Lecture08-Conservation of Energy-I

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CONTENTS

1. Nonisolated System – Other Energy Transfer
2. Isolated System
3. Energy Concept for Non-conservative forces - Kinetic Friction
4. Energy Quantization
5. Power

1. NONISOLATED SYSTEM – OTHER ENERGY TRANSFER

Energy transfer that we commonly use in the study:

Kinetic Energy: K

Potential Energy: U Note that work can be taken as the transfer media for different type of energy.

Internal Energy: E_{int}

Other ways of energy transfer, like work, across the system boundary:

- Mechanical Waves: $P = \frac{1}{2}\mu\omega^2 A^2 v$, μ: mass per unit length, ω: angular velocity, A: amplitude, v: speed of the wave
- Heat: Q, Thermal Energy: $E = k_B T$, $k_B = 1.38 \times 10^{-23}$ (J/K)
- Matter Transfer
- Electrical Transmission – Electric Currents: $P = IV$
- Electromagnetic Radiation: $E = hv = hf$, $h = 6.626 \times 10^{-34}$ (J s)
- Chemical Energy: battery
- Nuclear Energy: $E = mc^2$, $c = 3 \times 10^8$ (m/s)

1. NONISOLATED SYSTEM – OTHER ENERGY TRANSFER

Example: The wavelength of visible light is between 400 and 700 nm. Please calculate the photon energy of light with a wavelength of 400 nm.

$\lambda = 400 \times 10^{-9}$ (m)

$$E = hf = h\frac{c}{\lambda} = 6.626 \times 10^{-34} \times \frac{3 \times 10^{8}}{400 \times 10^{-9}} = 4.97 \times 10^{-19}(J) = 3.10\ (eV)$$

Example: Please calculate the thermal energy at 300 K.

$$E = k_B T = 1.38 \times 10^{-23} \times 300 = 4.14 \times 10^{-21}\ (J) = 25.8\ (meV)$$

Example: A typical nuclear fusion reaction is written $^2H + {}^3H \rightarrow {}^4He + n$. How much energy is released in this fusion reaction?

rest mass energy: $E = mc^2$

1875.613 + 2808.410 – 3727.379 – 939.565 ≅ 17.1 MeV

Particle	Symbol	Rest Energy (MeV)
Neutron	n	939.565
Deuteron	d	1875.613
Triton	t	2808.410
Helium-4	^{4}He	3727.379

1. NONISOLATED SYSTEM – OTHER ENERGY TRANSFER

Energy is one of several quantities in physics that are conserved. There are many physical quantities that do not have conservation feature.

Here we call the system energy E_{system} is the total energy of the kinetic K, the potential U, and the internal $E_{internal}$ energy.
The change of system energy is expressed as: ΔE_{system}

Other ways of energy transfer, such as mechanical waves or heat, are expressed as T_i. The symbol T means transfer.

The principle of conservation of energy is described by the conservation of energy equation as:

$$\Delta E_{system} = \sum_{i=1}^{N} T_i$$

The full expansion is: $\Delta K + \Delta U + \Delta E_{int} = W + Q + T_{MW} + T_{MT} + T_{ET} + T_{ER}$

2. ISOLATED SYSTEM

The work done by the gravitational force:

$$\vec{F} = -\frac{GMm}{r^2}\hat{r} \qquad U = -\int_{r_i}^{r_f} -\frac{GMm}{r^2}dr = \left[-\frac{GMm}{r}\right]_{r_i}^{r_f}$$

$$U_f - U_i = \Delta U = -\Delta W = -\Delta K = K_i - K_f$$

$$U_i + K_i = U_f + K_f$$

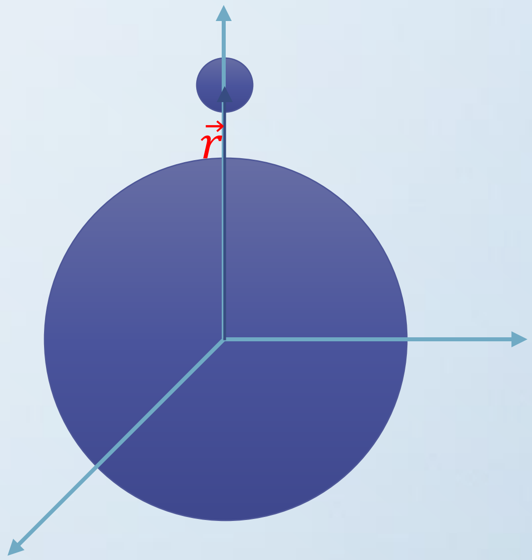
Here we define the mechanical enegy of $E_{mech} = K + U$

