Chapter 30 Faraday's Law

Physics II – Part II
Wen-Bin Jian
Department of Electrophysics, National Chiao Tung University

Summary of Electric and Magnetic Fields

The sources for generating electric fields are static charges. There are two polarities, positive and negative polarities, for the charges. Each polarity of charges can exist independently in the space.

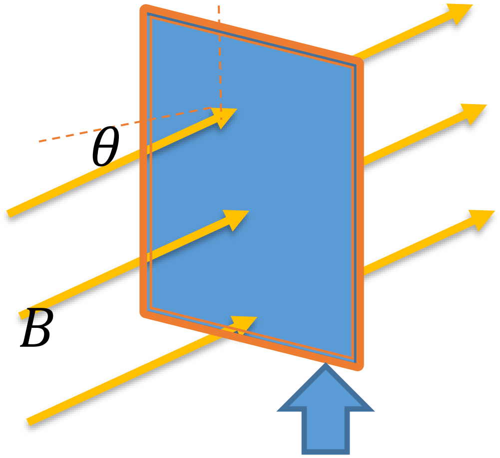
The sources for generating magnetic fields are electric currents. The current give a magnetic field thus creating north and south polarities for a virtue magnet. The two polarities exist simultaneously.

$$\oint \vec{E} \cdot d\vec{A} = 4\pi k Q_{enc} \qquad \oint \vec{B} \cdot d\vec{A} = 0$$

For an open surface, the magnetic flux is $\int \vec{B} \cdot d\vec{A}$. The unit of the magnetic flux is weber (T m^2).

For a circuit surrounding the open surface, the changing of the flux will induce an electromotive force thus generating an induced current in the circuit. This phenomena is named the Faraday's law.

Faraday's Law of Induction



The magnetic flux of the circuit loop:

$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos(\theta)$$

The induced emf (electromotive force) in the circuit is resulted from the change of magnetic flux:

$$\varepsilon = -\frac{d\Phi}{dt}$$

The ways to change the magnetic flux are:

1. increase or decrease the magnetic field at the circuit loop
2. a magnet moved close to or away from the circuit loop
3. rotate the circuit loop with respect to the magnetic field
4. move the circuit in a nonuniform magnetic field

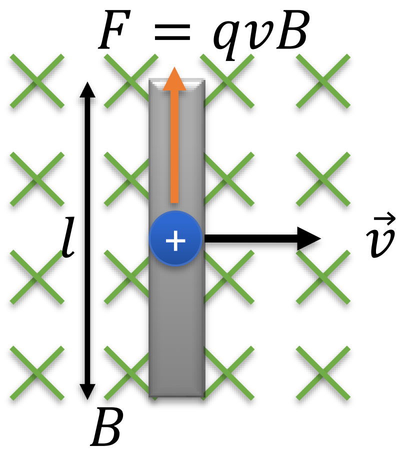
$$\varepsilon = -\frac{d\Phi}{dt} = \left(-\frac{dB}{dt}\right)A\cos\theta = -B\frac{dA}{dt}\cos\theta = -BA\left(\frac{d\cos\theta}{dt}\right)$$

Ref:

Lorentz's Law Induced emf

Motional emf

A conducting rod moves in a magnetic field:



The positive charges in the rod is pulled to move with a constant velocity $\vec{v}$.

The Lorentz law predicts an effective electric field in the direction along the rod.

