Chapter 9
Systems of particles and conservation of linear momentum-I

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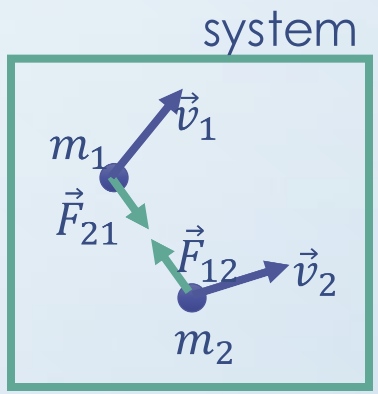
CONTENTS

1. LINEAR MOMENTUM AND ITS CONSERVATION

Start from Newton's 3rd Law, Action and Reaction Force

If there are no external force acting on the system,

$$\sum \vec{F}_{i,external} = \vec{F}_{net,external} = 0$$



$$\vec{F}_{12} = -\vec{F}_{21} \Rightarrow \vec{F}_{21} + \vec{F}_{12} = 0$$

$$m_1\vec{a}_1 + m_2\vec{a}_2 = 0 \Rightarrow m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} = 0$$

$$\frac{d}{dt}[m_1\vec{v}_1 + m_2\vec{v}_2] = 0$$

Note that the variation of the vector sum of $m_i\vec{v}_i$ is zero. Its shows the importance of mass and velocity.

Here we define the linear momentum: $\vec{p} = m\vec{v}$

$$\frac{d}{dt}[\vec{p}_1 + \vec{p}_2] = 0$$

1. LINEAR MOMENTUM AND ITS CONSERVATION

Characteristic of Linear Momentum

1. Separated to components

$$\vec{p} = p_x\hat{\imath} + p_y\hat{\jmath} + p_z\hat{k} = m\vec{v} = mv_x\hat{\imath} + mv_y\hat{\jmath} + mv_z\hat{k}$$

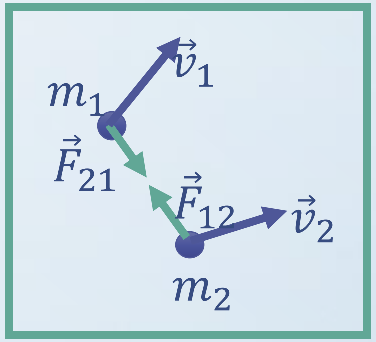
2. Related to the external force

$$\sum \vec{F}_i = \vec{F}_{net} = m\vec{a} = m\frac{d\vec{v}}{dt}$$ if the mass m is constant

$$\sum \vec{F}_i = \frac{dm\vec{v}}{dt} = \frac{d\vec{p}}{dt}$$

In the following studies, we will find that the change of mass, even without any changes of velocity, needs external forces.

1. LINEAR MOMENTUM AND ITS CONSERVATION



If the system of two particles is isolated (without external forces)

$$\vec{F}_{ext,net} = 0 = \frac{d}{dt}(\vec{p}_1 + \vec{p}_2) = \frac{d}{dt}\vec{p}_{total}$$

$$\Delta\vec{p}_{total} = 0 \rightarrow \vec{p}_{total} = constant\_vector$$

$$\vec{p}_{1,i} + \vec{p}_{2,i} = \vec{p}_{1,f} + \vec{p}_{2,f}$$

Separated to components:

$$p_{1xi} + p_{2xi} = p_{1xf} + p_{2xf}$$

$$p_{1yi} + p_{2yi} = p_{1yf} + p_{2yf}$$

$$p_{1zi} + p_{2zi} = p_{1zf} + p_{2zf}$$

It can also be used when one component of the force is zero, for example, $F_{x,ext} = 0 \rightarrow p_{1xi} + p_{2xi} = p_{1xf} + p_{2xf}$

1. LINEAR MOMENTUM AND ITS CONSERVATION

Example: A man of mass 60.0 kg stands on frictionless ice surface and he throws a baseball of 150 g horizontally at a speed of 150 km/h. What is his speed after throwing the baseball.

$$p_{man,i} + p_{ball,i} = p_{man,f} + p_{ball,f}$$

$$0 + 0 = 60v + 0.15 \times 150/3.6$$

$$v = -0.104(m/s)$$

https://giphy.com/gifs/justin-g-snow-cold-sad-john-xTcnTehwgRcbgymhTW

1. LINEAR MOMENTUM AND ITS CONSERVATION

Example: A man of mass 75 kg is jumping up against the ground at a speed of 4.85 m/s, where the mass of the Earth is 5.97X10^{24} kg. What's the ratio of linear momentum between the Earth and the man when the man is jumping up? What's the ratio of kinetic energy between the Earth and the man?

