Chapter 28 Magnetic Force – Lorentz Force Law

Physics II – Part II

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Lorentz Force

History

1865 AD – James Clerk Maxwell (Scottish)
formulations of field equations

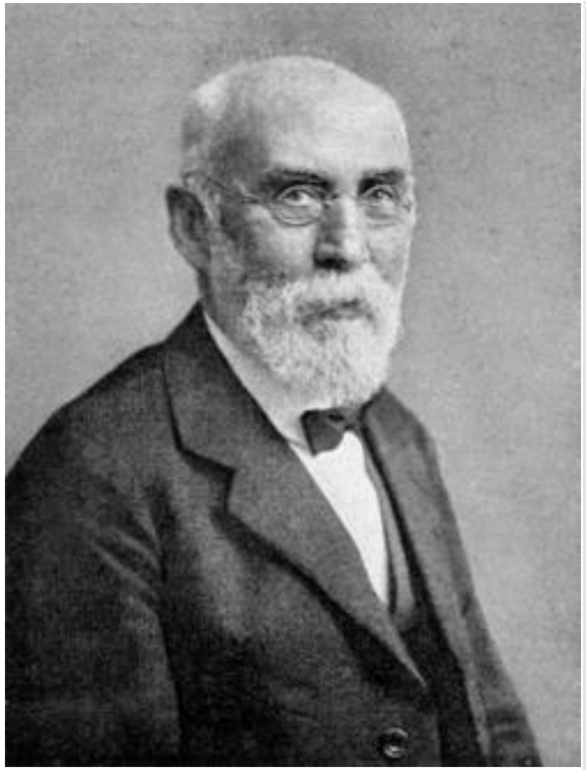
1881 AD – Sir Joseph John Thomson (English)
derive from Maxwell's equations

$$\vec{F} = \frac{1}{2} q\vec{v} \times \vec{B}$$

1881 AD – Oliver Heaviside (English)
derive the correct form of the force law

1892 AD – Hendrik Antoon Lorentz (Dutch)
modern form, named Lorentz force law due to the consideration of relative transformation

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$



http://en.citizendium.org/images/thumb/4/40/HALorentz.gif/250px-HALorentz.gif

Is Lorentz law incompatible with special relativity?
https://phys.org/news/2012-05-classical-electrodynamics-law-incompatible-special.html
Masud Mansuripur, Phys. Rev. Lett. 108, 193901 (2012).

Ref: https://en.wikipedia.org/wiki/Lorentz_force; http://ffden-2.phys.uaf.edu/webproj/212_spring_2017/Curtis_Fortenberry/curtis_fortenberry/page1/page1.html

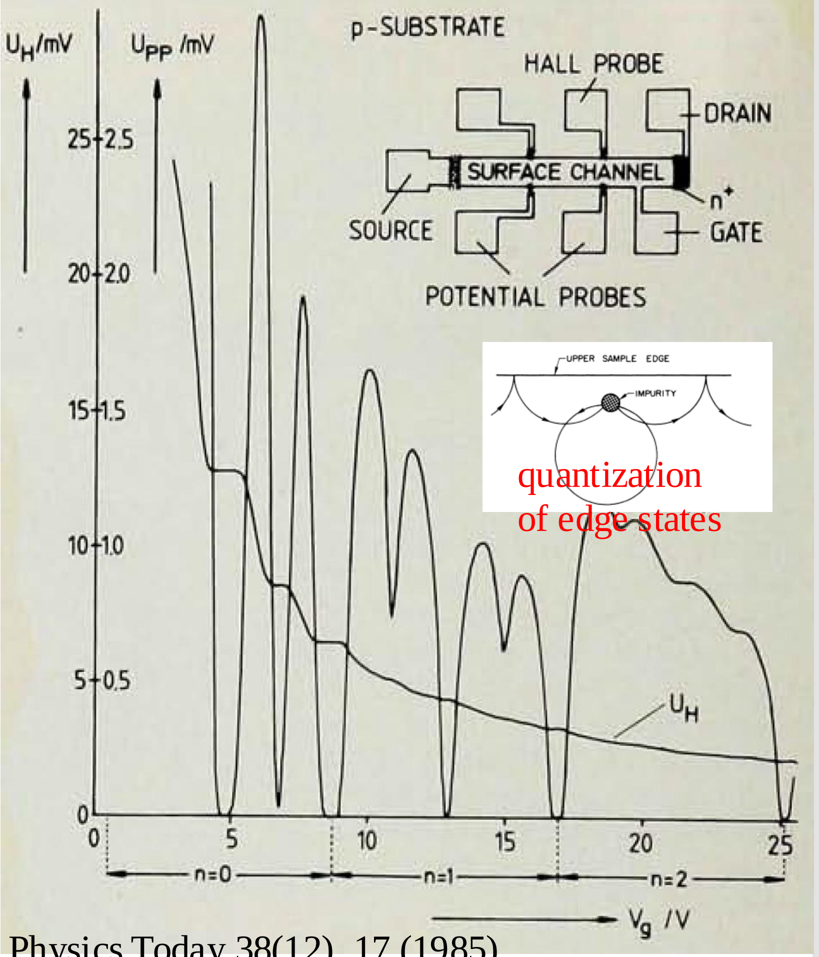
Hall Effect, Quantum Hall Effect, Fractional Quantum Hall Effect