$$F = qvB = qE \rightarrow E = vB$$

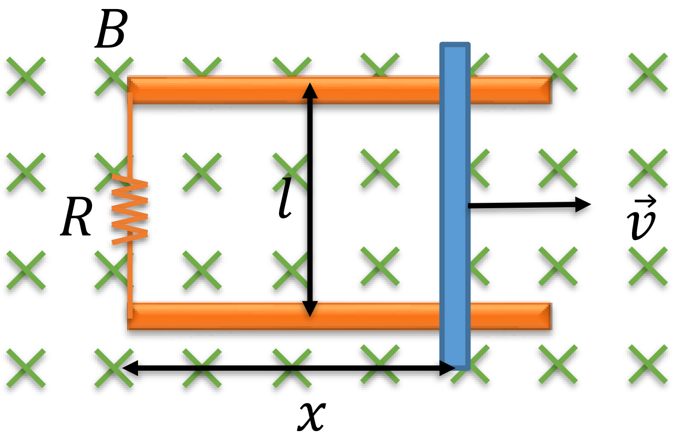
The induced emf in the rod is estimated by the line integration:

$$\varepsilon = -\int \vec{E} \cdot d\vec{l} = -vBl$$

Ref:

Motional emf

Compare the two physical models of the motional emf and the change of flux:



$$\Phi_B = Blx$$

$$\varepsilon = -\frac{d(Blx)}{dt} = -Bl\frac{dx}{dt} = -Blv$$

The induced current is in the counterclockwise direction.

$$qvB = qE \rightarrow E = vB$$

$$\varepsilon = El = vBl$$

The induced voltage also causes a current flow in the counterclockwise direction.

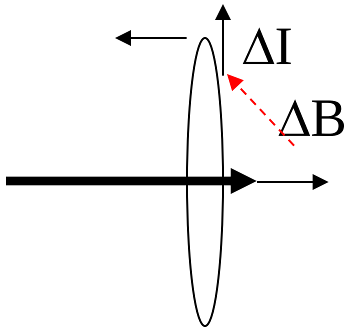
The induced voltage source is just across the moving rod.

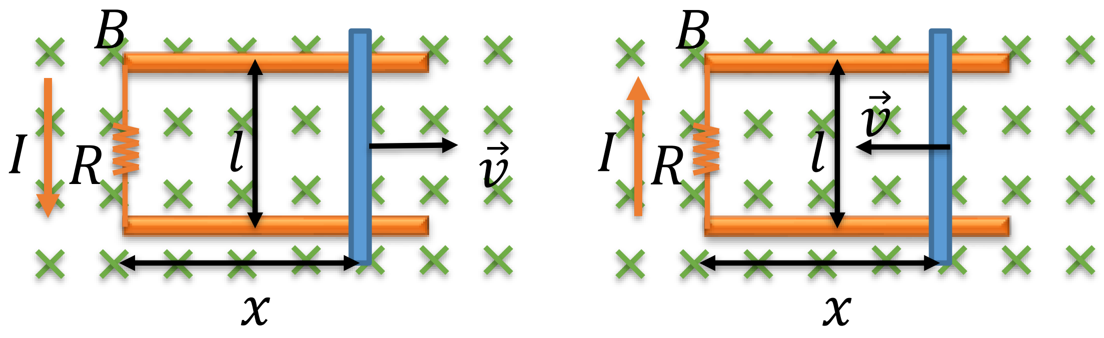
Ref:

The Direction of Induced Currents

The Lenz's Law

When the magnetic flux changes, an induced current produces additional magnetic flux through the same surface to resist the change of magnetic flux.





The preservation of magnetic flux in a region of area always exists. It can be observed after you put an electric circuit in the region. Without electric circuit, the resistor is so high thus the flux preservation is not observable.

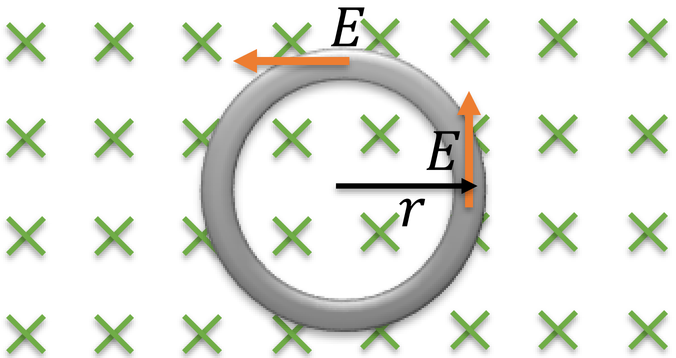
Ref:

The Electric Field Concept (Equivalent to The Lorentz Electric Field)

Induced Electric Fields

Equivalent concepts between the Lorentz electric field and the induced electromotive force:

The place where the induced emf occurs must be considered at first.



$$B = B(t) \& \frac{dB}{dt} > 0$$

$$\Phi_B = B\pi r^2$$

$$\varepsilon = -\frac{d(B\pi r^2)}{dt} = -\pi r^2 \frac{dB}{dt}$$

Here the induced electric field is uniformly distributed in the circular wire (rather than in a certain length of wire).

$$(2\pi r)E = \pi r^2 \frac{dB}{dt} \rightarrow E = \frac{r}{2}\frac{dB}{dt}$$

Note that the induced electric field always exists at the position while the metal circuit helps to see the induced current.

Ref:

Generators & Motors

The Faraday law is used to make generators (AC & DC generators).
The change of flux is generated by rotating a loop with respect to a constant magnetic field.

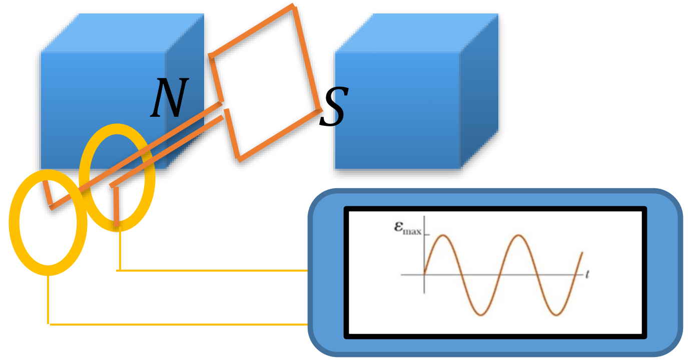
$$\Phi_B = NBA\cos\theta$$

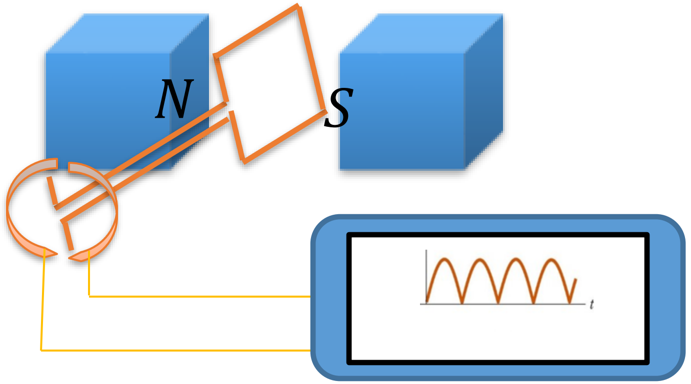
$$\varepsilon = -\frac{d\Phi_B}{dt} = NBA\sin\theta\frac{d\theta}{dt}$$

$$\varepsilon = \omega NBA\sin\theta$$

The DC generator is realized by

$$\varepsilon = \omega NBA|\sin\theta|$$





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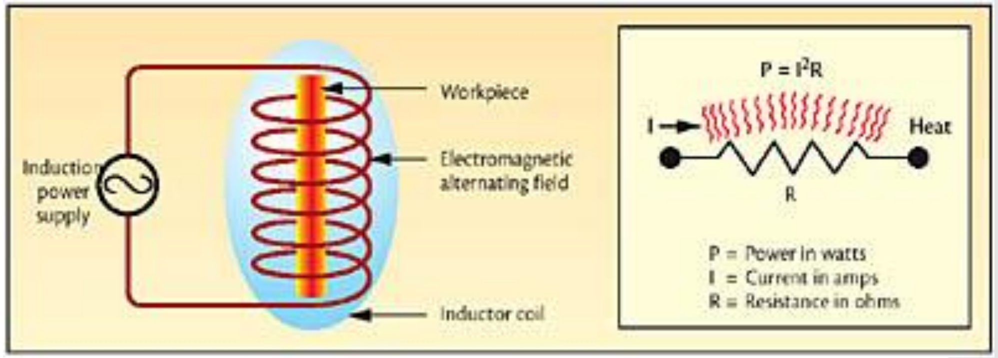
Eddy Currents

The Faraday law predicts an induced emf and current in a circuit loop due to the change of magnetic flux.