The conservation of the mechanical energy is given as

$$-\frac{GMm}{r_i} + \frac{mv_i^2}{2} = -\frac{GMm}{r_f} + \frac{mv_f^2}{2}$$

The principle of conservation of mechanical energy is:

$$\Delta E_{mech} = \Delta U + \Delta K$$

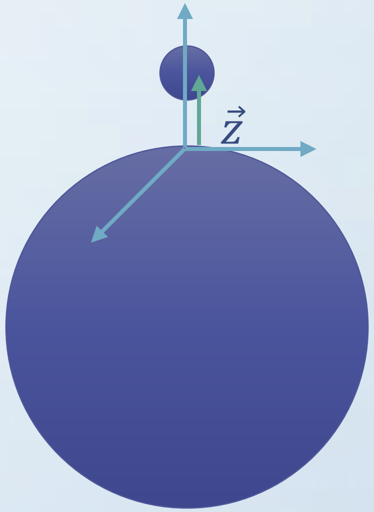


2. ISOLATED SYSTEM

The work done by the gravitational force near the Earth surface:

$$\vec{F} = -mg\hat{k} \qquad U = -\int_{z_i}^{z_f} -mg\,dz = [mgz]_{z_i}^{z_f}$$

$$mgz_i + \frac{mv_i^2}{2} = mgz_f + \frac{mv_f^2}{2}$$



The work done by the spring:

$$\vec{F} = -kx\hat{x} \qquad U = -\int_{x_i}^{x_f} -kx\,dx = \left[\frac{kx^2}{2}\right]_{x_i}^{x_f}$$

$$\frac{kx_i^2}{2} + \frac{mv_i^2}{2} = \frac{kx_f^2}{2} + \frac{mv_f^2}{2}$$

2. ISOLATED SYSTEM

Example: A child of mass m is released from rest at the top of a slide, at height h =2.0 m above the bottom of the slide. Assuming that the slide is frictionless, find the child's speed at the bottom of the slide.

$$U_i + K_i = U_f + K_f$$

$$mg \times 2.0 + 0 = 0 + \frac{mv_f^2}{2}$$

$$v_f = \sqrt{2 \times 2.0 \times 9.8} = 6.26\ (m/s)$$

https://giphy.com/gifs/boom-bsr8y78w5eBoY

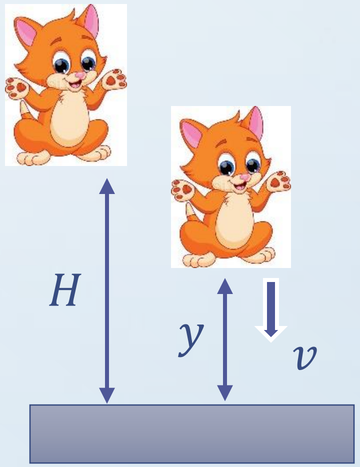
2. ISOLATED SYSTEM

Example: A cat of mass m is released from a height of H. Please calculate its speed, when it is at a height of y.

$$U_i + K_i = U_f + K_f$$

$$mgH + 0 = mgy + \frac{mv^2}{2}$$

$$v = \sqrt{2g(H - y)}$$



https://www.colourbox.com/vector/cat-cartoon-vector-9840641
https://giphy.com/gifs/QhaDJN80P3YqI

2. ISOLATED SYSTEM

Example: Two blocks are connected by a massless cord that passes over two frictionless pulleys. One end of the cord is attached to an object of mass m_1=3.00 kg that is a distance L=1.20 m from the pulley on the left. The other end of the cord is connected to a block of mass m_2=6.00 kg resting on a table. From what angle must the 3.00 kg mass be released in order to just lift the 6.00 kg block off the table?

Let zero potential energy at the height where m_1 is at the bottom.

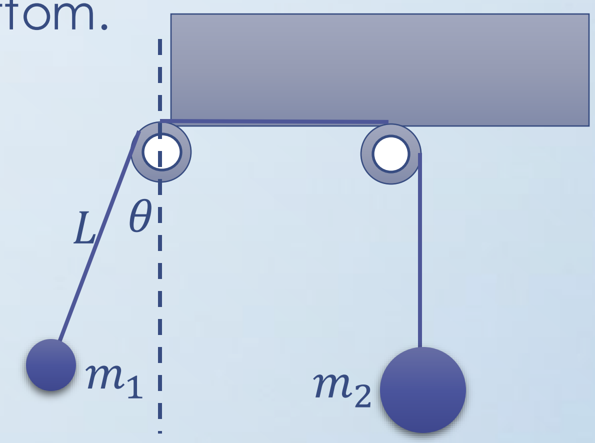
$$mgL(1-\cos\theta)+0=0+\frac{mv^2}{2}$$

$$v=\sqrt{2gL(1-\cos\theta)}$$

At the lowest position, the centripetal force and gravitation of m_1 need to be larger than the gravitation of m_2.

