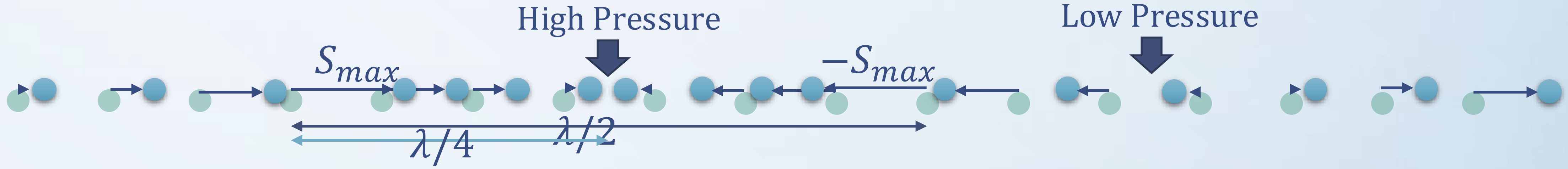
Chapter 17-1 Sound Waves

簡紋濱
國立交通大學 理學院 電子物理系

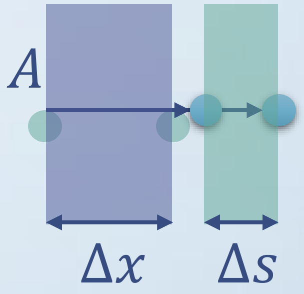
Outline

1. Sound Waves
2. Speed of Sound Waves
3. Intensity of Sound Waves
4. The Doppler Effect

1. SOUND WAVES



$$S(x,t) = s_{max}\cos(kx - \omega t)$$



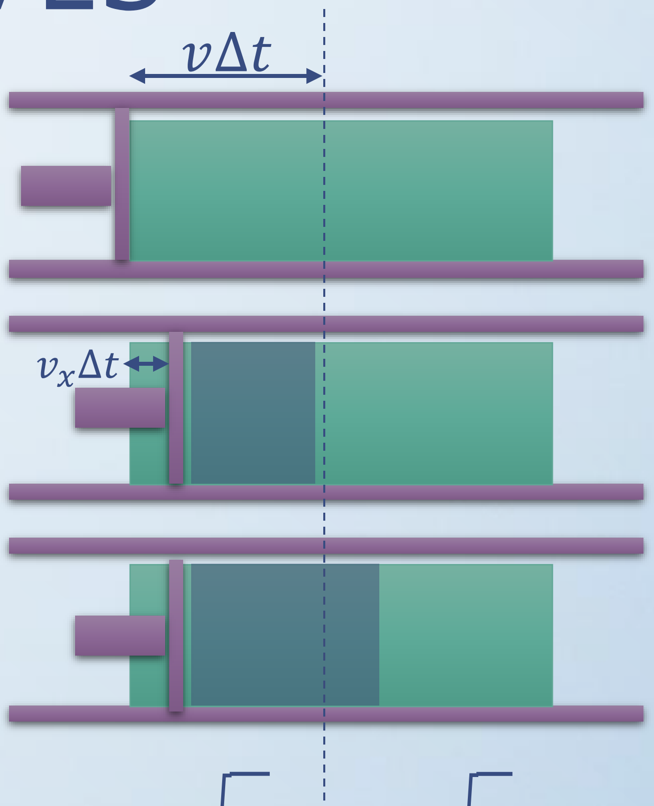
$$\Delta P = \Delta P_{max}\sin(kx - \omega t)$$

$$B = \frac{volume\ stress}{volume\ strain} = -\frac{\Delta F/A}{\Delta V/V}$$

Bulk modulus: $\Delta P = -B\frac{\Delta V}{V} = -B\frac{A\Delta s}{A\Delta x}$

$$\Delta P = -B\frac{\partial s}{\partial x} = Bs_{max}k\sin(kx - \omega t),\ \Delta P_{max} = Bs_{max}k$$

2. SPEED OF SOUND WAVES



Impulse from the change of volume:

$$I = F\Delta t = A(\Delta P)\Delta t$$

$$\Delta P = -B\frac{\Delta V}{V} = -B\frac{-Av_x\Delta t}{Av\Delta t} = B\frac{v_x}{v}$$

$$I = A\left(B\frac{v_x}{v}\right)\Delta t = AB\frac{v_x}{v}\Delta t$$

Momentum variation:

$$\Delta p = m\Delta v = \rho V v_x = \rho A v \Delta t v_x$$

$$I = \Delta p \to AB\frac{v_x}{v}\Delta t = \rho A v \Delta t v_x \to \quad \frac{B}{v} = \rho v \to v^2 = \frac{B}{\rho} \to v = \sqrt{\frac{B}{\rho}} \quad v = \sqrt{\frac{T}{\mu}}$$

$$v = \sqrt{\text{elastic property/inertial property}}$$

2. SPEED OF SOUND WAVES

For sound traveling through air, the speed of sound as a function of temperature is expressed as $v = 331\sqrt{\frac{273+T_C}{273}}$, where T_C is the air temperature in Celsius.

Medium	v (m/s)	Medium	v (m/s)	Medium	v (m/s)
Air (0°C)	331	Water	1493	Iron	5950
Oxygen (0°C)	317	Mercury	1450	Aluminum	6420
Helium(0°C)	972	Methyl Alcohol	1143	Copper	5010
Hydrogen(0°C)	1286			Gold	3240

$$B = \rho v^2 \qquad \Delta P_{max} = B s_{max} k \qquad \rightarrow \Delta P_{max} = \rho v^2 s_{max} k = \rho v^2 s_{max} \frac{\omega}{v} = \rho v \omega s_{max}$$

3. INTENSITY OF SOUND WAVES

The energy carried by the sound wave: $Power = Fv = (A\Delta P)\left(\frac{\partial s}{\partial t}\right), B = \rho v^2$

$$\Delta P = \Delta P_{max}\sin(kx - \omega t) = \rho v \omega s_{max}\sin(kx - \omega t)$$

$$S(x, t) = s_{max}\cos(kx - \omega t)$$

$$Power = A\rho v \omega s_{max}\sin(kx - \omega t)\,(\omega s_{max}\sin(kx - \omega t))$$

$$Power = A\rho v \omega^2 s_{max}^2 \sin^2(kx - \omega t)$$

$$\langle Power \rangle_{avg} = A\rho v \omega^2 s_{max}^2 \langle \sin^2(kx - \omega t)\rangle_{avg}$$

$$\langle \sin^2(kx - \omega t)\rangle_{avg} = \frac{1}{T}\int_0^T \sin^2(\omega t)\, dt = \frac{1}{2}$$

