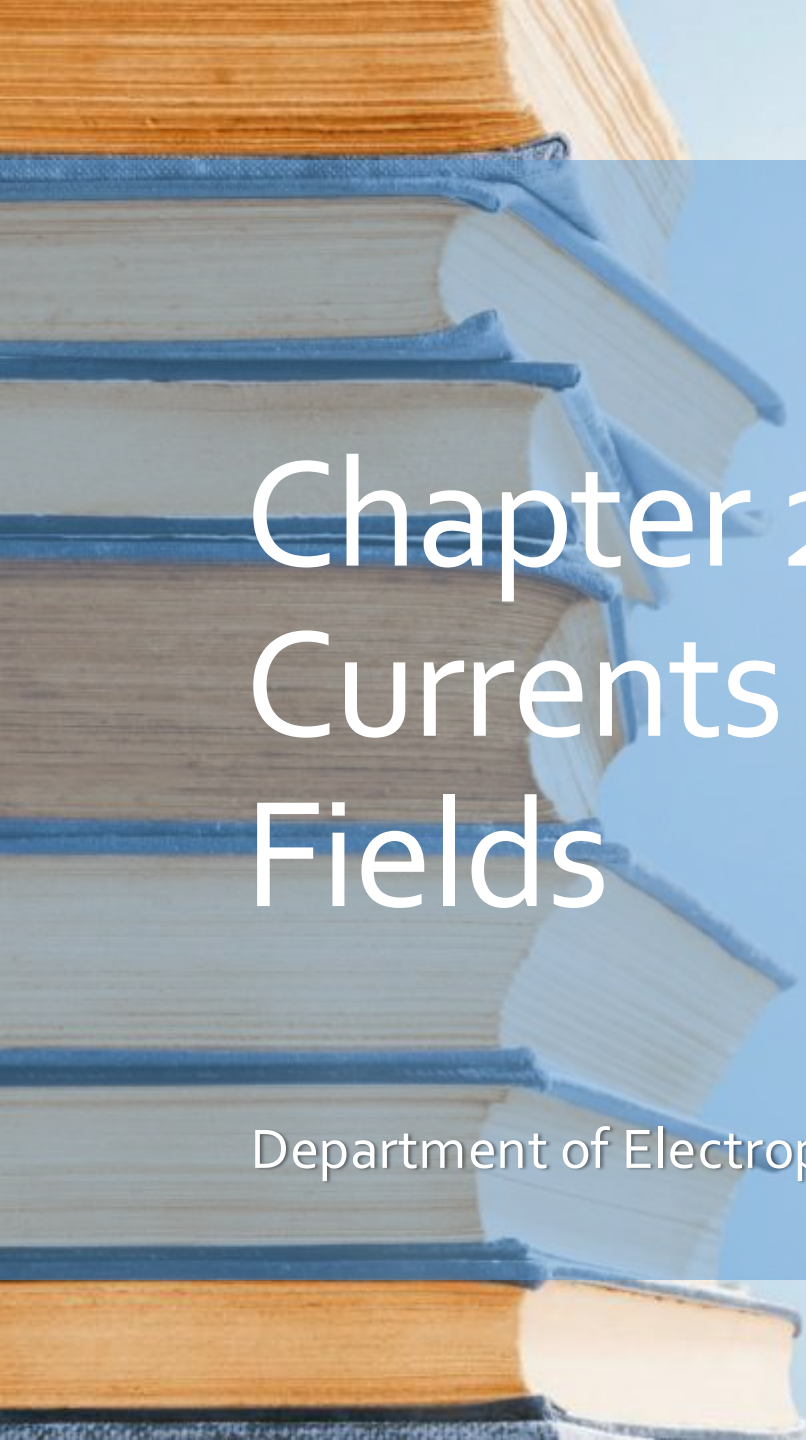




Chapter 29 Electric Currents and Magnetic Fields

Physics II – Part II
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Department of Electrophysics, National Chiao Tung University

Biot & Savart's Experiments

History

1820 AD – Hans Christian Ørsted (Danish)

Electric current generates magnetic fields.

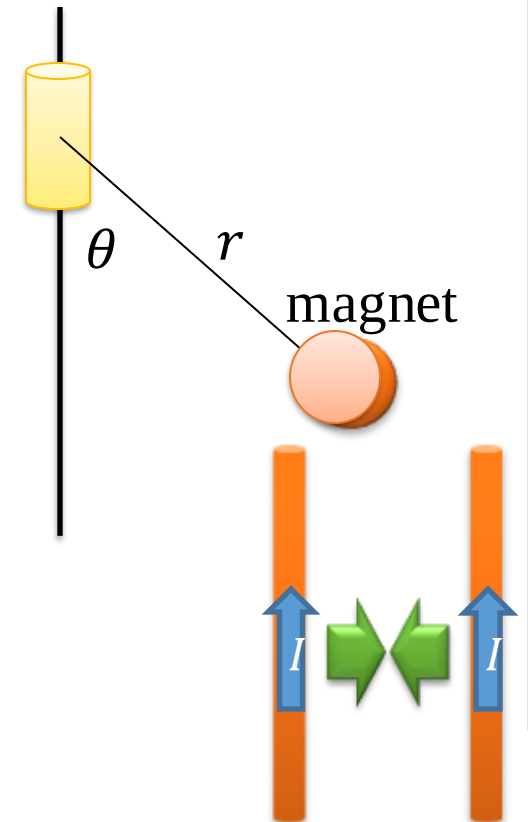
1820 AD (18 December) – Jean-Baptiste Biot (French) & Félix Savart (French)

Experimental determination of current generated magnetic fields. Treat currents as magnets with magnetic fields.