$$m_1g+m_1\frac{v^2}{L}\geq m_2g \qquad m_1g+2m_1g(1-\cos\theta)\geq m_2g$$

$$\cos\theta\leq\frac{3m_1-m_2}{2m_1} \qquad \theta\geq\cos^{-1}\left(\frac{3m_1-m_2}{2m_1}\right)=\frac{\pi}{3}$$



2. ISOLATED SYSTEM

Example: A bead slides without friction around a loop-the-loop. The bead is released from a height h = 3.50 R. (a) What is its speed at point A? (b) How large is the normal force on it if its mass is 5.00 g?

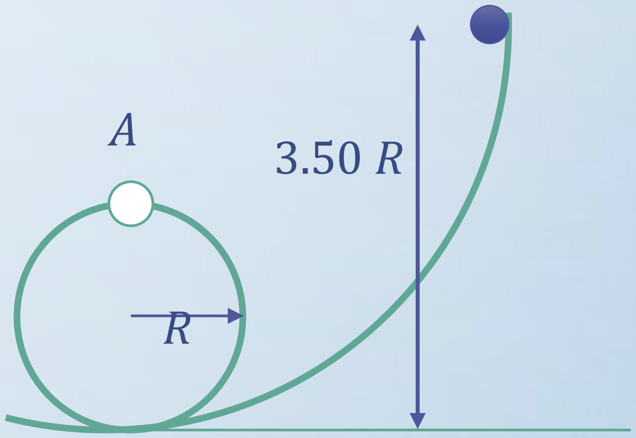
(a) Let zero energy of potential on the ground level.

$$mg(2R) + \frac{mv^2}{2} = mg(3.5R) + 0 \qquad v = \sqrt{3gR}$$

(b) The centripetal force is the total force of the normal force and the gravitational force.

$$mg + N = m\frac{v^2}{R}$$

$$N = 2mg = 2 \times 0.00500 \times 9.8 = 0.098 \text{ (N)}$$



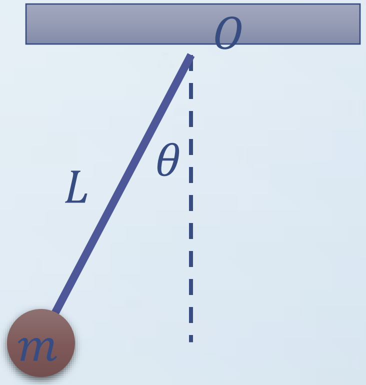
2. ISOLATED SYSTEM

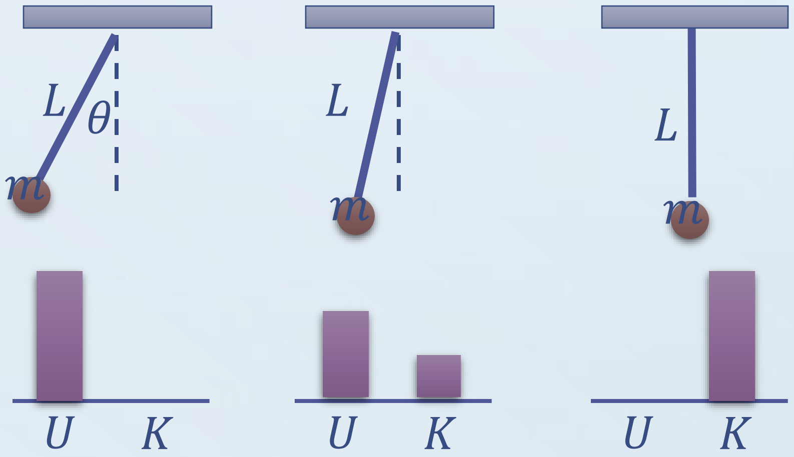
Example: A pendulum is assembled by a sphere of mass m and a string of length L. It is released at an angle θ to the vertical. Please calculate its maximum speed.