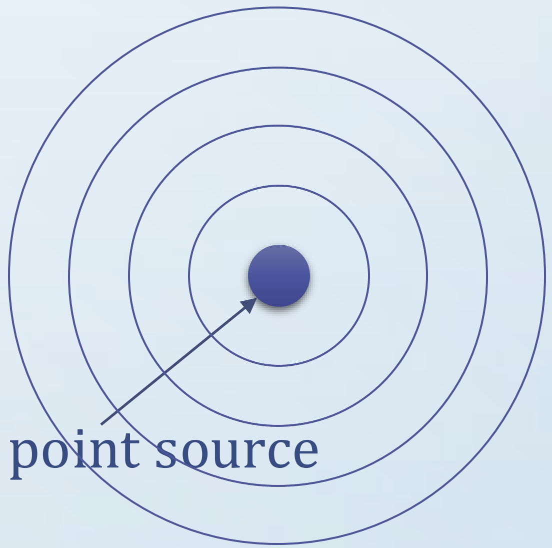
3. INTENSITY OF SOUND WAVES

$$\langle Power \rangle_{avg} = \frac{1}{2} A\rho v \omega^2 s_{max}^2 \rightarrow I = \frac{\langle Power \rangle_{avg}}{A} = \frac{1}{2} \rho v \omega^2 s_{max}^2$$

$$\Delta P_{max} = \rho v \omega s_{max} \rightarrow I = \frac{(\Delta P_{max})^2}{2\rho v}$$

For a point-source sound wave, the intensity is dependent on the distance and the power according to

$$I = \frac{Power}{area} = \frac{Power}{4\pi r^2}$$



3. INTENSITY OF SOUND WAVES

Example: The faintest sounds the human ear can hear at a frequency of 1,000 Hz have an intensity of ~$1.00 X 10^{-12}$ W/m^2. Determine the pressure and the displacement amplitude of the sound. ($\rho = 1.20 \frac{kg}{m^3}, v = 343 \frac{m}{s}$)

$$I = \frac{(\Delta P_{max})^2}{2\rho v} \rightarrow \Delta P_{max} = \sqrt{2I\rho v} = \sqrt{2 \times 1.00 \times 10^{-12} \times 1.20 \times 343} = 2.87 \times 10^{-5} N/m^2$$

$$\Delta P_{max} = \rho v \omega s_{max} \rightarrow s_{max} = \frac{\Delta P_{max}}{\rho v \omega} = \frac{2.87 \times 10^{-5}}{1.20 \times 343 \times (2\pi \times 1000)} = 1.11 \times 10^{-11} m$$

Example: A point-source sound wave has a power of 80.0 W. Find the intensity 3 m away from it.

$$I = \frac{P}{A} = \frac{80.0}{4\pi \times 3^2} = 0.707\, W/m^2$$

3. INTENSITY OF SOUND WAVES

Sound level in decibels (dB): $\beta = 10 \log(I/I_0)$

I_0 is of $1.00\text{X}10^{-12}$ W/m^2, the reference intensity as well as the threshold of hearing.

dB	0	40	50	60	70	90
Example	Threshold of hearing	Home, library	Office, soft music	Normal conversation	Noisy Office	Heavy truck, damage

Example: The noisy sound with an intensity of $6.00\text{X}10^{-7}$ W/m^2 is delivered to the students. What is the sound level?

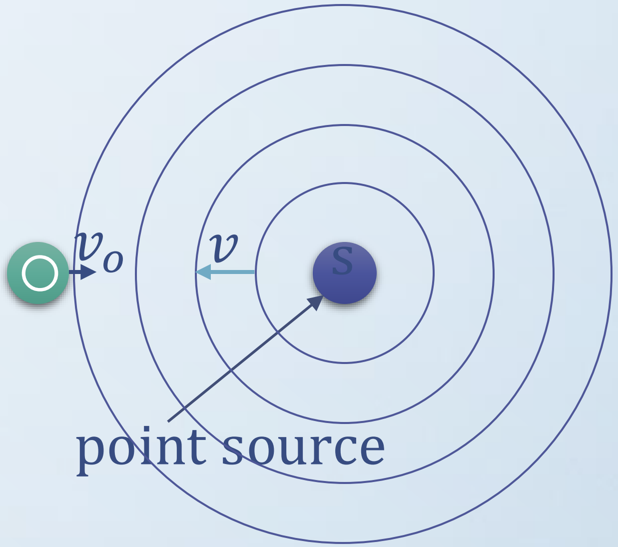
$$\beta = 10 \log\left(\frac{6.00 \times 10^{-7}}{1.00 \times 10^{-12}}\right) = 57.8\ dB$$

4. THE DOPPLER EFFECT

The observer is moving toward the source.

$$v' = v + v_o$$

$$f' = \frac{v'}{\lambda}, \lambda = \frac{v}{f} \rightarrow f' = \frac{v'}{v} f \rightarrow \frac{f'}{f} = \frac{v + v_o}{v}$$

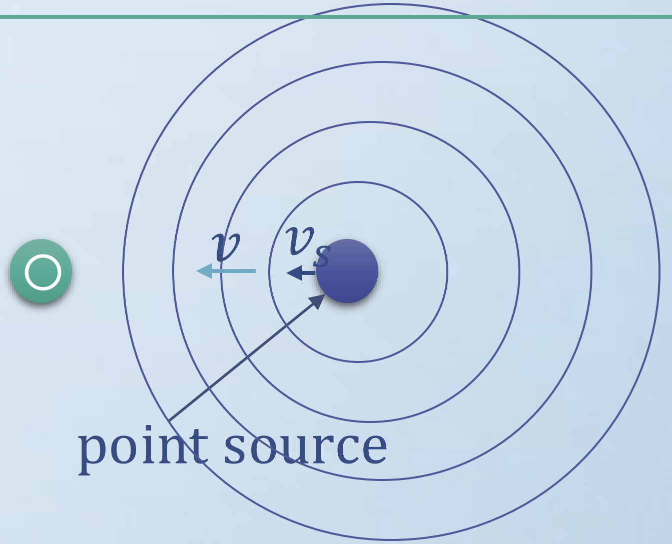


The observer is moving away from the source.

$$v' = v - v_o \rightarrow \frac{f'}{f} = \frac{v - v_o}{v}$$

The source is moving toward the observer.

$$\lambda' = \lambda \frac{v - v_s}{v} \rightarrow f' = \frac{v}{\lambda'} = \frac{v}{\lambda} \frac{v}{v - v_s} \rightarrow \frac{f'}{f} = \frac{v}{v - v_s}$$



The source is moving away from the observer.

$$\lambda' = \lambda \frac{v + v_s}{v} \rightarrow \frac{f'}{f} = \frac{v}{v + v_s}$$