$$p_{man,i} + p_{Earth,i} = p_{man,f} + p_{Earth,f}$$

$$0 + 0 = 75 \times 4.85 + 5.97 \times 10^{24} \times v \qquad \frac{|p_{Earth,f}|}{|p_{man,f}|} = 1$$

$$v = 6.09 \times 10^{-23} \text{ (m/s)}$$

$$\frac{E_{Earth,f}}{E_{man,f}} = \frac{5.97 \times 10^{24} \times (6.09 \times 10^{-23})^2}{75 \times (4.85)^2} = 1.26 \times 10^{-23} = \frac{m_{man}}{m_{Earth}}$$

2. IMPULSE

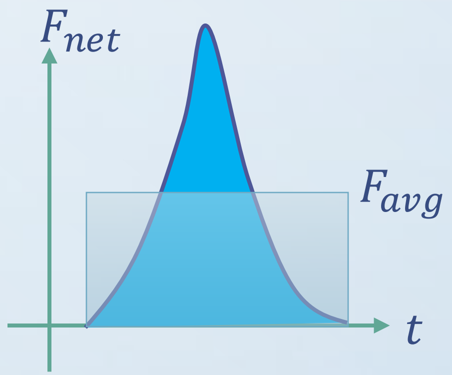
The short time action of forces:

$$\sum \vec{F}_i = \frac{d\vec{p}}{dt} \Rightarrow d\vec{p} = \left(\sum \vec{F}_i\right) dt \quad \text{or} \quad \Delta\vec{p} = \vec{F}_{net,ext}\Delta t$$

We define the short time action of forces as an impulse

$$\vec{I} = \left(\sum \vec{F}_i\right)\Delta t = \int_{t_i}^{t_i+\Delta t}\left(\sum \vec{F}_i\right) dt$$

$$\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i \qquad \vec{F}_{net,avg} = \frac{\vec{I}}{\Delta t}$$



2. IMPULSE

Example: In a crash test, an automobile of mass 1500 kg collides with a wall. The initial and final velocities of the automobile are $v_i = -15$ m/s and $v_f = 2.6$ m/s. If the collision lasts for 0.15 s, find the impulse due to the collision and the average force excerted on the automobile.

$$I = p_f - p_i = 1500 \times 2.6 - 1500 \times (-15) = 2.64 \times 10^4 \text{ (kg m/s)}$$

$$F_{avg} = \frac{2.64 \times 10^4}{0.15} = 1.76 \times 10^5 (N)$$

https://www.nytimes.com/2014/01/22/automobiles/minicars-are-worst-performers-in-small-overlap-front-crash-test.html

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3. COLLISION IN ONE DIMENSION

A collision is an isolated event in which two or more objects (the colliding objects) exert relatively strong forces on each other for a short time.

The collision do not need a real touch since the force is acting at a distance.

Rules:

1. The linear momentum must be conserved no matter that the collision is either elastic or inelastic.
2. For elastic collisions, the total kinetic energy must be conserved. For inelastic collisions, the total kinetic energy of the system is not conserved.

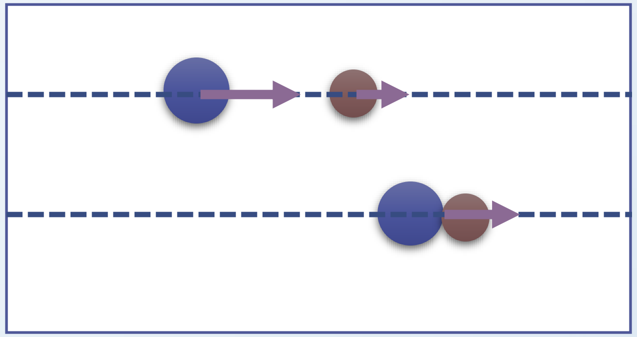
3. COLLISION IN ONE DIMENSION

Perfectly Inelastic Collision:

$$p_{1xi} + p_{2xi} = p_{1xf} + p_{2xf}$$

$$m_1 v_1 + m_2 v_2 = m_1 v' + m_2 v'$$

$$v' = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$



https://en.wikipedia.org/wiki/Inelastic_collision

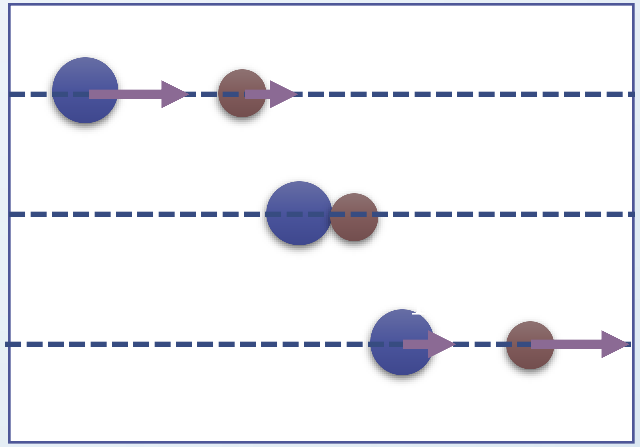
3. COLLISION IN ONE DIMENSION

Elastic Collision:

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$m_1 (v_1 - v_1') = m_2 (v_2' - v_2)$$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 {v_1'}^2 + \frac{1}{2} m_2 {v_2'}^2$$



$$m_1 (v_1 + v_1')(v_1 - v_1') = m_2 (v_2' + v_2)(v_2' - v_2)$$

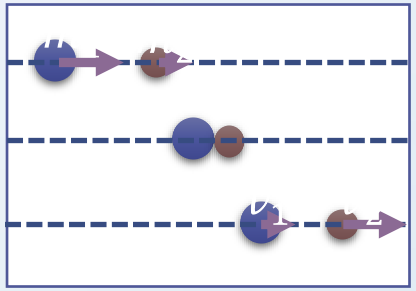
$$v_1 + v_1' = v_2 + v_2'$$

$$m_1 v_1' + m_2 v_2' = m_1 v_1 + m_2 v_2 \qquad (1)$$

$$v_1' - v_2' = v_2 - v_1 \qquad (2)$$

3. COLLISION IN ONE DIMENSION

Elastic Collision:



$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$

$$v_2' = \frac{m_2 - m_1}{m_1 + m_2} v_2 + \frac{2m_1}{m_1 + m_2} v_1$$

Special Case for Elastic Collision - $v_2 = 0$

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1$$

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1$$

3. COLLISION IN ONE DIMENSION

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 \qquad v_2' = \frac{2m_1}{m_1 + m_2} v_1$$

Special Case for Elastic Collision - $v_2 = 0$, Equal Mass: $m_1 = m_2$

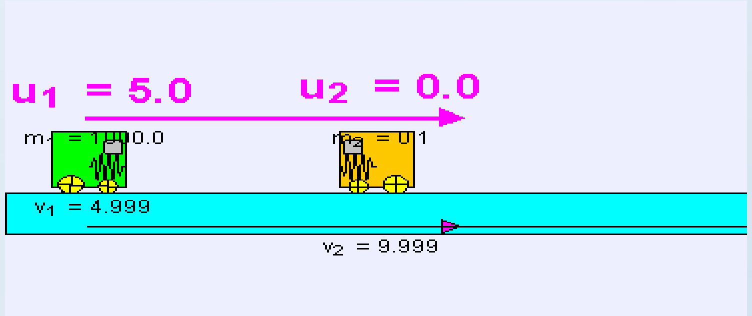
$v_1' = 0 \qquad v_2' = v_1$

Special Case for Elastic Collision - $v_2 = 0$, Massive Target: $m_1 \ll m_2$

$$v_1' = -v_1 \qquad v_2' = \frac{2m_1}{m_2} v_1$$

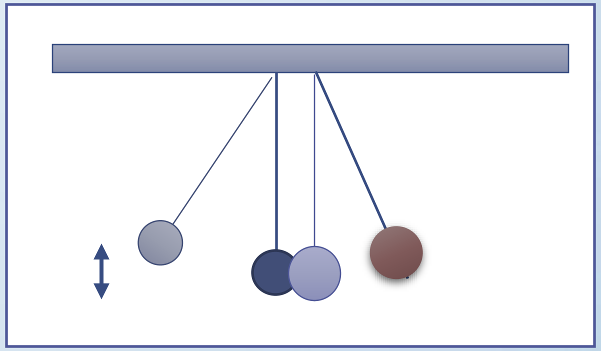
Special Case for Elastic Collision - $v_2 = 0$, Massive Projectile: $m_1 \gg m_2$

$v_1' = v_1 \qquad v_2' = 2v_1$



3. COLLISION IN ONE DIMENSION

Example: Two metal spheres, suspended by vertical cords, initially just touch. Sphere 1, with mass m_1, is pulled to the left to height h_1, and then released from rest. After swinging down, it undergoes an elastic collision with Sphere 2 of mass m_2. What is the velocity v_1' of Sphere 1 just after the collision?



