as $F_r = -mkv^2$, where k is a constant. Please find the speed of the object after $t = 0$.

$$-k \int_0^{t'} dt = \int_{v_i}^{v(t')} \frac{dv}{v^2}$$

$$-k[t]_{t=0}^{t=t'} = \left[-\frac{1}{v} \right]_{v=v_i}^{v=v(t')}$$

$$-kt' = -\left(\frac{1}{v(t')} - \frac{1}{v_i} \right)$$

$$\frac{1}{v(t)} = \frac{1}{v_i} + kt \rightarrow v(t) = \frac{v_i}{v_i kt + 1}$$

EXERCISE

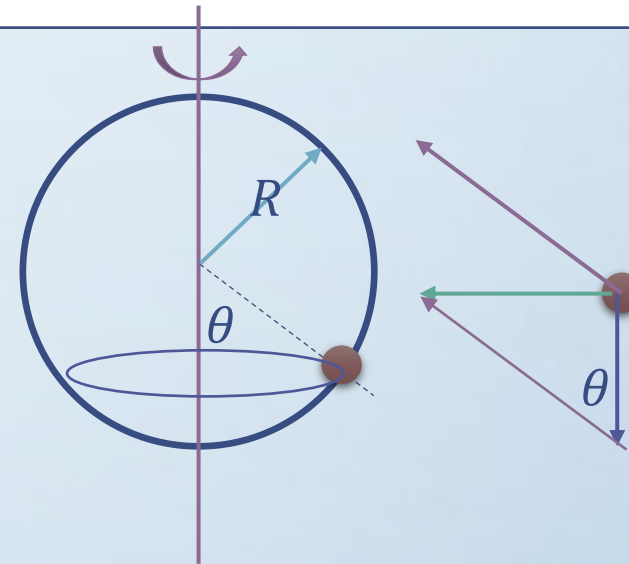
A single bead can slide with negligible friction on a stiff wire that has been bent into a circular loop of radius R as shown in the figure. The circle is always in the vertical plane and rotates steadily about its vertical diameter. The position of the bead is described by the angle θ that the radial line, from the center of the loop to the bead, makes with the vertical. Please calculate the angle θ when the circle is rotating with a period of T .

$$\omega = \frac{2\pi}{T}, r_h = R \sin \theta, a_T = r_h \omega^2 = \frac{4\pi^2 r_h}{T^2}$$

$$a_T = g \tan \theta = \frac{4\pi^2 r_h}{T^2} = \frac{4\pi^2 R \sin \theta}{T^2}$$

$$\rightarrow g \frac{\sin \theta}{\cos \theta} = \frac{4\pi^2 R \sin \theta}{T^2}$$

$\theta = 0$ is one solution



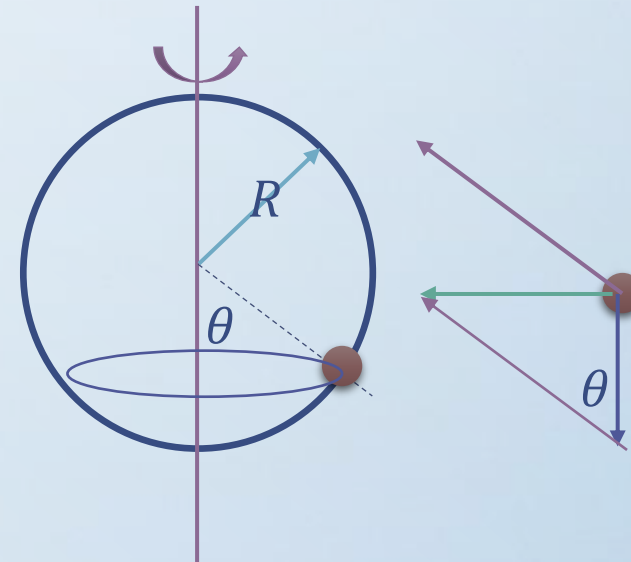
EXERCISE

A single bead can slide with negligible friction on a stiff wire that has been bent into a circular loop of radius R as shown in the figure. The circle is always in the vertical plane and rotates steadily about its vertical diameter. The position of the bead is described by the angle θ that the radial line, from the center of the loop to the bead, makes with the vertical. Please calculate the angle θ when the circle is rotating with a period of T .

$$\cos \theta = \frac{gT^2}{4\pi^2 R}$$

$$1 > \cos \theta > 0 \rightarrow \sqrt{\frac{4\pi^2 R}{g}} > T > 0$$

$$\text{if } R = 0.15 \text{ m, } T < \sqrt{\frac{4\pi^2 \times 0.15}{9.8}} = 0.78 \text{ s}$$



ACKNOWLEDGEMENT



國立交通大學理學院
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