Let the zero potential energy at O.

$$0 - mgL\cos\theta = \frac{mv^2}{2} - mgL$$

$$v = \sqrt{2gL(1-\cos\theta)}$$





https://giphy.com/gifs/cheezburger-cat-th-DP5bFAwGYLQha

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3. ENERGY CONCEPT FOR NON-CONSERVATIVE FORCES - KINETIC FRICTION

Without Kinetic Friction

$W_i = \int \vec{F}_i \cdot d\vec{r}$ $\qquad \sum W_i = \int \sum \vec{F}_i \cdot d\vec{r} = \int \vec{F}_{net} \cdot d\vec{r} = \Delta K$

Put The Kinetic Friction into The System. The kinetic energy is lower

$$\sum W_i + \int \vec{f}_k \cdot d\vec{r} = \int \sum \vec{F}_i \cdot d\vec{r} + \int \vec{f}_k \cdot d\vec{r} = \int \vec{F}_{net} \cdot d\vec{r} = \Delta K$$

Clarify the work done by kinetic friction, since $\vec{f}_k$ is always opposite to the displacement $d\vec{r}$, after working for a distance d, $\int \vec{f}_k \cdot d\vec{r} = -f_k d$

$$\sum W_i - f_k d = \Delta K$$

3. ENERGY CONCEPT FOR NON-CONSERVATIVE FORCES - KINETIC FRICTION

$$\sum W_i - f_k d = \Delta K$$

When no other work is done on the system, $\sum W_i = 0$, $-f_k d = \Delta K$

The kinetic energy is transferred to the friction and the frictional force do work to increase heat and atomic vibration at the interface $f_k d = \Delta E_{int}$.

Put the potential energy into the mechanical energy

$$\Delta E_{mech} = \Delta U + \Delta K = -f_k d = -\Delta E_{int}$$

If no other work or energy transfer exists, the conservation of energy is

$$\Delta U + \Delta K + f_k d = 0$$

$$\Delta U + \Delta K + \Delta E_{int} = 0$$

3. ENERGY CONCEPT FOR NON-CONSERVATIVE FORCES - KINETIC FRICTION

Example: A car traveling at a speed v slides a distance d to a complete stop after the driver brakes. If the car's initial speed is doubled to $2v$, please estimate the distance it slides.

$\Delta U + \Delta K + f_k d = 0,\ \Delta U = 0$

$$\frac{mv^2}{2} + 0 = 0 + f_k d \quad \Rightarrow \quad f_k = \frac{mv^2}{2d}$$

When the initial speed is doubled and the frictional force remains the same, the final distance d' is:

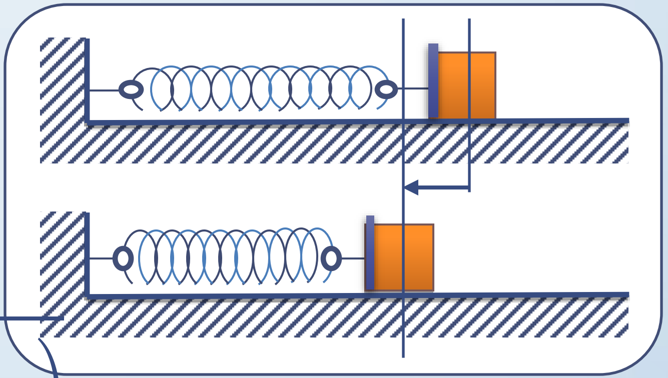
$$v' = 2v, \frac{m(2v)^2}{2} + 0 = 0 + f_k d'$$

$$d' = \frac{1}{f_k} 2mv^2 = \frac{2d}{mv^2} 2mv^2 = 4d$$

3. ENERGY CONCEPT FOR NON-CONSERVATIVE FORCES - KINETIC FRICTION

Example: A block of mass m=1.60 kg is attached to a horizontal spring that has a force constant of k=1.00 X 10^3 N/m. The spring is compressed d=2.00 cm and it is then released from rest. Calculate the speed of the block as it passes through the equilibrium position if a constant friction force of f_k=4.00 N retards its motion from the moment it is released.

$$\Delta U + \Delta K + f_k d = 0$$



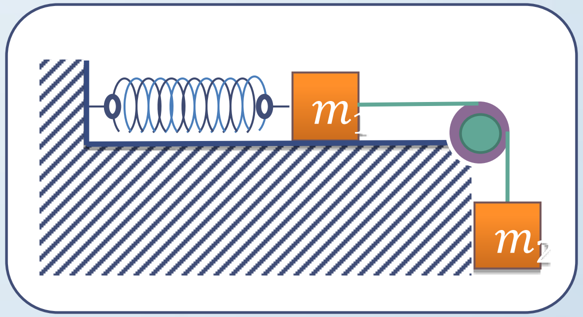
$$\frac{kd^2}{2} + 0 + 0 = 0 + \frac{mv^2}{2} + f_k d$$

$$v = \sqrt{\frac{2}{m}\left(\frac{kd^2}{2} - f_k d\right)} = \sqrt{\frac{2}{1.60}\left(\frac{1.00\times 10^3 \times (0.02)^2}{2} - 4\times 0.02\right)}$$

$$v = 0.387\ (m/s)$$

3. ENERGY CONCEPT FOR NON-CONSERVATIVE FORCES - KINETIC FRICTION

Example: Two blocks are connected by a light cord. The block of mass m_1 is connected to a spring of force constant k. The system is released from rest when the spring is unstreched. If the block of mass m_2 falls a distance of h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the table surface.