Energy transfer for m_1:

$$m_1 g h_1 = \frac{m_1 v_1^2}{2} \qquad v_1 = \sqrt{2gh_1}$$

Collision at bottom: $v_1 = \sqrt{2gh_1}$, $v_2 = 0$

$$m_1 v_1' + m_2 v_2' = m_1 v_1$$

$$v_1' - v_2' = 0 - v_1$$

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 = \frac{m_1 - m_2}{m_1 + m_2} \sqrt{2gh_1}$$

3. COLLISION IN ONE DIMENSION

Example: A block of mass m_1=1.60 kg initially moving to the right with a speed of 4.00 m/s on a frictionless horizontal track collides with a spring attached to a second block of mass m_2=2.10 kg initially moving to the left with a speed of 2.50 m/s. The spring constant is 600 N/m. (a) Find the velocities of the two blocks after the collision. (b) During the collision, at the instant Block 1 is moving to the right with a velocity of +3.00 m/s, determine the velocity of Block 2. (c) Determine the distance the spring is compressed at that instant.

(a) $m_1 = 1.60,\ v_1 = 4.00,\ m_2 = 2.10,\ v_2 = -2.50$

$1.60v_1' + 2.10v_2' = 1.60 \times 4.00 + 2.10 \times (-2.50) = 1.15$

$v_1' - v_2' = (-2.50) - 4.00 = -6.50$

$v_1' = -3.38,\ v_2' = 3.12$

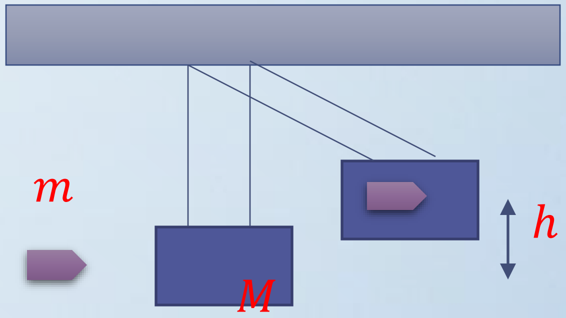
(b) $1.60 \times 3.00 + 2.10v_2' = 1.15$ ⇒ $v_2' = -1.74$

(c) $\frac{600x^2}{2} = \frac{1.60(4.00)^2}{2} + \frac{2.10(-2.50)^2}{2} - \frac{1.60(3.00)^2}{2} - \frac{2.10(-1.74)^2}{2}$

$x = 0.173$ (m)

3. COLLISION IN ONE DIMENSION

Example: The ballistic pendulum is used to measure the speed of a bullet. A bullet of mass m is fired into a large block of wood of mass M suspended from some light wires. The bullet imbeds in the block, and the entire system swings through a height h. How can we determine the speed of the bullet from a measurement of h?



The speed after collision:

$$\frac{(m+M)}{2}v'^2 = (m+M)gh \Rightarrow v' = \sqrt{2gh}$$

The collision:

$$mv_1 + 0 = (m+M)\sqrt{2gh}$$

$$v_1 = \frac{(m+M)}{m}\sqrt{2gh}$$

4. COLLISION IN TWO DIMENSIONS

Conservation of Linear Momentum $\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2'$

Expressed in Components $p_{1x} + p_{2x} = p_{1x}' + p_{2x}'$ $\quad p_{1y} + p_{2y} = p_{1y}' + p_{2y}'$

Conservation of Energy $K_1 + K_2 = K_1' + K_2'$

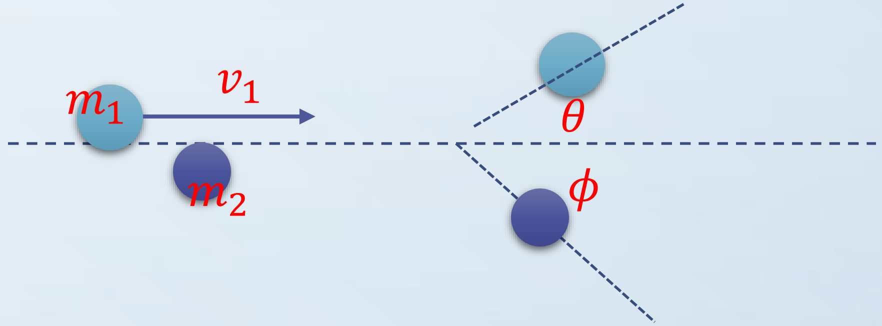
$$\frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} = \frac{{p_1'}^2}{2m_1} + \frac{{p_2'}^2}{2m_2} \qquad \frac{p_{1x}^2 + p_{1y}^2}{2m_1} + \frac{p_{2x}^2 + p_{2y}^2}{2m_2} = \frac{{p_{1x}'}^2 + {p_{1y}'}^2}{2m_1} + \frac{{p_{2x}'}^2 + {p_{2y}'}^2}{2m_2}$$