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \hat{r}}{r^2} \quad \frac{\mu_0}{4\pi} = 10^{-7} \text{ T m / A}$$

1820 AD (4 December) – André-Marie Ampère (French)
interaction between currents

1882 AD – William Thomson, Baron Kelvin (Scottish)
current balance design



The Biot-Savart Law

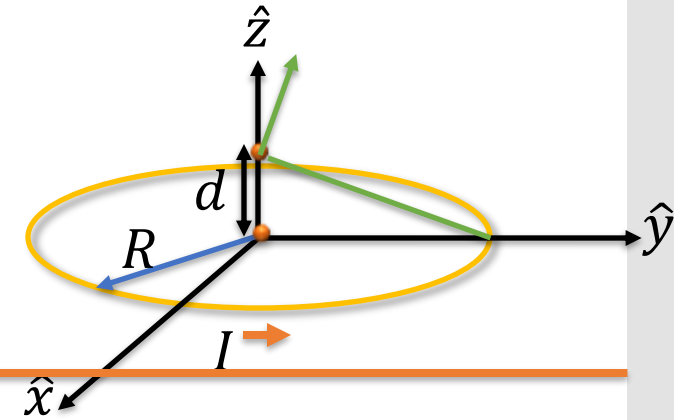
Magnetic Fields

Use the Biot-Savart law to calculate the magnetic field at the center of a circular loop with radius R and current I . Please calculate the magnetic field on the central axis and at a distant d away from the center.

$$\vec{B}_o = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times (-\hat{r})}{r^2} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{R d\theta}{R^2} \hat{k}$$

$$\vec{B}_o = \frac{\mu_0 I}{4\pi} \frac{2\pi}{R} \hat{k} = \frac{\mu_0 I}{2R} \hat{k}$$

$$\vec{B}_d = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times (-\hat{r})}{r^2} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{R d\theta}{R^2 + d^2} \frac{R}{\sqrt{R^2 + d^2}} \hat{k}$$



The Biot-Savart Law

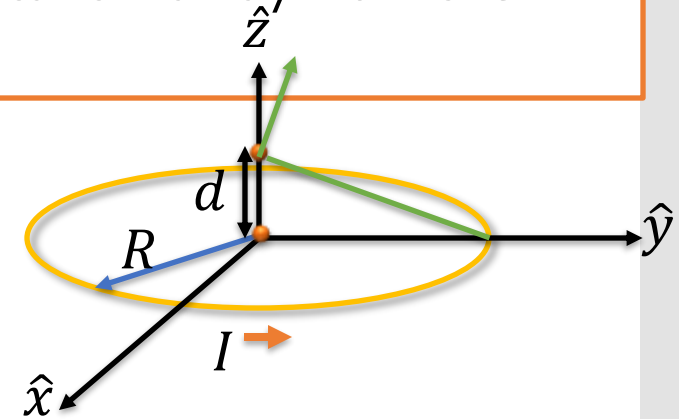
Magnetic Fields

Use the Biot-Savart law to calculate the magnetic field at the center of a circular loop with radius R and current I . Please calculate the magnetic field on the central axis and at a distant d away from the center.

$$\vec{B}_d = \frac{\mu_0 I}{4\pi} \frac{R^2 2\pi}{(R^2 + d^2)^{\frac{3}{2}}} \hat{k} = \frac{\mu_0 I R^2}{2(R^2 + d^2)^{\frac{3}{2}}} \hat{k}$$

$$B_d \xrightarrow{d \gg R} \frac{\mu_0 I R^2}{2d^3} = \frac{2\mu_0 \pi I R^2}{4\pi d^3} = \frac{\mu_0}{4\pi} \frac{2m}{d^3} \iff E = \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \hat{r}}{r^2}$$



The Biot-Savart Law

Magnetic Fields

Use the Biot-Savart law to calculate the magnetic field at point P a distance a away from the segment of line current I .