$$\Delta U_1 + \Delta U_2 + f_k d = 0$$

$$0 + m_2 g h + 0 = \frac{kh^2}{2} + 0 + f_k h \qquad f_k = m_1 g \mu_k$$

$$\mu_k = \frac{m_2 g h - \frac{kh^2}{2}}{m_1 g h}$$

3. ENERGY CONCEPT FOR NON-CONSERVATIVE FORCES - KINETIC FRICTION

Example: A 2.0 kg block slides along a floor with speed $v_1 = 4.0$ m/s. It then runs into and compresses a spring, until the block momentarily stops. Its path to the initially relaxed spring is frictionless, but as it compresses the spring, a kinetic frictional force of magnitude 15 N acts on it. The spring constant is 10,000 N/m. By what distance d is the spring compressed when the package stops?

$$\Delta U + \Delta K + f_k d = 0$$

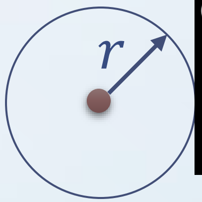
$$0 + \frac{mv^2}{2} + 0 = \frac{kd^2}{2} + 0 + f_k d$$

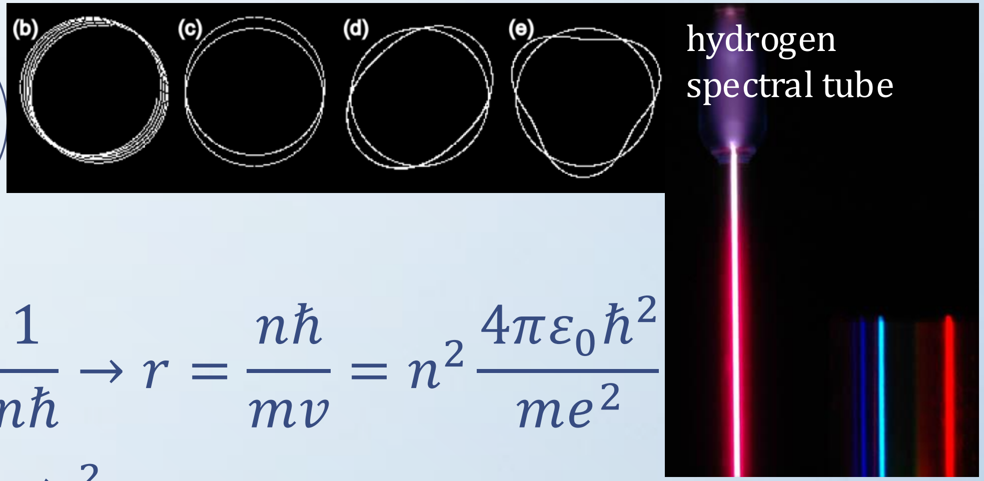
$$\frac{2.0 \times (4.0)^2}{2} = \frac{10000d^2}{2} + 15d$$

$5000d^2 + 15d - 16 = 0,\ d \cong 0.056$ (m)

4. ENERGY QUANTIZATION

1. Hydrogen Atom



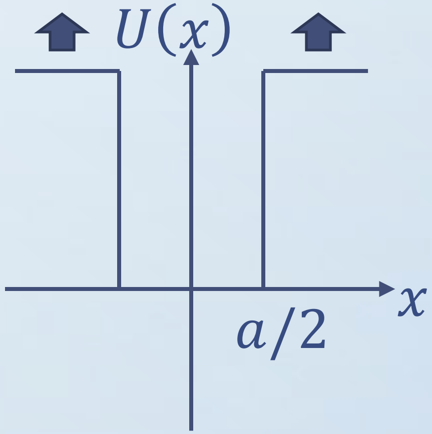


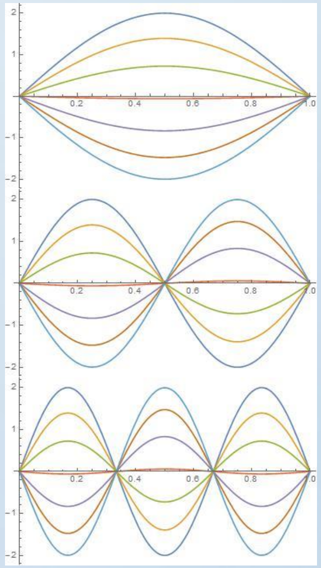
$$\frac{2\pi r}{\lambda} = n \rightarrow kr = n \rightarrow \hbar kr = n\hbar \rightarrow l = n\hbar$$