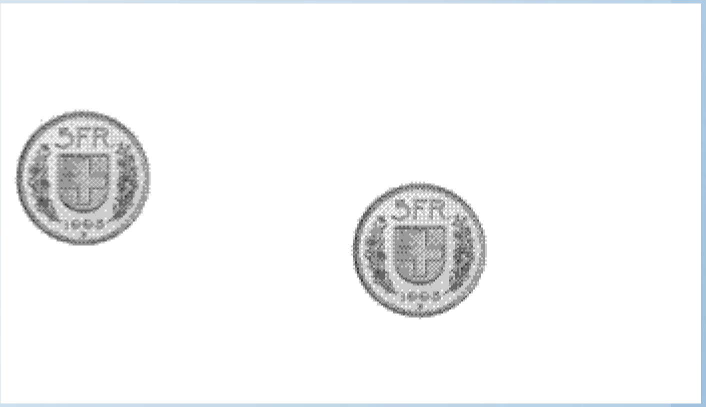
Simplified Example:

$$m_1 v_1 = m_1 v_1' \cos\theta + m_2 v_2' \cos\phi$$

$$m_1 v_1' \sin\theta = m_2 v_2' \sin\phi$$

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 {v_1'}^2 + \frac{1}{2} m_2 {v_2'}^2$$





https://en.wikipedia.org/wiki/Elastic_collision#/media/File:Elastischer_sto%C3%9F_2D.gif

4. COLLISION IN TWO DIMENSIONS

Example: A proton collides elastically with another proton that is initially at rest. The incoming proton has an initial speed of 3.5×10^5 m/s and makes a glancing collision with the second proton. After collision, one proton moves off at an angle of 37^o to the original direction of motion, and the second deflects at an angle of ϕ to the same axis. Find the final speeds of the two protons and the angle ϕ.

$$mv = mv_1' \cos 37^0 + mv_2' \cos \phi$$

$$mv_1' \sin 37^0 = mv_2' \sin \phi$$

$$\Rightarrow$$

$$v_2' \cos \phi = v - v_1' \cos 37^0$$

$$v_2' \sin \phi = v_1' \sin 37^0$$

$$v_2'^2 = v^2 - 2vv_1' \cos 37^0 + v_1'^2$$

$$mv^2 = mv_1'^2 + mv_2'^2 \Rightarrow v^2 - v_1'^2 = v_2'^2$$

$$v^2 - v_1'^2 = v^2 - 2vv_1' \cos 37^0 + v_1'^2 \Rightarrow 2v_1'(v_1' - v \cos 37^0) = 0$$

$$v_1' = v \cos 37^0 = 3.5 \times 10^5 \times \cos 37^0 = 2.80 \times 10^5 (\text{m/s})$$

$$v_2' = \sqrt{v^2 - v_1'^2} = v \sin 37^0 \qquad \sin \phi = \cos 37^0 = \sin 53^0$$

CONTENTS

5. THE CENTER OF MASS

The COM of Two Particles, m_1, x_1, m_2, x_2

$$x_{COM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

The COM of More Than Two Particles, $m_1, x_1, m_2, x_2, \ldots$

$$x_{COM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{1}{M}\sum_{i=1}^{N} m_i x_i$$

The COM of Many Particles in Three Dimensions

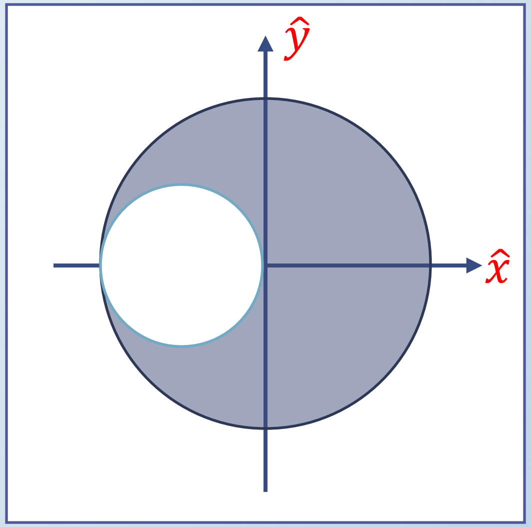
$$\vec{r}_{COM} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{1}{M}\sum_{i=1}^{N} m_i \vec{r}_i$$

5. THE CENTER OF MASS

Example: In the figure, it shows a uniform metal plate P of radius 2R from which a disk of radius R has been stamped out in an assembly line. Using the xy coordinate system shown to locate the center of mass of the plate.