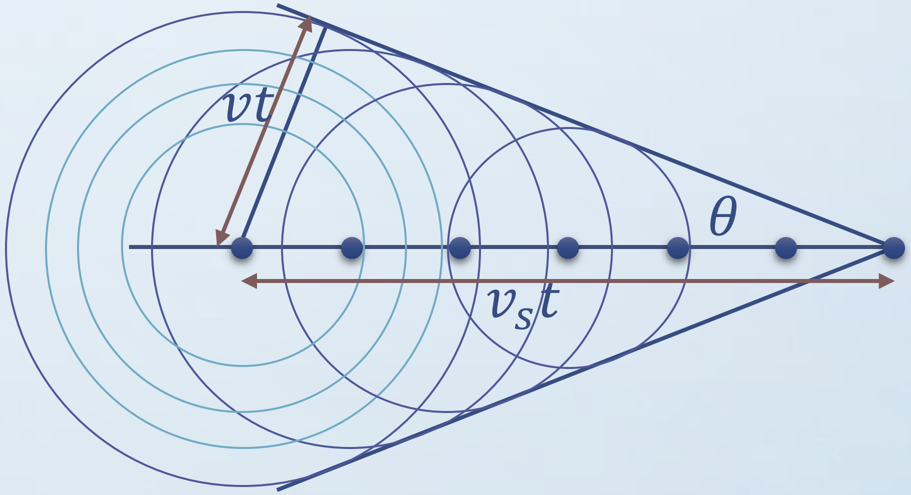
4. THE DOPPLER EFFECT

Example: A clock radio is sending a sound of frequency 600 Hz when it falls down a height of 15.0 m. The observer is staying at the original place of 15 m in height. What will its frequency be just before the clock radio strikes against the ground? Assume the speed of sound is 343 m/s.

$$v_s = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 15} = 17.1\ m/s$$

$$f = 600, \frac{f'}{f} = \frac{v}{v + v_s} \rightarrow f' = 600 \times \left(\frac{343}{343 + 17.1}\right) = 572\ Hz$$

Shock Waves:



$$\sin\theta = \frac{vt}{v_s t} = \frac{v}{v_s}$$

Physics I
Lecture17-2
Superposition and Standing Waves

簡紋濱
國立交通大學 理學院 電子物理系

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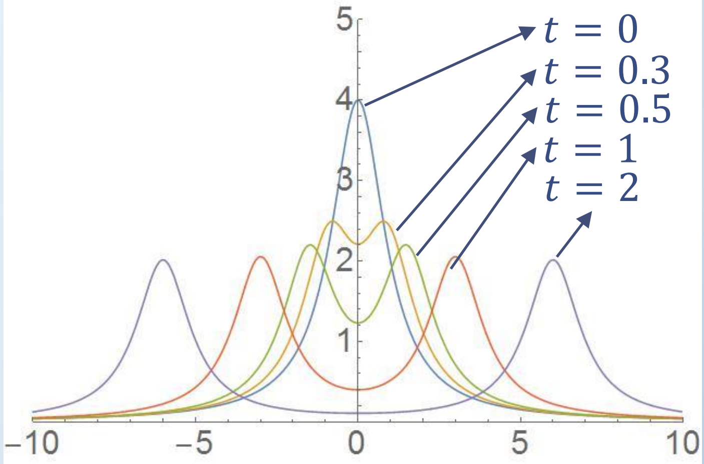
1. Superposition & Interference
2. Standing Waves
3. Standing Waves in Strings Fixed at Both Ends
4. Standing Waves in Air Columns
5. Beats: Interference in Time
6. Nonsinusoidal Wave Patterns

1. SUPERPOSITION & INTERFERENCE

Superposition: $y_1(x,t) = \frac{2.0}{(x-3.0t)^2+1}, y_2(x,t) = \frac{2.0}{(x+3.0t)^2+1},$

$$y_{total}(x,t) = \frac{2.0}{(x-3.0t)^2+1} + \frac{2.0}{(x+3.0t)^2+1}$$



Both waves do not alter or change the travel of each other.

Superposition of wave functions rather than the energy.

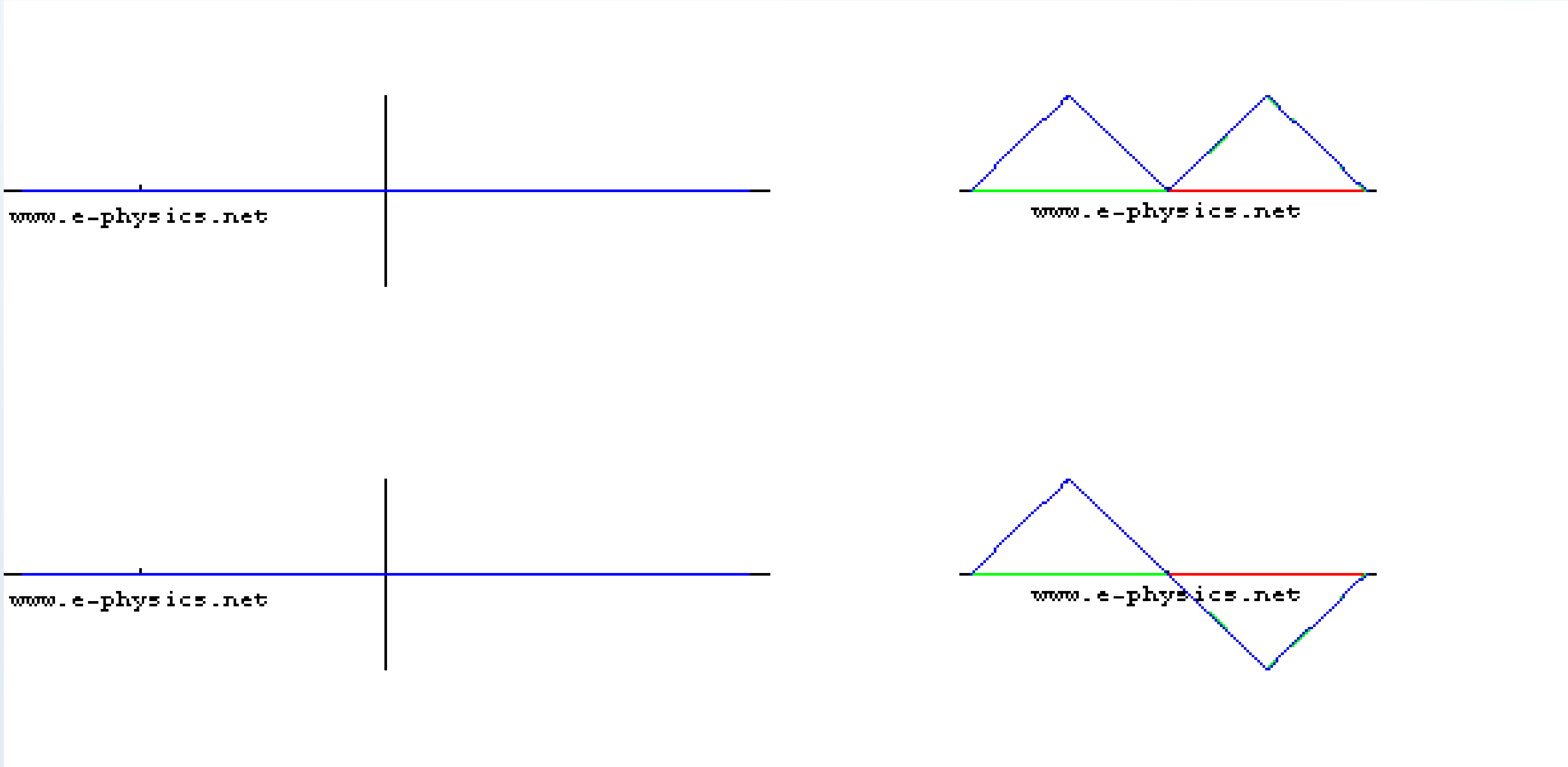
$$y_1(x,t) = A_1 \sin(k_1 x - \omega_1 t)$$

$$y_2(x,t) = A_2 \sin(k_2 x - \omega_2 t)$$

$$E_1 \propto y_1^2(x,t), E_2 \propto y_2^2(x,t), E_{total} \propto \left(y_1(x,t) + y_2(x,t)\right)^2$$

$$E_{total} \neq E_1 + E_2$$

1. SUPERPOSITION & INTERFERENCE



http://www.e-physics.net/Download/12wave/12wave.htm

1. SUPERPOSITION & INTERFERENCE

Superposition of two sinusoidal waves of the same amplitude, wave length, and frequency:

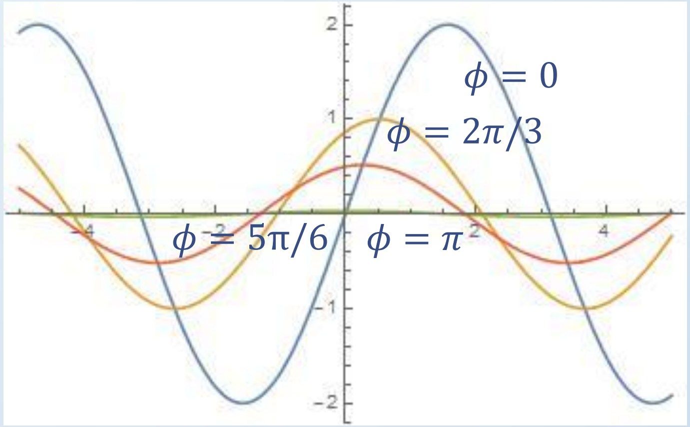
$$y_1(x,t) = A\sin(kx - \omega t)$$

$$y_2(x,t) = A\sin(kx - \omega t + \phi)$$

$$y_t(x,t) = A\sin(kx - \omega t) + A\sin(kx - \omega t + \phi)$$

$$y_t(x,t) = 2A\cos(\phi/2)\sin(kx - \omega t + \phi/2) \rightarrow A' = 2A\cos(\phi/2)$$

Phase diff	Phase diff (λ)	Type of Interference
0	0	fully constructive
$2\pi/3$	0.33	intermediate
π	0.5	fully destructive
$5\pi/6$	0.42	partially destructive



1. SUPERPOSITION & INTERFERENCE

Example: Two identical sinusoidal waves, moving in the same direction along a stretched string, interfere with each other. The amplitude of each wave is 9.8 mm, and the phase difference between them is 100°. (a) What is the amplitude of the resultant wave due to the interference of these two waves, and what type of interference occurs? (b) What phase difference, in radius and wavelength, will give the resultant wave an amplitude of 4.9 mm?

$A = 9.8\,mm, \phi = 100^0$

$y_t(x,t) = 2A\cos(\phi/2)\sin(kx - \omega t + \phi/2)\,, A' = 2A\cos(\phi/2)$

(a) $A' = 2 \times 9.8 \times \cos(50^0) = 13\,mm$

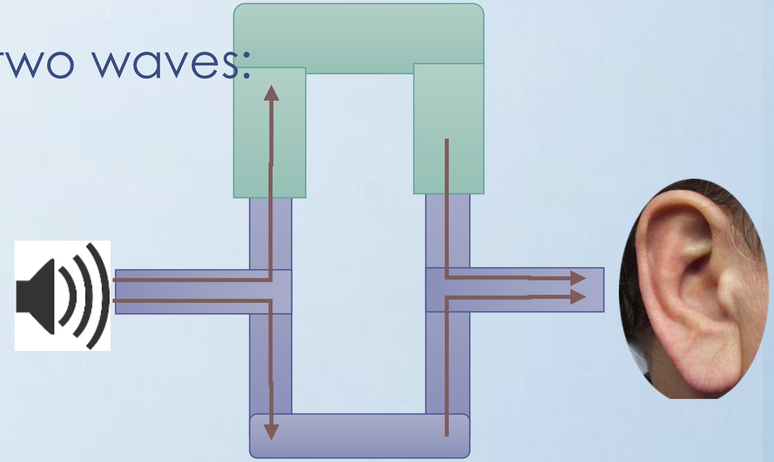
(b) $4.9 = 2 \times 9.8 \times \cos(\phi/2) \rightarrow \cos(\phi/2) = 0.25$

$\phi = 150^0 \rightarrow 5\lambda/12$

1. SUPERPOSITION & INTERFERENCE

Interference of sound waves: human audio spectrum is 20-20k Hz, the speed of sound is 340 m/s, the wavelength of 4k Hz sound is 8.5 cm

Use the path difference to give the phase difference between two waves:



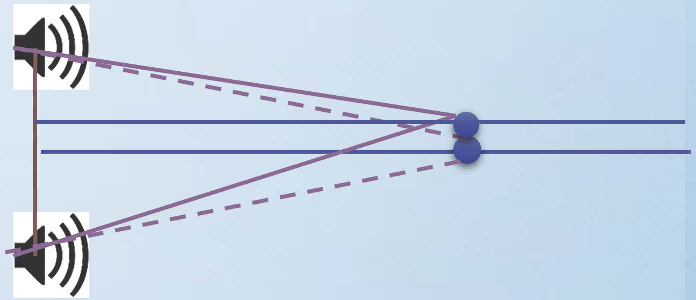
$$\frac{\Delta r}{\lambda} = \frac{\phi}{2\pi} \rightarrow \phi = 2\pi\frac{\Delta r}{\lambda}$$

$\Delta r = n\lambda$ → constructive interference

$\Delta r = \left(n + \frac{1}{2}\right)\lambda$ → destructive interference

1. SUPERPOSITION & INTERFERENCE

Example: Two identical speakers placed 3.00 m apart are driven by the same oscillator. A listener is originally at point O, located 8.00 m from the center of the line connecting the two speakers. The listener then moves to point P, which is a perpendicular distance 0.350 m from O, and she experiences the first minimum in sound intensity. What is the frequency of the oscillator? (The speed of sound is 343 m/s.)