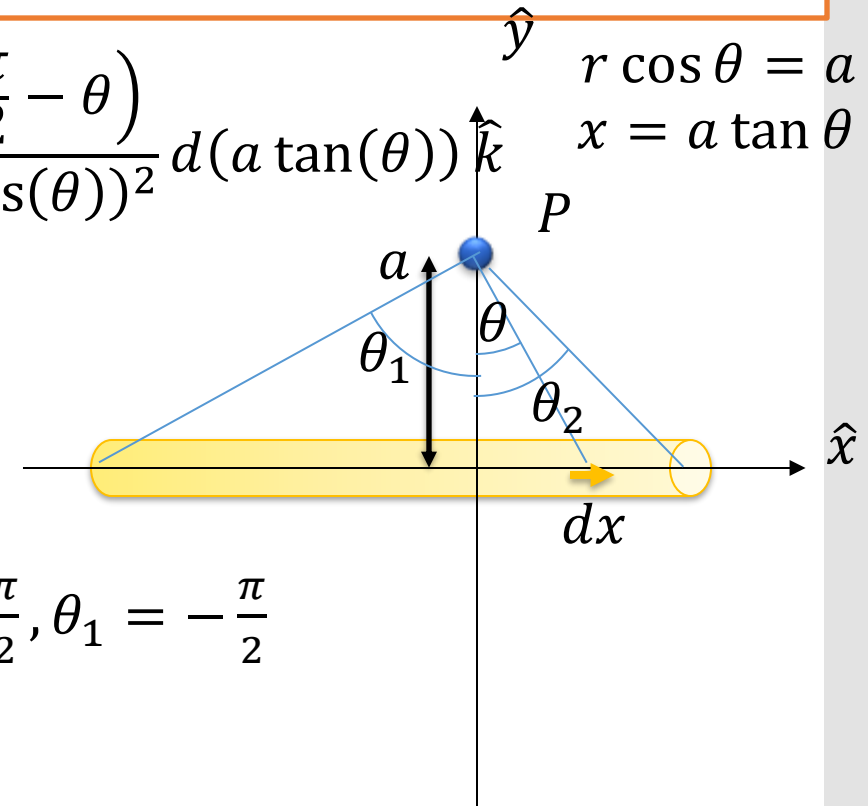
$$\vec{B}_P = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \int_{\theta_1}^{\theta_2} \frac{\sin\left(\frac{\pi}{2} - \theta\right)}{(a/\cos(\theta))^2} d(a \tan(\theta)) \hat{k}$$

$$\vec{B}_P = \frac{\mu_0 I}{4\pi} \int_{\theta_1}^{\theta_2} \frac{\cos(\theta) a}{a^2} d\theta \hat{k}$$

$$\vec{B}_P = \frac{\mu_0 I}{4\pi a} (\sin(\theta_2) - \sin(\theta_1)) \hat{k}$$

An infinitely long line current, $\theta_2 = \frac{\pi}{2}$, $\theta_1 = -\frac{\pi}{2}$

$$\vec{B}_P = \frac{\mu_0 I}{2\pi a} \hat{k}$$



Ampere's Law & Applications for The Calculation of Magnetic Fields

Ampere's Law

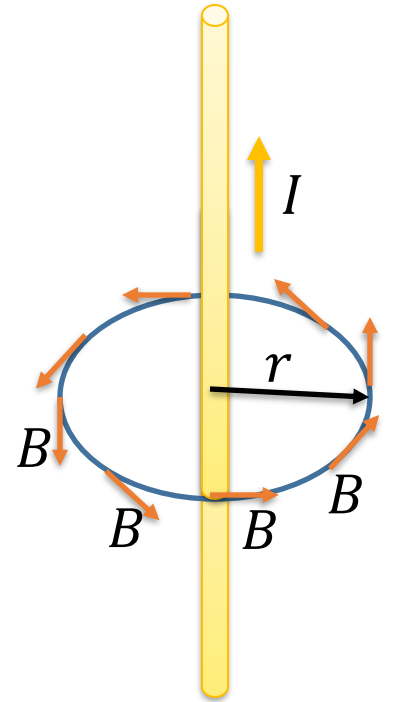
Magnetic fields make a circular rotation around the current lead. The magnitude of the magnetic field is of the same value of $\mu_0 I / 2\pi r$. Line integral of the field gives

$$\oint \vec{B} \cdot d\vec{l} = \int_0^{2\pi} \frac{\mu_0 I}{2\pi r} r d\theta = \mu_0 I$$

Ampere's Law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$



There Are No Magnetic Monopoles.

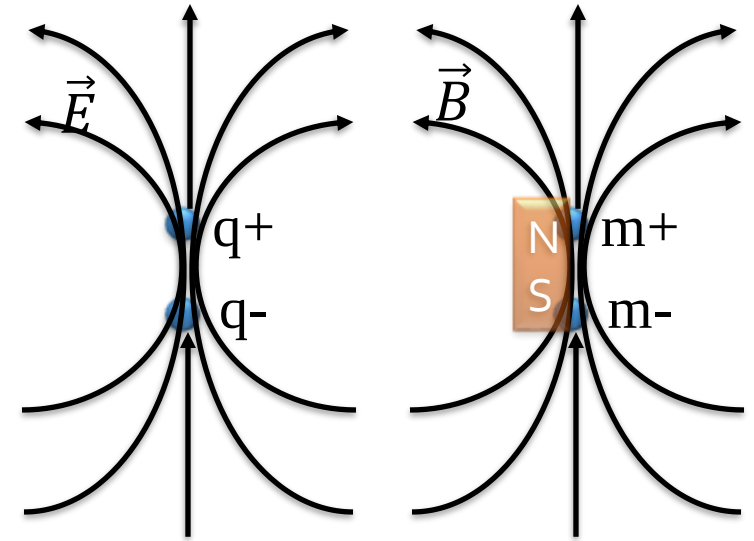
Gauss's Law for Magnetic Fields

Magnetic moment (magnetization) possesses the same concept as magnetic dipoles.