Assume that the mass of the circular disk of radius R is m and the big one has mass 4m.

$$x_{COM} = \frac{4m \times 0 + (-m) \times (-R)}{4m + (-m)} = \frac{R}{3}$$



5. THE CENTER OF MASS

The COM of a Body of Continuous Mass Distribution

$$\vec{r}_{COM} = \frac{1}{M}\int \vec{r}dM \qquad M = \rho V,\ dM = \rho dV \qquad \vec{r}_{COM} = \frac{1}{V}\int \vec{r}dV$$

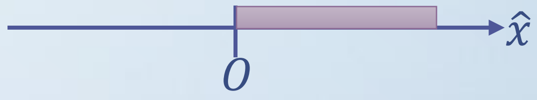
Expressed in x, y, z Components:

$$x_{COM} = \frac{1}{M}\int xdM \qquad y_{COM} = \frac{1}{M}\int ydM \qquad z_{COM} = \frac{1}{M}\int zdM$$

$$x_{COM} = \frac{1}{V}\int xdV \qquad y_{COM} = \frac{1}{V}\int ydV \qquad z_{COM} = \frac{1}{V}\int zdV$$

5. THE CENTER OF MASS

Example: Show that the center of mass of a rod of mass M and length L lies midway between its ends, assuming the rod has a uniform mass per unit length.



$\lambda = M/L$

$$x_{COM} = \frac{1}{M}\int_0^L x\lambda dx = \frac{L}{2}$$

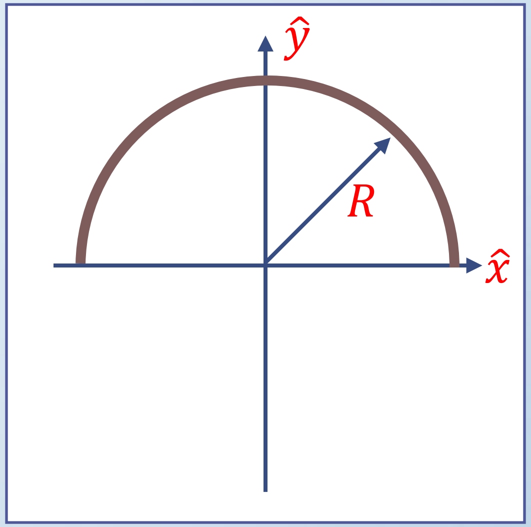
Example: Suppose the rod is non-uniform with its mass density varies as $\lambda = \alpha x$. Find the center of mass.

$$x_{COM} = \frac{\int_0^L x\alpha x dx}{\int_0^L \alpha x dx} = \frac{\alpha L^3/3}{\alpha L^2/2} = \frac{2}{3}L$$

5. THE CENTER OF MASS

Example: Please find out the center of mass of a semicircular hoop of radius R. The hoop has a uniform density of λ.

$$\vec{r}_{COM} = \frac{\int_0^{\pi} (R\cos\theta\,\hat{\imath} + R\sin\theta\,\hat{\jmath})\lambda R d\theta}{\int_0^{\pi} \lambda R d\theta} = \frac{2R}{\pi}\hat{\jmath}$$



6. MOTION OF A SYSTEM OF PARTICLES

The Motion of Center of Mass

$$\vec{r}_{COM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{r}_i$$

$$M\vec{r}_{COM} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots = \sum_{i=1}^{N} m_i\vec{r}_i$$

$$M\left(\frac{d}{dt}\right)\vec{r}_{COM} = m_1\left(\frac{d}{dt}\right)\vec{r}_1 + m_2\left(\frac{d}{dt}\right)\vec{r}_2 + m_3\left(\frac{d}{dt}\right)\vec{r}_3 + \cdots \quad \boldsymbol{M\vec{v}_{COM} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots}$$

$$M\left(\frac{d}{dt}\right)\vec{v}_{COM} = m_1\left(\frac{d}{dt}\right)\vec{v}_1 + m_2\left(\frac{d}{dt}\right)\vec{v}_2 + m_3\left(\frac{d}{dt}\right)\vec{v}_3 + \cdots \quad \boldsymbol{M\vec{a}_{COM} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots}$$

$$M\vec{a}_{COM} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = \sum \vec{F}_i = \vec{F}_{net,external}$$

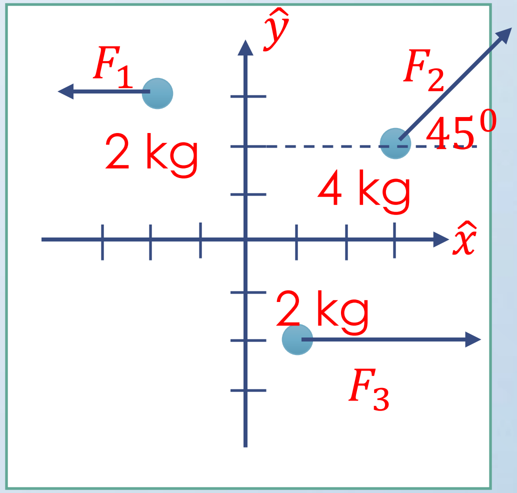
6. MOTION OF A SYSTEM OF PARTICLES

Example: Three particles are initially at rest. Each experiences an external force due to bodies outside the three-particle system. The directions are indicated, and the magnitudes are $F_1 = 6.0$ N, $F_2 = 12$ N, and $F_3 = 14$ N. What is the acceleration of the center of mass of the system, and in what direction does it move?