The induced current in a metal plate is named the eddy currents.

The induced current and the induced magnetic field can be used for applications.

For example,

1. In the transformer, eddy current which generates Joule heating should be reduced to be negligibly small.
2. The eddy current in the induction cooker (induction oven) is used to generate heat.
3. The eddy current is used for rapid heating.

Ref:

Examples

A uniform magnetic field makes an angle of 30° with the axis of a circular coil of 300 turns and a radius of 4 cm. The magnitude of the magnetic field increases at a rate of 85 T / s while its direction remains fixed. Find the magnitude of the induced emf in the coil.

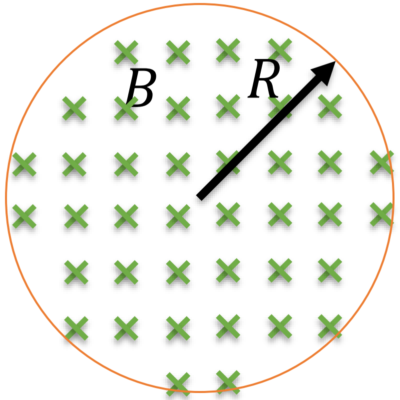
$$\Phi_B = \vec{B} \cdot \vec{A} = NB\pi r^2 \cos\theta$$

$$\varepsilon = -\frac{d\Phi_B}{dt} = -N\pi r^2 \cos\theta \frac{dB}{dt}$$

Examples

A magnetic field B is perpendicular to the plane of the page. B is uniform throughout a circular region of radius R. Outside this region, B equals zero. The direction of B remains fixed and the rate of change of B is $\frac{dB}{dt}$. What are the magnitude of the induced electric field in the plane of the page (a) a distance r < R from the center of the region and a distance r > R from the center, where B = o.

$$r < R:\ \Phi_B = B\pi r^2, 2\pi r E = \pi r^2 \frac{dB}{dt} \rightarrow E = \frac{r}{2}\frac{dB}{dt}$$



$$r > R:\ \Phi_B = B\pi R^2, 2\pi r E = \pi R^2 \frac{dB}{dt} \rightarrow E = \frac{R^2}{2r}\frac{dB}{dt}$$

If $\frac{dB}{dt} > 0$, the induced current is in the counterclockwise direction.

The Charge Integrator

Examples

A small coil of N turns has its plane perpendicular to a uniform static magnetic field . The coil is connected to a current integrator. Find the charge passing through the coil if the coil is rotated through 180° about the axis.

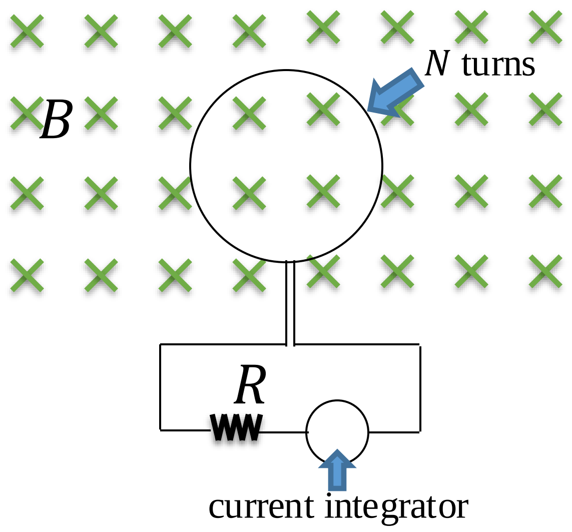
$$\Phi_B = NBA\cos\theta$$

$$\varepsilon = IR \rightarrow \frac{dQ}{dt} = \frac{\varepsilon}{R}$$

$$\varepsilon = -\frac{d\Phi_B}{dt} = NBA\sin\theta\,\frac{d\theta}{dt}$$

$$\frac{dQ}{dt} = \frac{NBA\sin\theta}{R}\frac{d\theta}{dt} \rightarrow dQ = \frac{NBA\sin\theta}{R}d\theta$$