In analogy to electric dipole, the Gauss's law gives us zero magnetic charges in a box due to the same number of $m +$ and $m -$.

$$\oiint \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} (+Nq + (-Nq)) = 0$$

$$\oiint \vec{B} \cdot d\vec{A} = \mu_0 (+Nm + (-Nm)) = 0$$



Magnetic Force Between Two Parallel, Current-Carrying Wires

Use Ampere's law to find the magnetic field around an infinite long wire carrying current I . Use Lorentz law to find the force per unit length between two parallel current-carrying and straight wires.

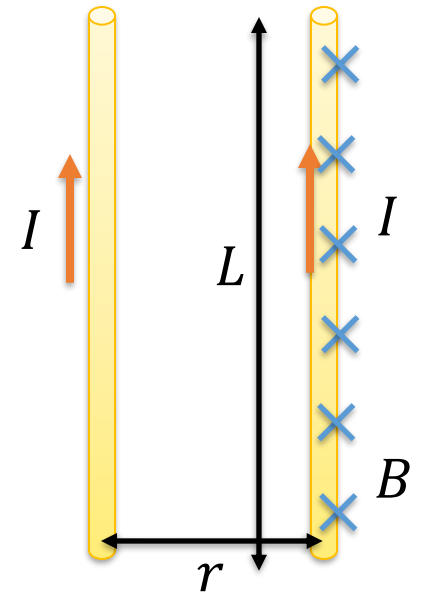
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I \rightarrow 2\pi r B = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$F = ILB = \frac{\mu_0 I^2}{2\pi r} L \quad \Rightarrow \quad \frac{F}{L} = \frac{\mu_0 I^2}{2\pi r}$$

If $r = 1 \text{ m}$, $I = 1 \text{ A}$,

$$\frac{F}{L} = 2(10^{-7}) \frac{1^2}{1} = 2 \times 10^{-7} \text{ N/m}$$

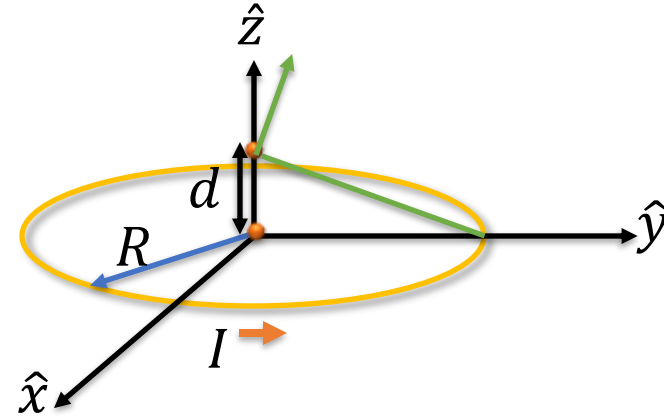


Magnetic
Force

The Magnetic Field of a Solenoid

Solenoid

$$\vec{B}_d = \frac{\mu_0 I R^2}{2(R^2 + d^2)^{\frac{3}{2}}} \hat{k}$$

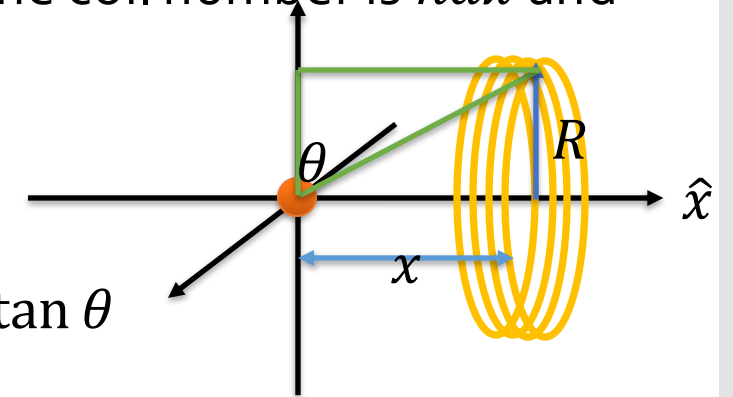


Let the current be I and the number of coils per unit length n for the solenoid. With a small displacement dx , the coil number is ndx and the current is $Indx$.

$$d\vec{B} = \hat{i} \frac{\mu_0 R^2}{2(R^2 + x^2)^{\frac{3}{2}}} Indx$$

$$B = \frac{\mu_0 n I R^2}{2} \int_{x_1}^{x_2} \frac{dx}{(R^2 + x^2)^{\frac{3}{2}}}$$