$\vec{F}_1 = -6\hat{\imath},\ \vec{F}_2 = 6\sqrt{2}\hat{\imath} + 6\sqrt{2}\hat{\jmath},\ \vec{F}_3 = 14\hat{\imath}$

$(2+4+2)\vec{a}_{COM} = \vec{F}_{net} = \left(8 + 6\sqrt{2}\right)\hat{\imath} + 6\sqrt{2}\hat{\jmath}$

$\vec{a}_{COM} = \left(1 + \frac{3\sqrt{2}}{4}\right)\hat{\imath} + \frac{3\sqrt{2}}{4}\hat{\jmath}$



6. MOTION OF A SYSTEM OF PARTICLES

Kinetic Energy of a System $K = \sum K_i = \sum \frac{m_i v_i^2}{2}$

Change to relative velocity with reference to the COM

$\vec{v}_i = \vec{v}_{COM} + \vec{u}_i \qquad M\vec{v}_{COM} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots$

$$M\vec{v}_{COM} = m_1(\vec{v}_{COM} + \vec{u}_1) + m_2(\vec{v}_{COM} + \vec{u}_2) + m_3(\vec{v}_{COM} + \vec{u}_3) + \cdots$$

$$M\vec{v}_{COM} = (m_1 + m_2 + m_3 + \cdots)\vec{v}_{COM} + (m_1\vec{u}_1 + m_2\vec{u}_2 + m_3\vec{u}_3 + \cdots)$$

$$m_1\vec{u}_1 + m_2\vec{u}_2 + m_3\vec{u}_3 + \cdots = 0$$

$$K = \sum K_i = \sum \frac{m_i}{2}(\vec{v}_{COM} + \vec{u}_i)\cdot(\vec{v}_{COM} + \vec{u}_i) = \frac{1}{2}Mv_{COM}^2 + \frac{1}{2}\sum m_i u_i^2$$

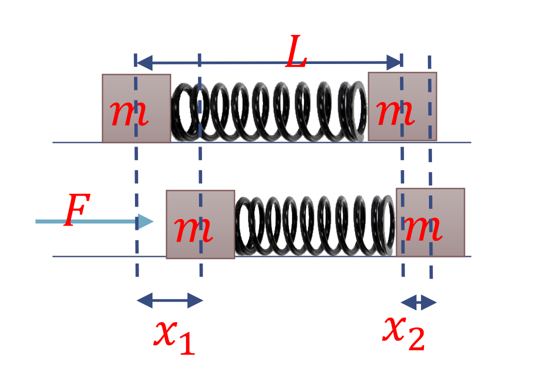
The Reference Frame at Center of Mass

If the external force is zero, the velocity of the center of mass is constant.

$M\frac{d\vec{v}_{COM}}{dt} = \vec{F}_{net,external} = 0 \Rightarrow \qquad \vec{v}_{COM} = const\ vector$

7. DEFORMABLE SYSTEMS

Example: As shown in the figure, two blocks are at rest on a frictionless table. Both blocks have the same mass m, and they are connected by a spring of negligible mass. The separation distance of the blocks when the spring is relaxed is L. During a time interval Δt, a constant force of magnitude F is applied horizontally to the left block, moving it through a distance x_1. During the time interval, the right block moves through a distance x_2. At the end of this time interval, the force F is removed. (a) Find the resulting speed v_{com} of the center of mass of the system.



Momentum: $F\Delta t = 2mv_{COM,final}$

Displacement: $v_{COM,avg}\Delta t = \frac{x_1 + x_2}{2}$

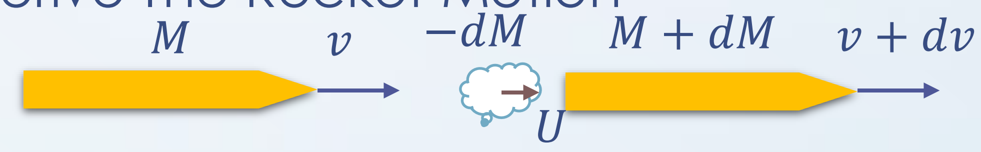
Constant Acceleration Motion: $v_{COM,avg} = \frac{v_{COM,final}}{2}$

$$F\frac{x_1 + x_2}{v_{COM,final}} = 2mv_{COM,final} \qquad v_{COM,final} = \sqrt{\frac{F(x_1 + x_2)}{2m}}$$