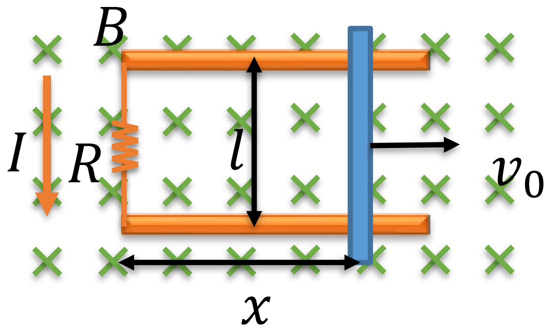
$$Q = \frac{NBA}{R}\int_0^{\pi}\sin\theta\,d\theta = \frac{2NBA}{R}$$



Examples

A rod of mass m and resistance R slides on frictionless conducting rails with a separation distance of l in a region of static uniform magnetic field B. An external agent is pushing the rod, maintaining its motion to the right at constant speed v_0. At time t = o, the agent abruptly stops pushing the rod continuous forward. The rod is slowed down by the magnetic force. Find the speed $v(t)$ of the rod as a function of time.

$$F = -IBl, I = \frac{\varepsilon}{R} = \frac{vBl}{R} \rightarrow F = -\frac{B^2 l^2}{R} v$$



$$F = -\frac{B^2 l^2}{R} v = ma \rightarrow m\frac{dv}{dt} + \frac{B^2 l^2}{R} v = 0$$

$$m\frac{dv}{dt} = -\frac{B^2 l^2}{R} v \rightarrow \frac{dv}{v} = -\frac{B^2 l^2}{mR} dt \rightarrow \int_{v_0}^{v} \frac{dv}{v} = -\frac{B^2 l^2}{mR} \int_0^t dt$$

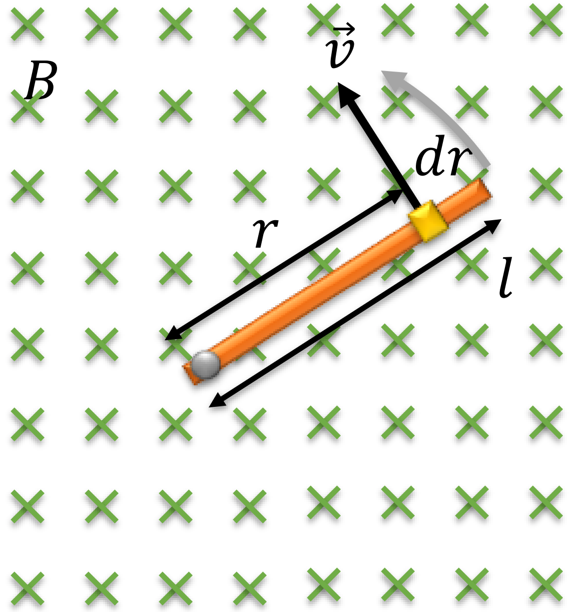
$$\ln v - \ln v_0 = -\frac{B^2 l^2}{mR} t \rightarrow \ln\left(\frac{v}{v_0}\right) = -\frac{B^2 l^2}{mR} t \qquad v = v_0 e^{-\frac{B^2 l^2}{mR} t}$$

Examples

Please calculate the emf of the rotating bar of length l in a magnetic field B. The angular speed is ω.