Let $x = R \tan \theta$



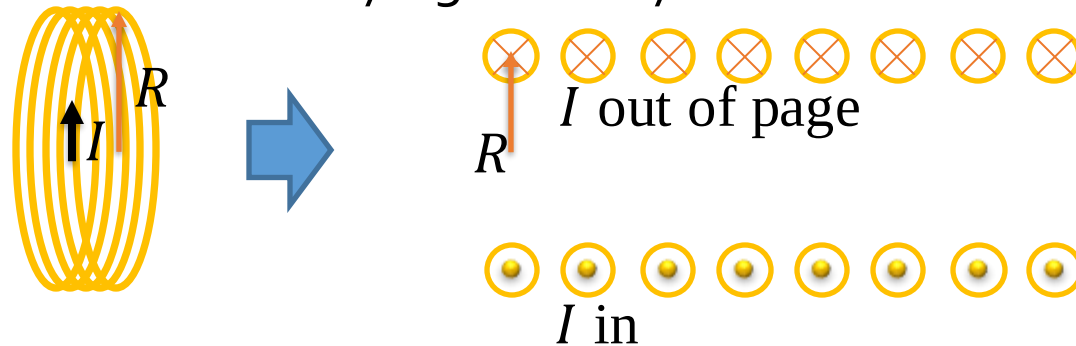
$$B = \frac{\mu_0 n I R^2}{2} \int_{\theta_1}^{\theta_2} \cos \theta d\theta = \frac{1}{2} \mu_0 n I (\sin \theta_2 - \sin \theta_1)$$

For an infinite long solenoid, $\theta_1 = -\pi/2$ and $\theta_2 = \pi/2$, we get $B = \mu_0 n I$

The Magnetic Field of a Solenoid

Solenoid

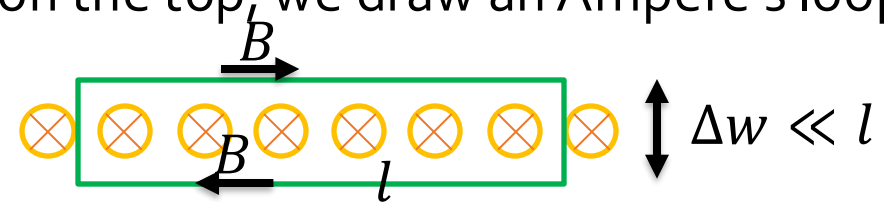
Using the Ampere's law to find the magnetic field of a very long solenoid. Consider n closely wound turns per unit length on a cylinder of radius R and carrying a steady current I . Turn to see the crosssection.



For the wires of current on the top, we draw an Ampere's loop and find the magnetic field by

$$Bl + Bl = \mu_0 n l I$$

$$B = \frac{1}{2} \mu_0 n I$$



Add up the magnetic field due to the wires of current on the bottom:

$$B = \mu_0 n I$$

The Magnetic Field of a Solenoid

Magnetism in Atoms

Estimate the orbital magnetic moment of the electron in an hydrogen atom.

$\vec{L} = \vec{r} \times (m\vec{v}) \rightarrow L = mvr$, the quantization rule gives you $L = n\hbar$

Estimate the current due to the charge $-e$.

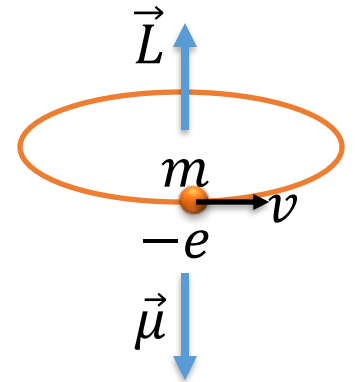
$$I = -\frac{e}{T} = -\frac{e}{\frac{2\pi r}{v}} = -\frac{ev}{2\pi r}$$

Estimate the magnetic moment.

$$\mu = IA \rightarrow \mu = -\frac{ev}{2\pi r} \pi r^2 = -\frac{evr}{2} = -\frac{e}{2m} mvr = -\frac{e}{2m} L$$

Consider the quantization effect, you have $\mu = -n \frac{e\hbar}{2m}$. Here we define the Bohr magneton $\mu_B = \frac{e\hbar}{2m}$ as a unit for atomic magnetic moments.

$$\mu = -n\mu_B$$



Magnetism in Atoms

Similar to the quantization of orbital angular momentum as:

$$\mu_L = -\frac{e}{2m} L = -n \frac{e\hbar}{2m} = -\frac{\mu_B}{\hbar} n\hbar = -g_L \frac{\mu_B}{\hbar} L$$