$$F = m\frac{v^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2} \rightarrow mvr = \frac{e^2}{4\pi\varepsilon_0}\frac{1}{v} \rightarrow v = \frac{e^2}{4\pi\varepsilon_0}\frac{1}{n\hbar} \rightarrow r = \frac{n\hbar}{mv} = n^2\frac{4\pi\varepsilon_0\hbar^2}{me^2}$$

$$E = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\varepsilon_0 r} = -\frac{1}{2}mv^2 \rightarrow E_n = -\frac{m}{2}\left(\frac{e^2}{4\pi\varepsilon_0\hbar}\right)^2\frac{1}{n^2}$$

2. Particle in a Box





$$n\frac{\lambda}{2} = a \rightarrow \frac{2\pi}{\lambda}a = n\pi$$

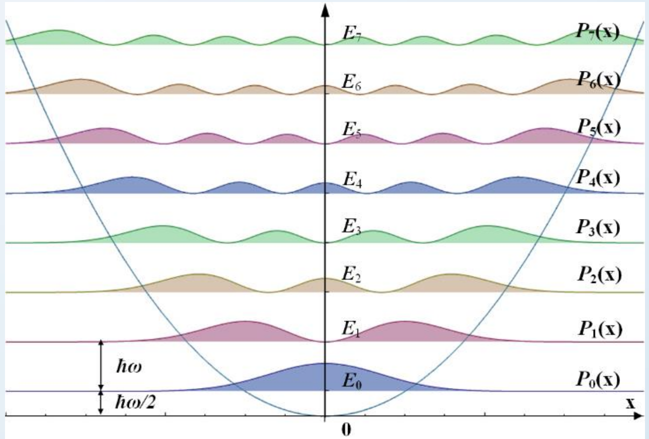
$$E = \frac{p^2}{2m} = \frac{k^2\hbar^2}{2m} \qquad E_n = \frac{\hbar^2\pi^2}{2ma^2}n^2$$

4. ENERGY QUANTIZATION

3. Quantum Harmonic Oscillator

$$F = -kx = ma \rightarrow \omega = \sqrt{\frac{k}{m}}$$

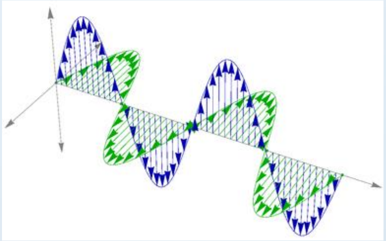
$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

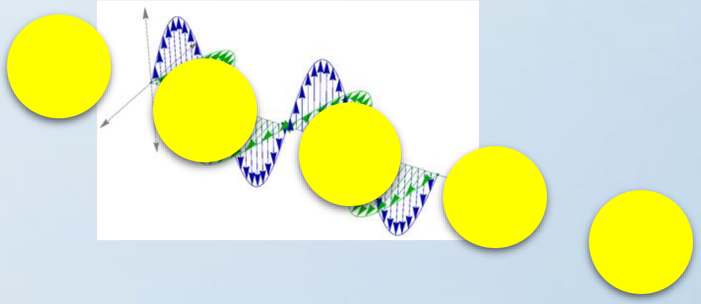


https://en.wikipedia.org/wiki/Quantum_harmonic_oscillator

4. Light, Electromagnetic Waves, Photons

$$E_n = nhf$$





5. POWER

Average Power: $P_{avg} = \frac{\Delta W}{\Delta t}$, ΔW: work

Instantaneous Power: $P = \lim_{t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$

Related to force and velocity: $P = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{r}}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$

The unit of power: 1 W = 1 J / s = 1 kg m^2 / s^3

One horsepower (hp) is 746 W. Typically, the engine power of a sedan is about 150 hp (112 kW). The other important spec is engine torque.