$$d\varepsilon = vBdr = \omega rBdr$$

$$\varepsilon = \omega B \int_0^l rdr = \frac{1}{2}\omega Bl^2$$



Examples

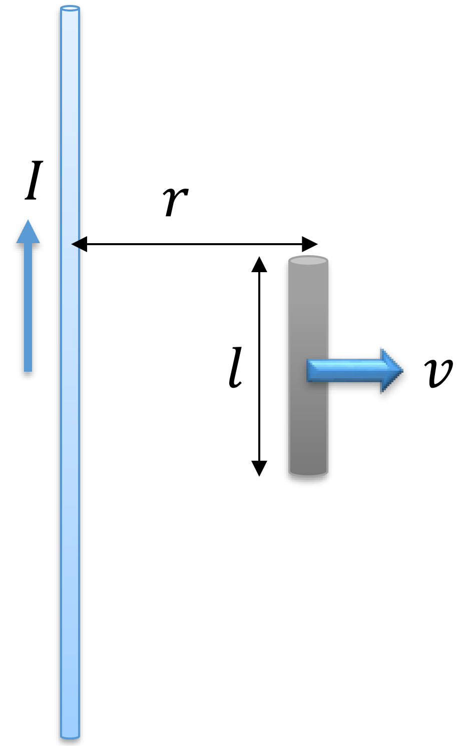
A conducting rod of length l is arranged in parallel with a long and straight wire, carrying a current I, and it moves with a constant speed v in a direction perpendicular to the wire. The initial distance between the rod and the wire is r. Calculate the induced emf generated in the rod.

$$2\pi r B = \mu_0 I \rightarrow B = \frac{\mu_0 I}{2\pi r}$$

motional emf

$$qvB = qE \rightarrow E = vB$$

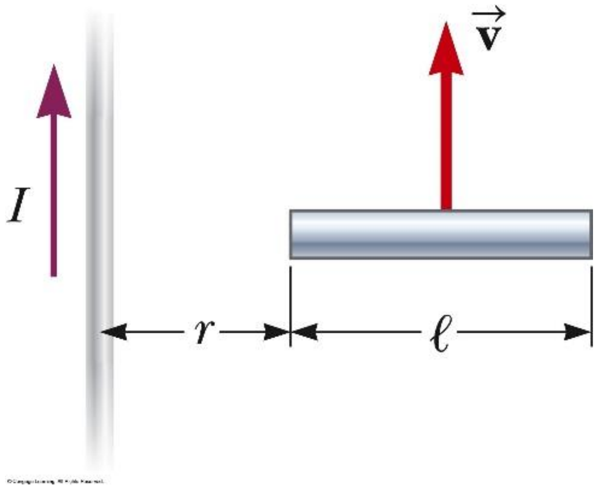
$$\Delta V = \int_0^l E dy = vBl = \frac{\mu_0 I v l}{2\pi r}$$



The Induced emf

Examples

A conducting rod of length l moves with velocity $\vec{v}$ parallel to a long wire carrying a current I. The axis of the rod is maintained perpendicular to the wire with the near end a distance r away. Please calculate the emf induced in the rod.



$$2\pi aB = \mu_0 I \rightarrow B = \frac{\mu_0 I}{2\pi a}$$

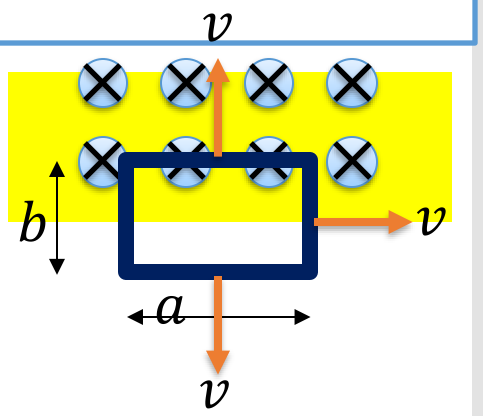
$$qvB = qE \rightarrow E = vB$$

$$\Delta V = \int_r^{r+l} E da$$

$$\Delta V = \int_r^{r+l} \frac{\mu_0 I v}{2\pi a} da = \frac{\mu_0 I v}{2\pi} \ln\left(\frac{r+l}{r}\right)$$

Examples

A rectangular coil of N turns, each of width a and length b is located in a magnetic field B directed into the page, with only half of the coil in the region of the magnetic field. The resistance of the coil is R. Find the magnitude and direction of the induced current if the coil is moved with a speed v (a) to the right, (b) up, and (c) down.