History

1879 AD – Edwin Herbert Hall (American)
an experimentalist for his doctoral degree at Johns Hopkins University in Baltimore, Maryland

1978 AD – Klaus von Klitzing (German)
discover the quantum Hall effect

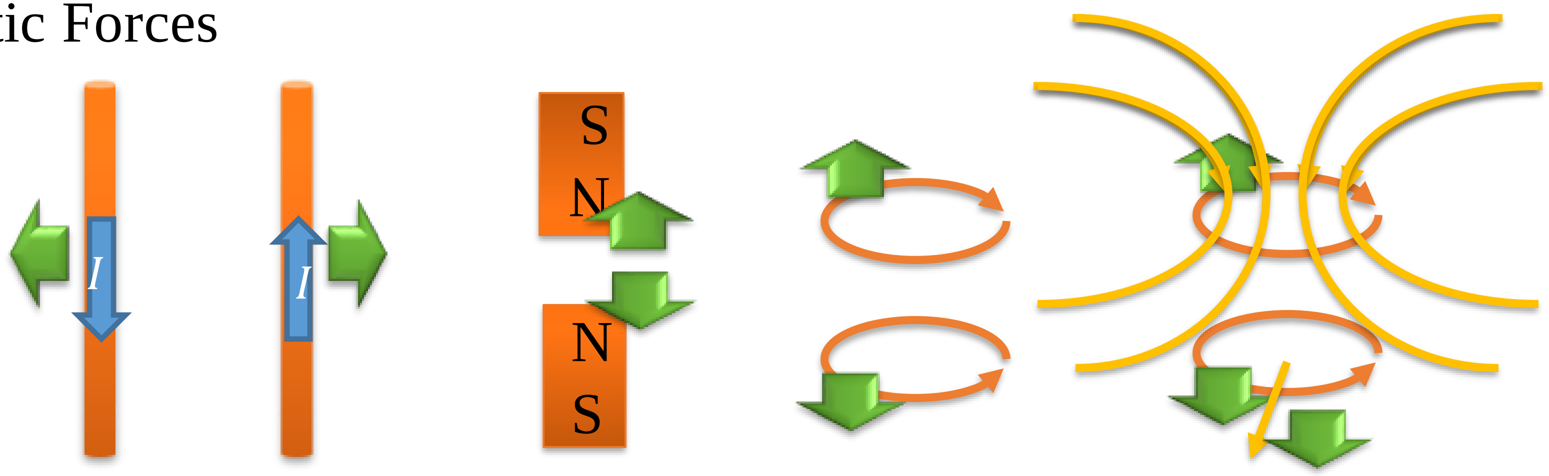
1982 AD – Robert B. Laughlin, Daniel C. Tsui, Horst L. Störmer (American)
Laughlin, a theorist, explained the experimental results of fractional quantum Hall effect done by Tsui & Störmer.
https://www.britannica.com/biography/Robert-B-Laughlin



Physics Today 38(12), 17 (1985).

Ref: https://en.wikipedia.org/wiki/Hall_effect; https://www.nist.gov/pml/engineering-physics-division/hall-effect-measurements-introduction

Lorenz Force Law



The Ampere’s repulsive force between two opposite current flows

Alternative description for the repulsive force between two magnets

The mechanism could be separated to the interaction between the current and the magnetic force.

The Lorentz force law describes: charge in motion is exerted by a force from magnetic field

$$\vec{F}_{Lorentz} = q\vec{v} \times \vec{B}$$

Magnetic Fields

Lorentz Force Law

Source of Magnetic Field	Magnitude of Magnetic Field (T)
Transient Superconducting Magnet	100
Static Lab Superconducting Magnet	20
Conventional Electromagnet	2
Conventional Bar Magnet	0.01-1.5 (short range)
Medical MRI	1.5
Magnetic Field on The Earth	0.44×10^{-4} (0.5 Gauss)
Human Brain	10^{-15}

Static charges do not response to magnetic fields.

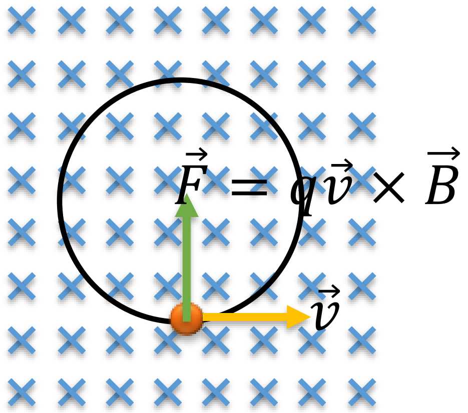
Magnetic force is perpendicular to both the magnetic field and the charge motion direction thus producing cycloid motion.