$$\Delta r = \sqrt{(1.5+0.35)^2+8^2} - \sqrt{(1.5-0.35)^2+8^2} = 0.136\, m$$

destructive interference: $\phi = \pi = 2\pi \frac{\Delta r}{\lambda} \rightarrow \lambda = 2(\Delta r) = 0.272\, m$

$$f = \frac{v}{\lambda} = \frac{343}{0.272} = 1260\, Hz$$

2. STANDING WAVES

Two waves propagate in the opposite directions:

$$y_1(x,t) = A\sin(kx - \omega t), \quad y_2(x,t) = A\sin(kx + \omega t)$$

$$y_t(x,t) = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$

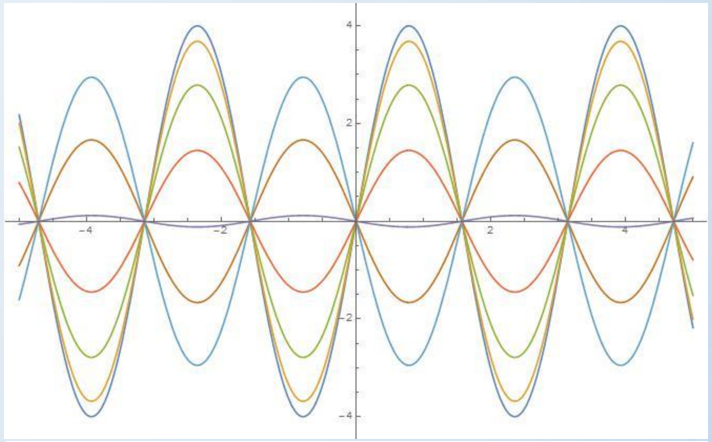
$$y_t(x,t) = 2A\sin(kx)\cos(\omega t)$$

$2A\sin(kx)$ is maximum when $kx = \left(n + \frac{1}{2}\right)\pi$

$x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$, the position of antinodes

$2A\sin(kx)$ is minimum when $kx = n\pi$

$x = n\frac{\lambda}{2}$, the position of nodes



http://www.e-physics.net/Download/12wave/12wave.htm

3. STANDING WAVES IN STRINGS FIXED AT BOTH ENDS

General mathematical form of a standing wave: $y(x,t) = A\sin(kx)\cos(\omega t)$

For standing waves in strings of length L fixed at two ends,
the boundary conditions are:

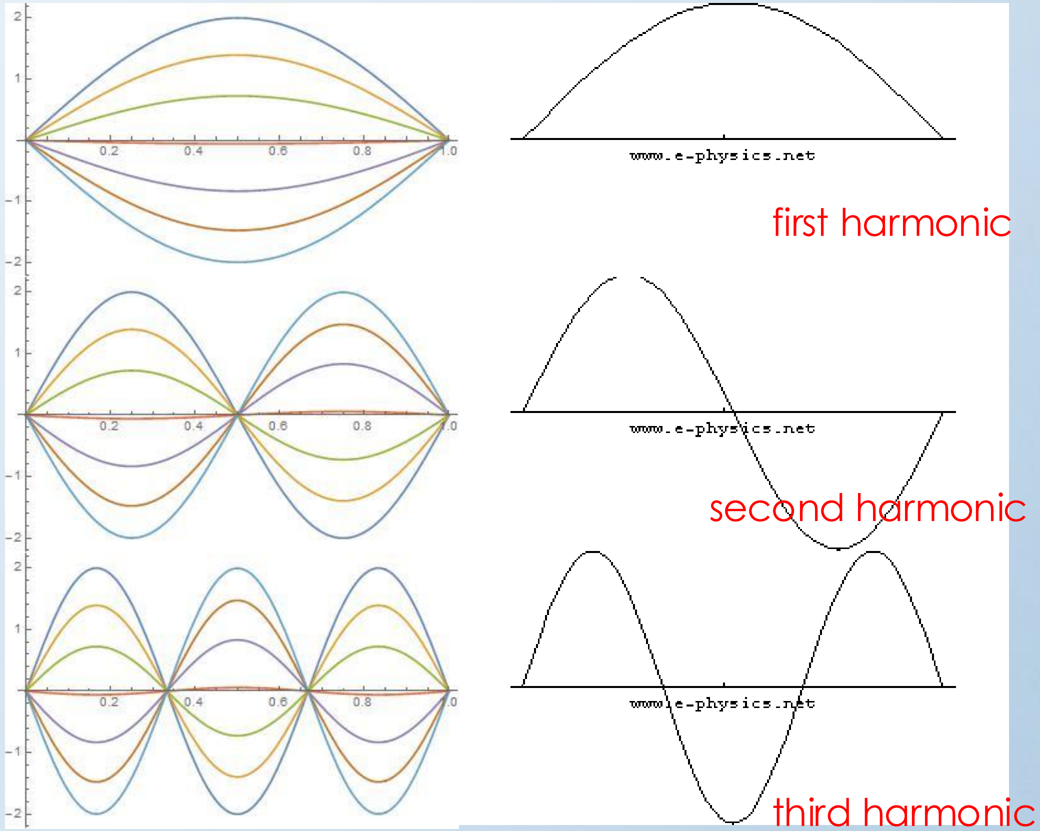
$$y(0,t) = A\sin(k \times 0)\cos(\omega t) = 0$$

$$y(L,t) = A\sin(k \times L)\cos(\omega t) = 0 \rightarrow kL = n\pi$$

$$k_1 = \frac{\pi}{L}, k_2 = \frac{2\pi}{L}, k_3 = \frac{3\pi}{L}, \ldots$$

$$\frac{2\pi}{\lambda}L = n\pi \rightarrow \lambda = \frac{2L}{n} \rightarrow \lambda_1 = 2L, \lambda_2 = L, \lambda_3 = \frac{2L}{3}, \ldots$$

$$f_1 = \frac{v}{2L}, f_2 = \frac{v}{L}, \ldots, f_n = \frac{v}{\frac{2L}{n}} = \frac{nv}{2L}$$



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3. STANDING WAVES IN STRINGS FIXED AT BOTH ENDS

Example: A string, tied to a sinusoidal vibrator at P and running over a support at Q, is stretched by a block of mass m. The separation between P and Q is 1.2 m, the linear density of the string is 1.6 g/m, and the frequency of the vibrator is fixed at 120 Hz. The amplitude of the motion at P is small enough for that point to be considered a node. A node also exists at Q. What mass m allows the vibrator to set up the fourth harmonic on the string?