8. ROCKET PROPULSION

Use Collision to Derive The Rocket Motion

$F_{ext,horizontal} = 0$



$dM < 0$, for a correct variation of mass to give correct differential equation

$Mv = -UdM + (M + dM)(v + dv) \qquad Mv = -UdM + Mv + Mdv + vdM + dvdM$

$0 = (v + dv - U)dM + Mdv$

Let the speed of the gas expelled by the rocket engine be v_{rel}

$v + dv - U = v_{rel} \qquad -v_{rel}dM = Mdv$

(a) $M\dfrac{dv}{dt} = -v_{rel}\dfrac{dM}{dt}$ let $R = -dM/dt$ $M\dfrac{dv}{dt} = Rv_{rel}$

(b) $dv = -v_{rel}\dfrac{dM}{M} \quad \int_{v_0}^{v} dv = -\int_{M_0}^{M} v_{rel}\dfrac{dM}{M} \qquad v = v_0 + v_{rel}\ln\left(\dfrac{M_0}{M}\right)$

8. ROCKET PROPULSION

Example: A rocket whose initial mass M_i is 850 kg consumes fuel at the rate R = 2.3 kg/s. The speed v_{rel} of the exhaust gases relative to the rocket engine is 2800 m/s. (a) What thrust does the rocket engine provide? (b) What is the initial acceleration of the rocket? (c) If the rocket is launched from a spacecraft where gravitation is negligible small. What is its speed relative to the spacecraft when the mass of the rocket is 180 kg?

(a) Thrust: $v_{rel}R = 2800 \times 2.3 = 6440$ N

(b) $$a = \frac{v_{rel}R}{M_i} = \frac{6440}{850} = 7.6(m/s^2)$$

(c) $$v = v_0 + v_{rel}\ln\left(\frac{M_0}{M}\right)$$

$$v = 0 + 2800 \times \ln\left(\frac{850}{180}\right) = 4300(m/s)$$

https://giphy.com/gifs/producthunt-rocket-launch-26xBEamXwaMSUbV72

EXERCISE

A bullet of mass m moving with an initial speed v_i is fired into and passes through a block of mass M. The block initially at rest on a frictionless, horizontal surface, is connected to a spring with a force constant k. The block moves d to the right after impact before brought to rest by the spring. Find (a) the speed at which the bullet emerges from the block.

$$\frac{1}{2}Mv^2 = \frac{1}{2}kd^2$$

$$v = \sqrt{\frac{k}{M}}d$$

$$mv_i = Mv + mv_f$$

$$v_f = v_i - \frac{M}{m}v = v_i - \frac{M}{m}\sqrt{\frac{k}{M}}d$$

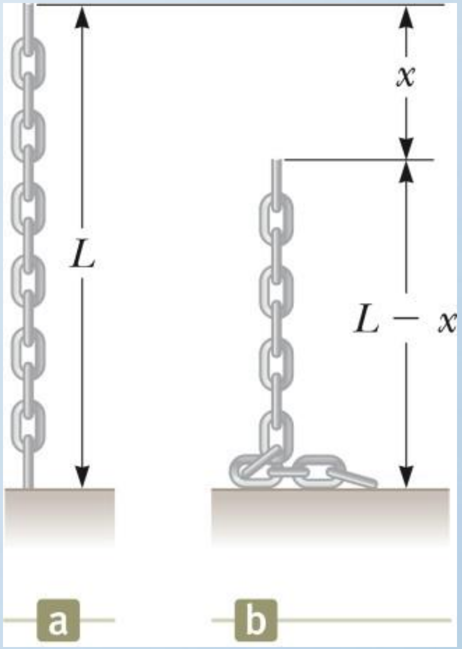
EXERCISE

A chain of length L and total mass M is released from rest with its lower end just touching the top of a table. Find the force exerted by the table on the chain after the chain has fallen through a distance x. (Assume each link comes to rest the instant it reaches the table.)

$$\frac{dv}{dt} = g, v^2 = 2gx \rightarrow v = \sqrt{2gx}$$

$$F = \frac{d}{dt}(mv) = \frac{d}{dt}\left(\frac{M}{L}xv\right) = \frac{M}{L}\frac{dx}{dt}v + \frac{M}{L}x\frac{dv}{dt}$$

$$F = \frac{M}{L}v^2 + \frac{M}{L}xg = 3\frac{M}{L}gx$$



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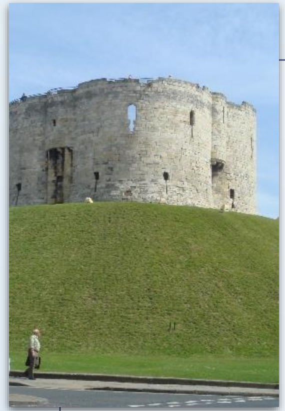

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