Considering the relativistic effect, the Dirac equation results in another quantization, spin quantization, giving another similar equation:

$$\mu_S = -g_S \frac{\mu_B}{\hbar} S$$

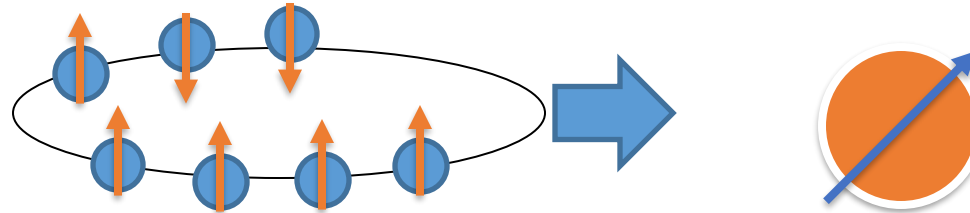
The differences are listed below.

$$g_L = 1, g_S = 2, L = n\hbar, S = \frac{n}{2} \hbar$$

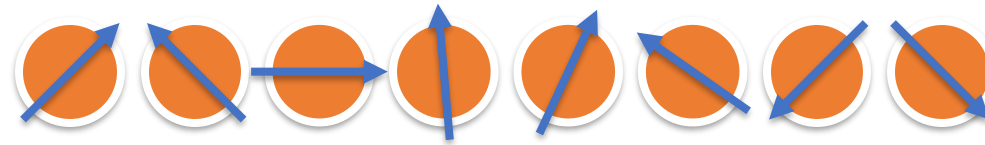
	Fe (BCC)	Co (HCP)	Ni (FCC)
Bohr Magnetons per Atom	2.22	1.72	0.60

Magnetism in Matter

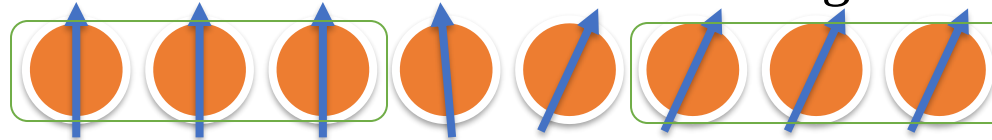
Unfilled orbitals, especially d -orbital electrons, give atomic magnetic moment:



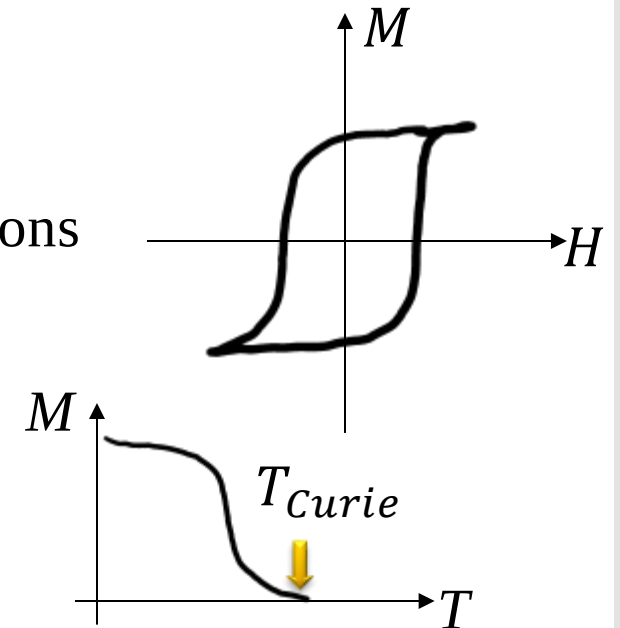
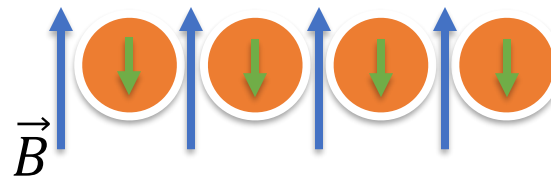
Paramagnetic materials:



Ferromagnetic materials: exchange interactions



Diamagnetic materials – Lenz' rule:



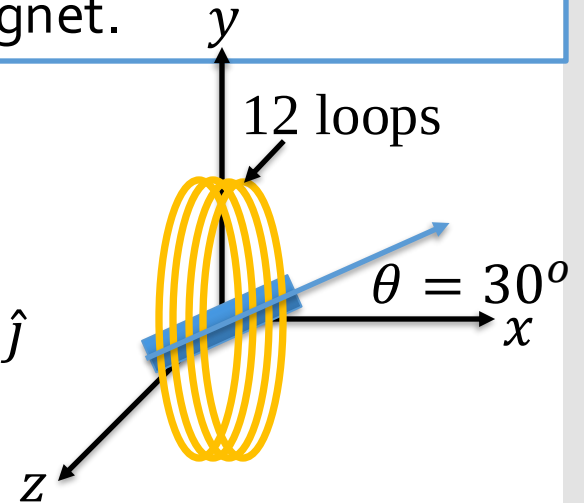
The Magnetic Torque

Examples

A magnet of magnetic moment m is placed at the origin on the xz plane with a direction making an angle of $\theta = 30^\circ$ to the x -axis. A coil of 12 loops is fixed at the origin with its central axis along the x -axis. Please calculate the magnetic torque on the magnet.

$$\vec{B} = 12 \frac{\mu_0 I}{2R} \hat{i}$$

$$\vec{\tau} = \vec{m} \times \vec{B} = -\frac{6m\mu_0 I}{R} \sin(30^\circ) \hat{j} = -\frac{3m\mu_0 I}{R} \hat{j}$$



Calculation of The Magnetic Field

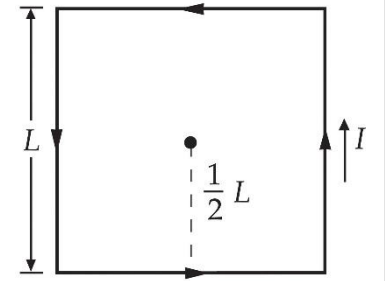
Find the magnetic field at the center of a square loop of edge length L , which carries a current I .