$L = 1.2\, m, \mu = 1.6\frac{g}{m} = 0.0016\frac{kg}{m}, f = 120\, Hz$

fourth harmonic → $kL = 4\pi,\ \lambda = \frac{2L}{4} = \frac{L}{2} = 0.60\ \mathrm{m}$

$v = f\lambda = 120 \times 0.60 = 72\, m/s$

$T = v^2\mu = 8.3\, N = 0.85\, kgw \rightarrow m = 0.85\, kg$

3. STANDING WAVES IN STRINGS FIXED AT BOTH ENDS

Example: One end of a horizontal string is attached to a vibrating blade, and the other end passes over a pulley. A sphere of mass 2.00 kg hangs on the end of the string. The string is vibrating in its second harmonic. A container of water is raised under the sphere so that the sphere is completely submerged. After this is done, the string vibrates in its fifth harmonic. What is the radius of the sphere?

$$T_0 = 2.00 \times 9.8 = 19.6\,N$$

$$\lambda_0 = \frac{2L}{2} = L \rightarrow \frac{T}{\mu} = v^2 = f^2\lambda^2 \rightarrow T_0 = \mu f^2 (L)^2$$

$$\lambda' = \frac{2L}{5} \rightarrow T' = \mu f^2 \left(\frac{2}{5}L\right)^2$$

$$\frac{T'}{T_0} = \frac{4}{25} \rightarrow T' = 3.14\,N \quad T_0 - T' = \varrho V g = (998)(9.8)\frac{4}{3}\pi R^3$$

$$R = 0.0738\,m = 7.38\,cm$$

3. STANDING WAVES IN STRINGS FIXED AT BOTH ENDS

Example: A middle C string on a piano has a fundamental frequency of 262 Hz, and the A note has a fundamental frequency of 440 Hz. (a) Calculate the frequency of the next two harmonics of the C string. (b) If the strings for A and C notes are assumed to have the same mass per unit length and the same length, determine the ratio of tensions in the two string. (c) In a real piano, the assumption we made in part (b) is only partial true. The string densities are equal, but the A string is 64% as long as the C string. What is the ratio of their tensions?

(a) 1st harmonic of the C string: $\lambda_1 = \frac{2L}{1}, f_1 = \frac{v}{2L} = 262\ Hz$

2nd harmonic: $\lambda_2 = \frac{2L}{2}, f_2 = \frac{v}{L} = 2f_1 = 524\ Hz$

3rd harmonic: $\lambda_3 = \frac{2L}{3}, f_3 = \frac{v}{2L/3} = 3f_1 = 786\ Hz$

(b)$T = \mu v^2 = \mu f^2 \lambda^2, f_C = 262\ Hz, f_A = 440\ Hz, \lambda_C = \lambda_A = 2L$

$$\frac{T_A}{T_C} = \left(\frac{f_A}{f_C}\right)^2 = 2.82$$

(c)$\lambda_C = 2L, \lambda_A = 2(0.64L) \rightarrow \frac{T_A}{T_C} = \left(\frac{f_A \lambda_A}{f_C \lambda_C}\right)^2 = 2.82 \times (0.64)^2 = 1.16$

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4. STANDING WAVES IN AIR COLUMNS

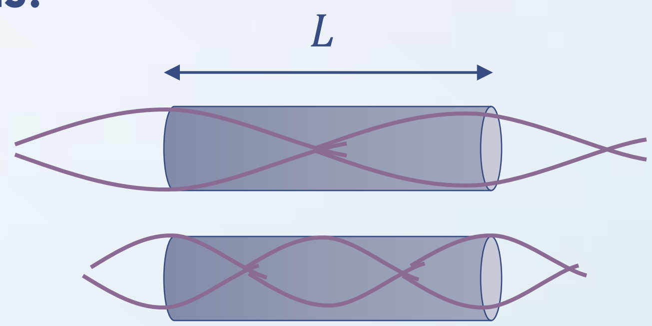
Pipes open at both ends:

$$n\frac{\lambda}{2} = L \rightarrow \lambda = \frac{2L}{n}$$

1st harmonic: $\lambda_1 = 2L$

2nd harmonic: $\lambda_2 = \frac{2L}{2} = L$

3rd harmonic: $\lambda_3 = \frac{2L}{3}$



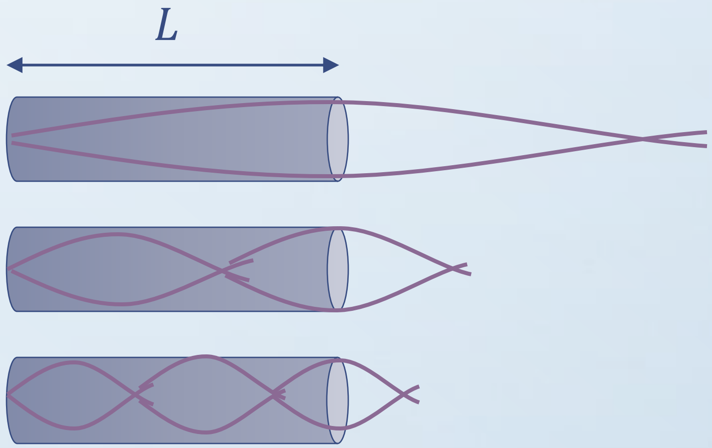
Pipes open at one end:

$$\frac{2n-1}{4}\lambda = L \rightarrow \lambda = \frac{4L}{2n-1}$$

1st harmonic: $\lambda_1 = 4L$

2nd harmonic: $\lambda_2 = \frac{4L}{3}$

3rd harmonic: $\lambda_3 = \frac{4L}{5}$



4. STANDING WAVES IN AIR COLUMNS

Example: A section of drainage culvert 1.23 m in length makes a howling noise when the wind blows across its open ends. (a) Determine the frequencies of the first three harmonics of the culvert if it is cylindrical in shape and open at both ends. Take v=343 m/s as the speed of sound in air. (b) What are the three lowest natural frequencies of the culvert if it is blocked at one end?

$L = 1.23\ m, v = 343\ m/s$

(a) $\lambda_n = \frac{2L}{n}, f_n = \frac{v}{\lambda_n} \rightarrow f_n = \frac{nv}{2L}$

1st harmonics: $f_1 = 139\ Hz$

2nd harmonics: $f_2 = 279\ Hz$

3rd harmonics: $f_3 = 418\ Hz$

(b) $\lambda_n = \frac{4L}{2n-1}, f_n = \frac{v}{\lambda_n} \rightarrow f_n = \frac{(2n-1)v}{4L}$

1st harmonics: $f_1 = 69.7\ Hz$

2nd harmonics: $f_2 = 209\ Hz$

3rd harmonics: $f_3 = 349\ Hz$

5. BEATS: INTERFERENCE IN TIME

Tempos generated by interference of waves in time, the wavelength and the frequency of the two waves have very small differences

$$\lambda_1 = \lambda_2 \rightarrow k_1 = k_2 = k, \qquad f_1 - f_2 = \Delta f \rightarrow \omega_1 - \omega_2 = 2\pi\Delta f$$