The unit used for electric power plant or for the electric bill is

kilowatt-hour = 1000 W X 1 h = 1000 W X 3600 s = 3.6 X 10^6 J

The kW-h is an unit of enrgy, not power.

https://giphy.com/gifs/one-just-horsepower-ECcdNDHImuJBC

5. POWER

Example: A 1000-kg, gasoline-powered car is running at a constant speed of 100 km/h up a 10-percent grade. (a) If the efficiency of the car engine is 15%, what is the rate at which the chemical energy of the car engine changes?

ten percent grade (slope, incline): $\tan\theta = 10\% = 0.1,\ \theta \cong 5.71^0$

the sliding force of the car on the grade is $mg\sin\theta = 1000 \times 9.8 \times \sin 5.71^o = 975$ N

the power to push the car is $P = Fv = 975N \times 100\left(\frac{km}{h}\right)$

$P = 975 \times \frac{100}{3.6} = 27.1\ kW$

It is only 15% of the engine power so the engine power is

$P_{eng} = \frac{27.1}{0.15} = 181$ kW

EXERCISE

Use the potential energy $U(x) = \frac{Ex}{x_0}$, for $E > 0$, to find the position x of a particle as a function of time t. Use the equation $v = \frac{dx}{dt} = \sqrt{\frac{2(E-U)}{m}}$.

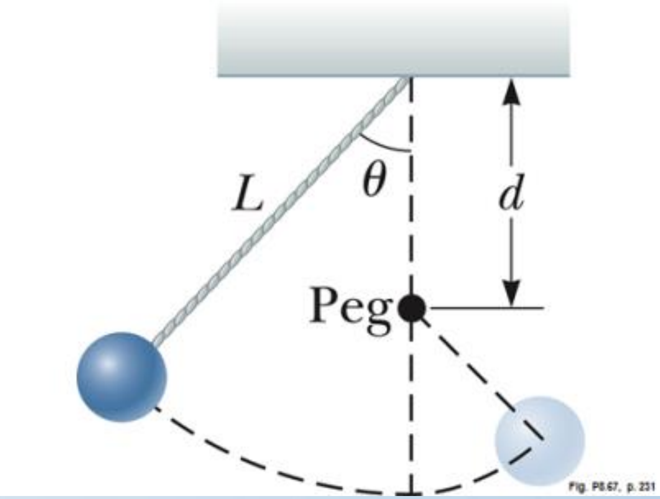
$$\frac{\sqrt{m}dx}{\sqrt{2\left(E - U(x)\right)}} = dt \qquad \int_{x_0}^{x} \frac{\sqrt{m}dx}{\sqrt{2\left(E - \frac{Ex}{x_0}\right)}} = \int_0^t dt$$

$$t = \int_{x_0}^{x} \frac{\sqrt{m}dx}{\sqrt{\frac{2E}{x_0}(x_0 - x)}} = \sqrt{\frac{2mx_0}{E}}(-\sqrt{x_0 - x})$$

$$x = x_0 - \frac{E}{2mx_0}t^2$$

EXERCISE

A pendulum, comprising a light string of length L and a small sphere, swings in the vertical plane. The string hits a peg located a distance d below the point of suspension. (a) Show that if the sphere is released from a height below that of the peg, it will return to this height after the string strikes the peg. (b) Show that if the pendulum is released from rest at the horizontal position ($\theta = 90^o$) and is to swing in a complete circle centered on the peg. The minimum value of d must be $3L/5$.



(a) Energy conservation. No other consumption.

(b) Potential energy: zero at bottom, potential position at horizontal place is mgL.

When swinging to the highest position,

$$\Delta U = mg\big(L - 2(L-d)\big) = mg(2d - L)$$

The potential energy is converted to kinetic energy,

$$\Delta U = mg(2d - L) = mv^2/2 \qquad v = \sqrt{2g(2d-L)}$$

EXERCISE

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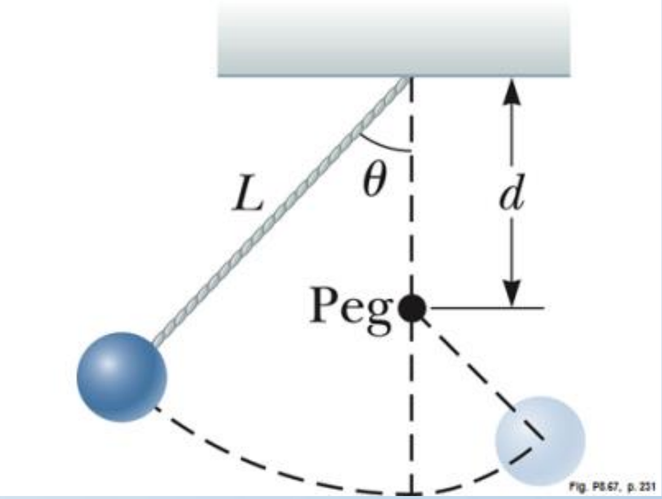


$$v = \sqrt{2g(2d - L)}$$

When will the centripetal acceleration larger than g?