Use the previously derived equation with $\theta_1 = -\pi/4$ and $\theta_2 = \pi/4$ for one edge of the square loop.

$$B = \frac{\mu_0 I}{4\pi r} (\sin(\theta_2) - \sin(\theta_1))$$

$$B_{\text{one edge}} = \frac{\mu_0 I}{4\pi(L/2)} (\sin(\pi/4) - \sin(-\pi/4)) = \frac{\mu_0 I}{2\pi L} \sqrt{2}$$

$$B_{\text{total}} = 4 \frac{\mu_0 I}{2\pi L} \sqrt{2} = \frac{2\mu_0 I}{\pi L} \sqrt{2}$$



Examples

The Magnetic Torque

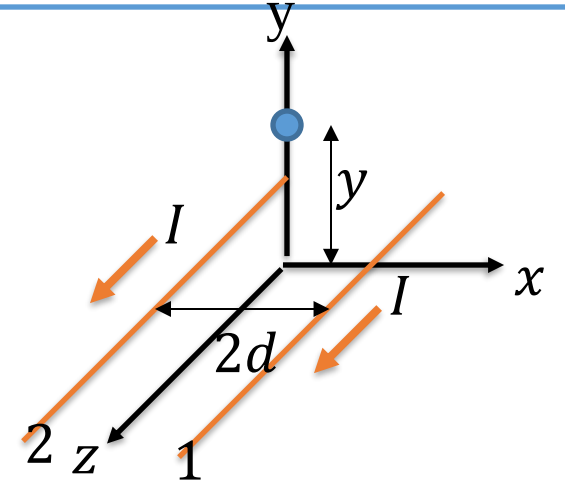
Examples

Two infinite wires carrying current I are placed on the xz plane with their central axes parallel to the z -axis. They are symmetrically arranged on both sides of the z -axis with a separation of $2d$. Please calculate the magnetic field at the position of $(0, y, 0)$.

$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} \quad r = \sqrt{d^2 + y^2}$$

$$\vec{B}_1 = \frac{\mu_0 I}{2\pi\sqrt{d^2 + y^2}} \left(-\frac{\sqrt{2}}{2} \hat{i} - \frac{\sqrt{2}}{2} \hat{j} \right)$$

$$\vec{B}_{\text{total}} = \frac{\mu_0 I}{2\pi\sqrt{d^2 + y^2}} (-\sqrt{2} \hat{i})$$



The Lorentz Force

Examples

Two straight rods of L in length are separated at a distance d apart in a current balance. They carry currents I each in opposite directions. What mass must be placed on the upper rod?

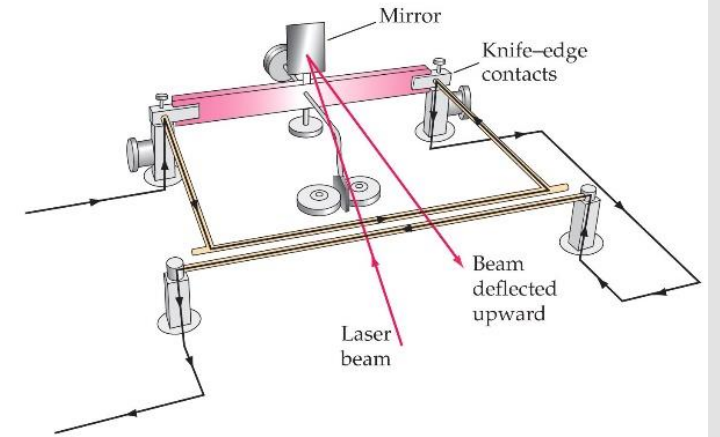
The magnetic field due to an infinite wire is

$$B = \frac{\mu_0 I}{2\pi d}$$

The repulsive Lorentz force is

$$F = IlB = IL \frac{\mu_0 I}{2\pi d}$$

$$mg = \frac{\mu_0 I^2 L}{2\pi d} \rightarrow m = \frac{\mu_0 I^2 L}{2\pi g d}$$



Ampere's Law & Applications for The Calculation of Magnetic Fields

A long, straight wire of radius R carries a current I that is uniformly distributed over the circular cross section of the wire. Find the magnetic field both outside and inside the wire.