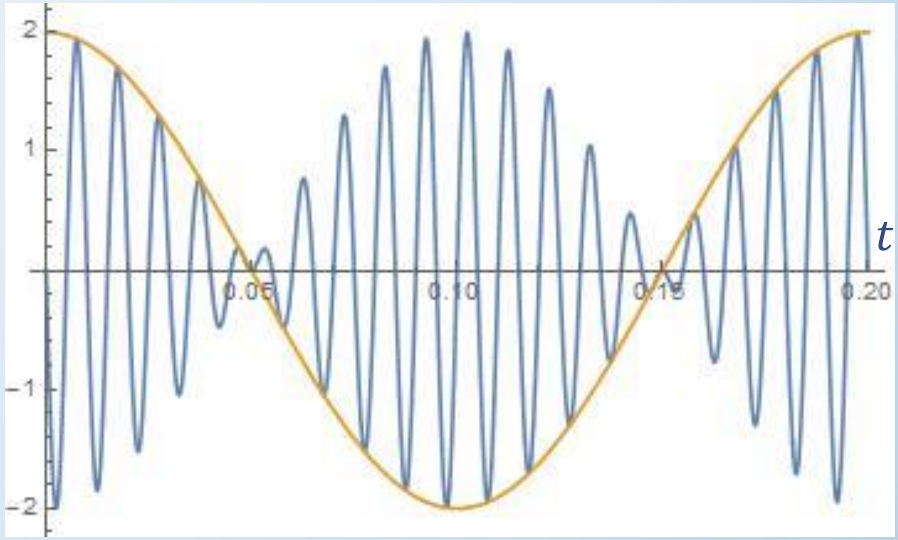
$$y_1(x,t) = A\sin(kx - \omega_1 t), y_2(x,t) = A\sin(kx - \omega_2 t)$$

$$y_t(x,t) = A\sin(kx - \omega_1 t) + A\sin(kx - \omega_2 t)$$

$$y_t(x,t) = 2A\sin\left(kx - \frac{\omega_1 + \omega_2}{2}t\right)\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)$$

$$A' = 2A\cos\left(\frac{\omega_1 - \omega_2}{2}t\right) = 2A\cos(\pi(\Delta f)t)$$

the frequency of beats is Δf rather than $\Delta f/2$



6. NON-SINUSOIDAL WAVE PATTERNS

Music of frequency, tone, tempo, … What about timbre? - the wave form

The mathematical form of a musical wave – standing wave

$$y(x,t) = A\sin(kx)\cos(\omega t)\,, y(0,t) = y(L.t) = 0$$

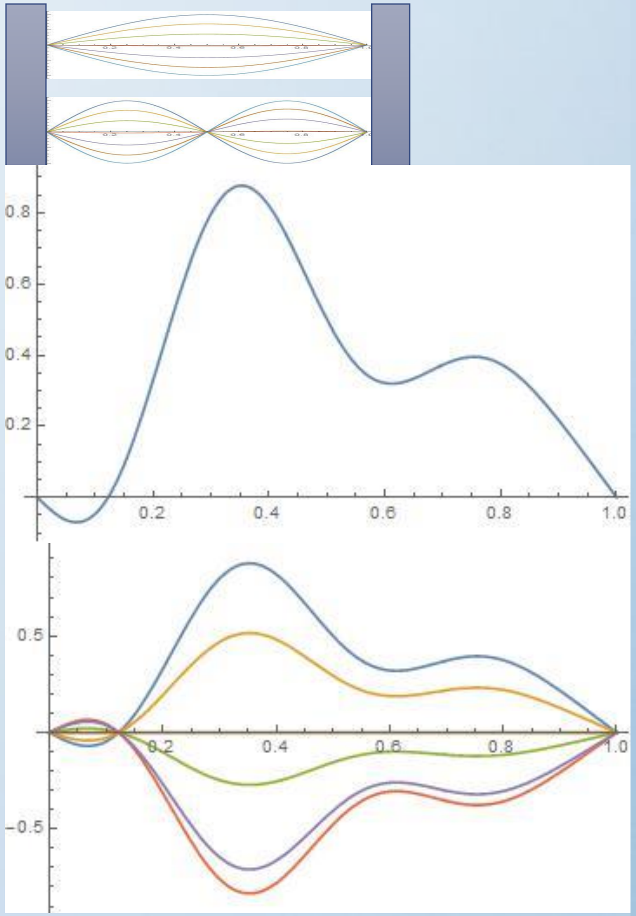
$$kL = n\pi \rightarrow k_1 = \frac{\pi}{L}, k_2 = \frac{2\pi}{L}, k_3 = \frac{3\pi}{L}, \ldots$$

$$\mathrm{y}_1(x,t) = A_1 \sin\left(\frac{\pi x}{L}\right)\cos(\omega t) \qquad \mathrm{y}_2(x,t) = A_2 \sin\left(\frac{2\pi x}{L}\right)\cos(\omega t)$$

$$\mathrm{y}_3(x,t) = A_3 \sin\left(\frac{3\pi x}{L}\right)\cos(\omega t)\,, \ldots$$

The timbre is dependent on wave form. Let's generate a new wave form:

$$y(x,t) = \left(0.6\sin\left(\frac{\pi x}{L}\right) + 0.1\sin\left(\frac{2\pi x}{L}\right) - 0.2\sin\left(\frac{4\pi x}{L}\right) - 0.1\sin\left(\frac{5\pi x}{L}\right)\right)\cos(\omega t)$$

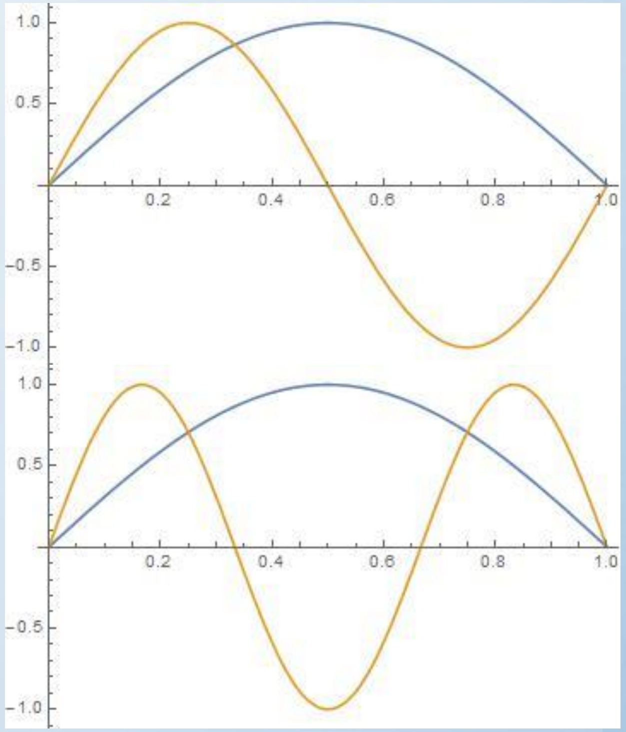


6. NON-SINUSOIDAL WAVE PATTERNS

Separation of variables: $y(x,t) = f(x)\cos(\omega t)$, wave form: $f(x)$

The wave form is the superposition of all harmonic waves – a Fourier series expansion.

$$f(x) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{L}\right)$$



Orthogonal properties of the harmonics in the range of $0 < x < L$

$$\int_0^L \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)dx = \frac{1}{2}\int_0^L \left[\cos\left(\frac{(n-m)\pi x}{L}\right) - \cos\left(\frac{(n+m)\pi x}{L}\right)\right]dx$$

$$= \frac{1}{2}\left[\frac{L}{(n-m)\pi}\sin\left(\frac{(n-m)\pi x}{L}\right) - \frac{L}{(n+m)\pi}\sin\left(\frac{(n+m)\pi x}{L}\right)\right]_{x=0}^{x=L} = 0$$

$$\int_0^L \sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)dx = \frac{L}{2}$$

6. NON-SINUSOIDAL WAVE PATTERNS

Example: The wave form $f(x) = 1, 0 \le x \le L$ is composed be a harmonic series of sinusoidal waves $A_n \sin\left(\frac{n\pi x}{L}\right)$, please evaluate the amplitudes A_n.