(a) 0

(b) $\Phi_m = NBax \rightarrow \varepsilon = -NBa\frac{dx}{dt} = -NBav$

The current inside the loop is $I = \frac{NBav}{R}$.

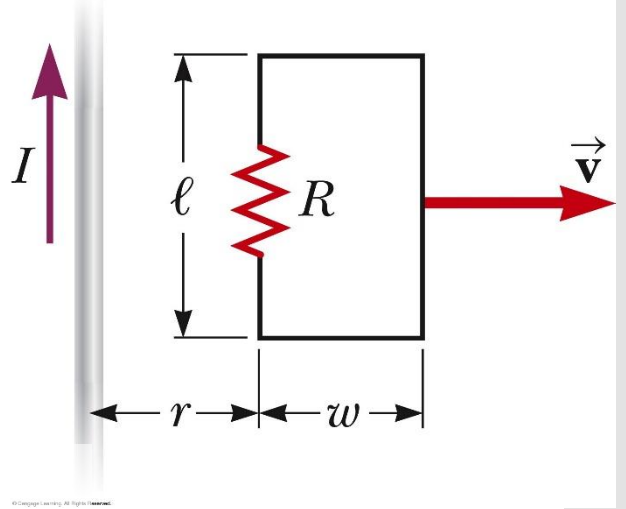
Use the Lenz's law, the current is in the counterclockwise direction.

(c) The current inside the loop is $I = \frac{NBav}{R}$.

Use the Lenz's law, the current is in the clockwise direction.

Examples

A rectangular loop of dimensions l and w moves with a constant velocity $\vec{v}$ away from a long wire that carries a current I in the plane of the loop. The total resistance of the loop is R. Calculate the current at the instance the near side is a distance r away from the wire.



$$2\pi a B = \mu_0 I \rightarrow B = \frac{\mu_0 I}{2\pi a}$$

$$\Phi_B = l\int_r^{r+w} \frac{\mu_0 I}{2\pi a} da = \frac{\mu_0 I l}{2\pi} \ln\left(1 + \frac{w}{r}\right)$$

$$v = \frac{dr}{dt}$$

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{\mu_0 I l}{2\pi} \frac{\left(-\frac{w}{r^2}\right)}{1 + \frac{w}{r}} \frac{dr}{dt} = \frac{\mu_0 v I l}{2\pi} \frac{w}{r(r+w)}$$

$$I = \frac{\varepsilon}{R} = \frac{\mu_0 v I l}{2\pi R} \frac{w}{r(r+w)}$$

Examples

A long solenoid of radius R has n turns of wire per unit length and carries a time-varying current that varies sinusoidally as $I(t) = I_0 \cos(\omega t)$, where I_0 is the maximum current and ω is the angular frequency of the ac current source. (a) Determine the magnitude of the induced electric field outside the solenoid at a distance $r > R$ from its long central axis. (b) What is the magnitude of the induced electric field inside the solenoid, a distance r from its axis?

The magnetic field inside the solenoid is $B = \mu_0 n I(t)$.

(a)

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\Phi_m \rightarrow 2\pi r E = -\frac{d}{dt}\left(\pi R^2 \mu_0 n I_0 \cos(\omega t)\right)$$

$$E = \frac{\omega R^2 \mu_0 n I_0 \sin(\omega t)}{2r}$$

(b)

$$E = \frac{\omega r \mu_0 n I_0 \sin(\omega t)}{2}$$