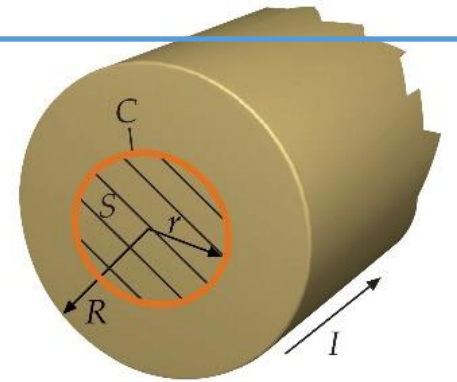
Current & Current Density:

$$I = JA \rightarrow J = \frac{I}{A} = \frac{I}{\pi R^2}$$

$$r > R: \int \vec{B} \cdot d\vec{l} = B2\pi r = \mu_0 I \rightarrow B = \frac{\mu_0 I}{2\pi r}$$

$$r < R: \int \vec{B} \cdot d\vec{l} = \mu_0 I \rightarrow 2\pi r B = \mu_0 (J\pi r^2)$$

$$B = \frac{\mu_0 J r}{2} = \frac{\mu_0 I r}{2\pi R^2}$$



Examples

Ampere's Law & Applications for The Calculation of Magnetic Fields

A wire carrying a current I is bent to form a toroid with N loops. Please find the magnetic field inside and outside the toroid.

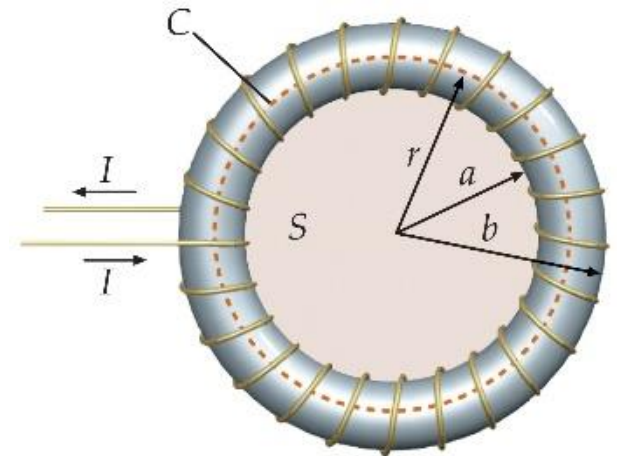
Outside the toroid:

$$r > b: 2\pi r B = \mu_0 \times 0 \rightarrow B = 0$$

$$r < a: 2\pi r B = \mu_0 \times 0 \rightarrow B = 0$$

Inside the toroid:

$$b > r > a: 2\pi r B = \mu_0 \times NI \rightarrow B = \frac{\mu_0 NI}{2\pi r}$$



Examples

Ampere's Law & Applications for The Calculation of Magnetic Fields

A very long, thin strip of metal of width w carries a current I along its length. The current is distributed uniformly across the width of the strip. Find the magnetic field at point P in the diagram. Point P is in the plane of the strip at distance b away from its edge.

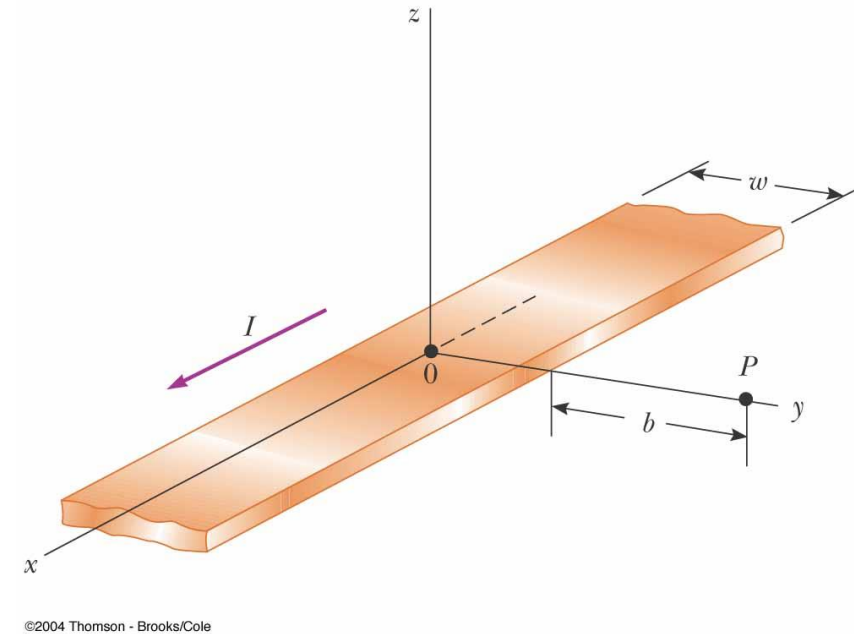
Examples

Use Ampere's law

$$2\pi y(dB) = \mu_0 \frac{I}{w} dy$$

$$B = \int_b^{b+w} \frac{\mu_0 I}{2\pi y w} dy$$

$$B = \frac{\mu_0 I}{2\pi w} \ln\left(\frac{b+w}{b}\right)$$



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