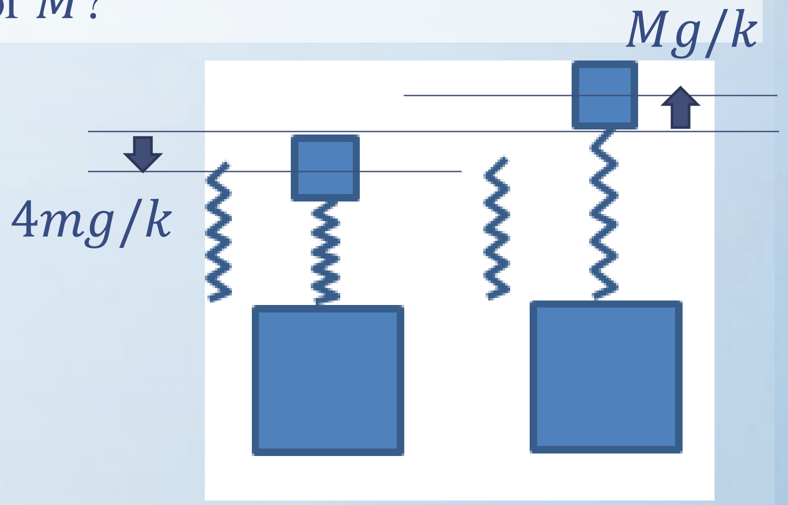
$$\frac{v^2}{r} = \frac{v^2}{(L-d)} > g$$

$$\frac{2g(2d-L)}{(L-d)} > g \rightarrow 2(2d-L) > L-d$$

$$5d > 3L \rightarrow d > \frac{3L}{5}$$

EXERCISE

A block of mass M rests on a table. It is fastened to the lower end of a light, vertical spring. The upper end of the spring is fastened to a block of mass m. The upper block is pushed down by an additional force $3mg$, so the spring compression is $4mg/k$. In this configuration, the upper block is released from rest. The spring lifts the lower block off the table. In terms of m, what is the greatest possible value for M?



The lower block is lifted if the spring force is larger than Mg. It is that the extended distance shall be longer than Mg/k.

$$-mg\left(\frac{4mg}{k}\right)+\frac{1}{2}k\left(\frac{4mg}{k}\right)^2 \geq mg\left(\frac{Mg}{k}\right)+\frac{1}{2}k\left(\frac{Mg}{k}\right)^2$$

$$-\frac{4m^2g^2}{k}+\frac{8m^2g^2}{k}\geq\frac{Mmg^2}{k}+\frac{M^2g^2}{2k}$$

$$M^2+2Mm\leq 8m^2$$

EXERCISE

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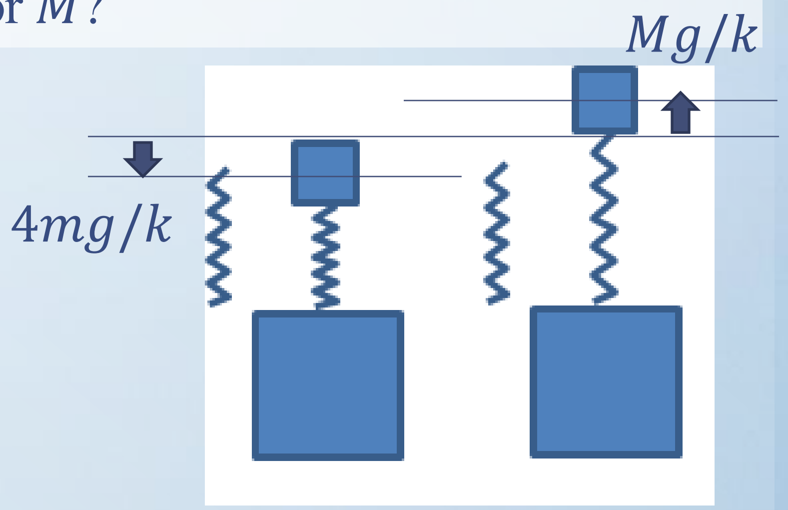

$$M^2 + 2Mm - 8m^2 \leq 0$$

The maximum M exists in the condition of $M^2 + 2Mm - 8m^2 = 0$

$$M = \frac{-2m \pm \sqrt{4m^2 + 32m^2}}{2}$$

$$= -m \pm 3m = 2m \text{ or } -4m$$

$$M = 2m$$

ACKNOWLEDGEMENT

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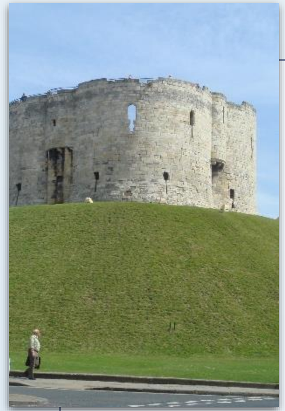
自主愛學習計畫



【科技部補助】