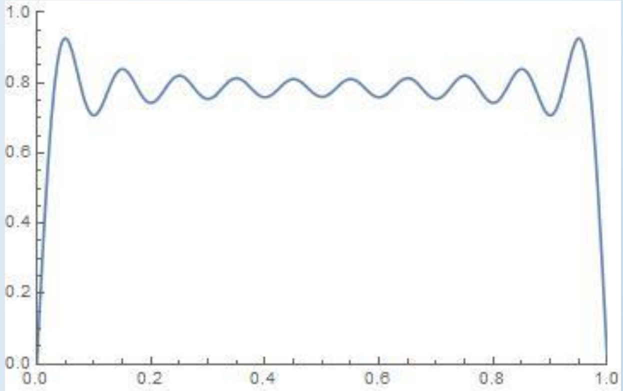
$$1 = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{L}\right), 0 \le x \le L$$

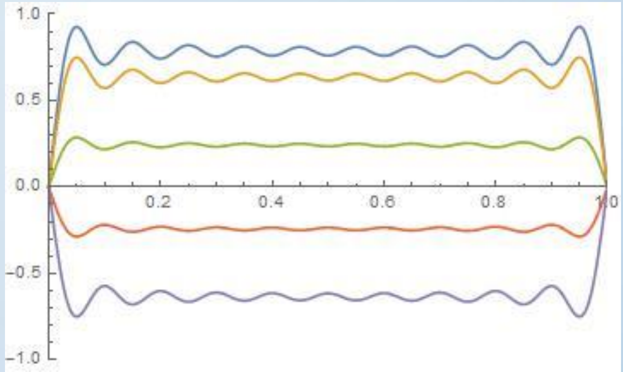
Use the orthogonal properties:

$$\int_0^L 1 \times \sin\left(\frac{m\pi x}{L}\right) dx = \sum_{n=1}^{\infty} A_n \int_0^L \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi x}{L}\right) dx$$

$$\frac{L}{m\pi}\left[-\cos\left(\frac{m\pi x}{L}\right)\right]_0^L = A_m \frac{L}{2} \rightarrow A_m = \frac{2}{\pi}\frac{(1-(-1)^m)}{m}$$

$$f(x) = 1 = \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{(1-(-1)^n)}{n} \sin\left(\frac{n\pi x}{L}\right), 0 \le x \le L$$





EXERCISE

One period of the square wave is described by $y(t)=\begin{cases} A, 0<t<\frac{T}{2} \\ -A, \frac{T}{2}<t<T \end{cases}$. Find out the amplitudes of the Fourier series transformation of the form $y(t)=\sum_{n=1}^{\infty}(A_n\sin(n\omega t)+B_n\cos(n\omega t))$. Show that the Fourier series expansion is $y(t)=\sum_{n=1}^{\infty}\frac{4A}{n\pi}\sin(n\omega t)$.

Check that the period is T, the angular frequency is $\omega=\frac{2\pi}{T}$.

$$\sum_{n=1}^{\infty}(A_n\sin(n\omega t)+B_n\cos(n\omega t))=\begin{cases} A, 0<t<\frac{T}{2} \\ -A, \frac{T}{2}<t<T \end{cases}$$

$$n\neq m\rightarrow\int_0^T\sin(n\omega t)\sin(m\omega t)\,dt=\int_0^T\cos(n\omega t)\sin(m\omega t)\,dt$$

$$=\int_0^T\cos(n\omega t)\cos(m\omega t)\,dt=0$$

EXERCISE

One period of the square wave is described by $y(t) = \begin{cases} A, 0 < t < \frac{T}{2} \\ -A, \frac{T}{2} < t < T \end{cases}$. Find out the amplitudes of the Fourier series transformation of the form $y(t) = \sum_{n=1}^{\infty}(A_n \sin(n\omega t) + B_n \cos(n\omega t))$. Show that the Fourier series expansion is $y(t) = \sum_{n=1}^{\infty} \frac{4A}{n\pi} \sin(n\omega t)$.

$$\int_0^T \sin^2(n\omega t) dt = \int_0^T \cos^2(n\omega t) dt = \frac{T}{2}$$

$$\int_0^T \sum_{n=1}^{\infty} (A_n \sin(n\omega t) + B_n \cos(n\omega t)) \times \sin(m\omega t) = \int_0^T \begin{cases} A, 0 < t < \frac{T}{2} \\ -A, \frac{T}{2} < t < T \end{cases} \times \sin(m\omega t)$$

$$\int_0^T A_m \sin^2(m\omega t)\, dt = \int_0^{\frac{T}{2}} A \sin(m\omega t)\, dt + \int_{\frac{T}{2}}^T -A \sin(m\omega t)\, dt$$

EXERCISE

One period of the square wave is described by $y(t) = \begin{cases} A, 0 < t < \frac{T}{2} \\ -A, \frac{T}{2} < t < T \end{cases}$. Find out the amplitudes of the Fourier series transformation of the form $y(t) = \sum_{n=1}^{\infty}(A_n \sin(n\omega t) + B_n \cos(n\omega t))$. Show that the Fourier series expansion is $y(t) = \sum_{n=1}^{\infty} \frac{4A}{n\pi} \sin(n\omega t)$.

$$A_m \frac{T}{2} = \frac{A}{m\omega}\left[-\cos\left(m\frac{2\pi}{T}t\right)\right]_{t=0}^{t=T/2} + \frac{A}{m\omega}\left[\cos\left(m\frac{2\pi}{T}t\right)\right]_{t=T/2}^{t=T}$$

$$A_m \frac{T}{2} = \frac{A}{m\omega}(1 - \cos(m\pi)) + \frac{A}{m\omega}(\cos(2\pi m) - \cos(m\pi)) \qquad A_m = 4\frac{A}{mT\omega}(1 - \cos(m\pi))$$

$$A_m = \frac{2A}{m\pi}(1 - (-1)^m) \rightarrow m = 2n + 1 \rightarrow A_n = \frac{4A}{(2n+1)\pi}$$

$$y(t) = \sum_{n=0}^{\infty} \frac{4A}{(2n+1)\pi} \sin((2n+1)\omega t) = \sum_{n=1,3,5,\dots}^{\infty} \frac{4A}{n\pi} \sin(n\omega t)$$

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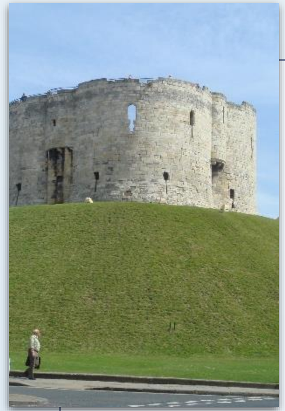

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