The Lorenz force law is a relative transformation of the static Coulomb law.

Cycloid Motion

Centripetal force is the driving force to make a circular motion.
Lorentz forc is the centripetal force for moving charges.

Cyclotron frequency is independent of the velocity of the charged particle.

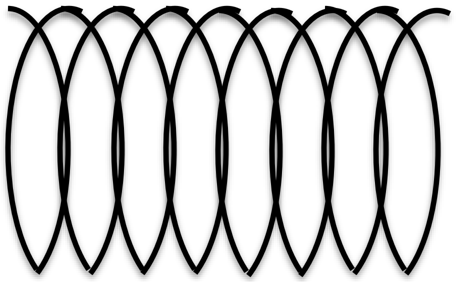


$$F = qvB = ma_r \qquad \vec{v} \perp \vec{B}$$

$$q(r\omega)B = m(r\omega^2) \rightarrow \omega = \frac{qB}{m}$$

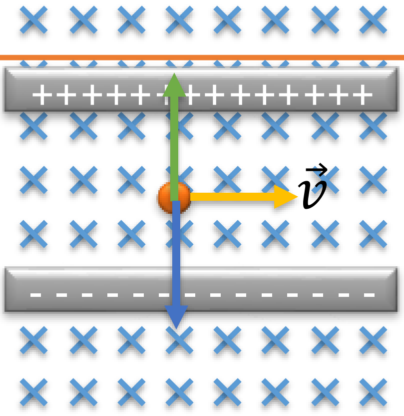
only depends on the charge, the mass, and the magnetic field

If the motional particle possess a velocity component parallel to the magnetic field, it will undergoe a constant velocity motion in that direction.



Application of Lorenz Force

The velocity selector. Use magnetic and electric fields to select the charged particles with specified velocities.



$$qE = qvB \rightarrow v = \frac{E}{B}$$

https://www.nyu.edu/classes/tuckerman/adv.chem/lectures/lecture_3/node1.html

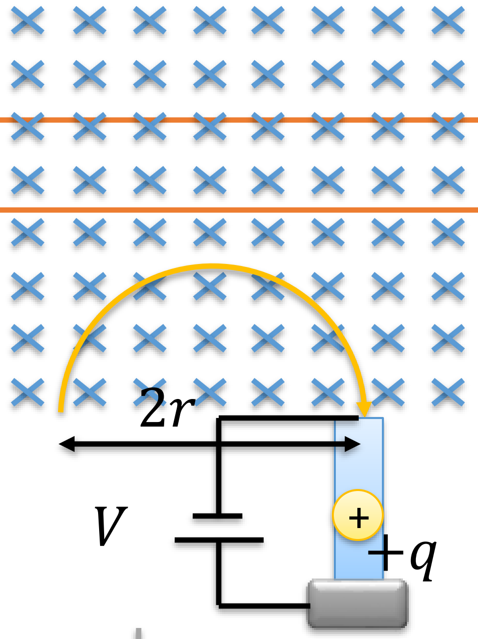
Application of Lorenz Force

Mass spectrometer

$$qvB = \frac{mv^2}{r} \rightarrow v = \frac{qBr}{m}$$

$$K = qV = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{qBr}{m}\right)^2$$

$$V = \frac{1}{2}\frac{qB^2r^2}{m} \rightarrow m = \frac{q}{2}\frac{B^2r^2}{V}$$

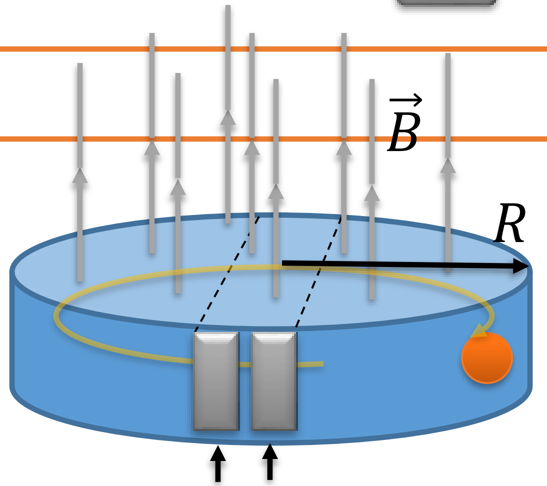


Cyclotron accelerator

$$\omega = \frac{qB}{m}$$

Large radius gives high energy for the charged particles.

$$K = \frac{1}{2}m(\omega R)^2 = \frac{q^2B^2}{2m}R^2$$



AC electric field

Another Form of Lorentz Force Law

Lorentz Force on Current-Carrying Wires

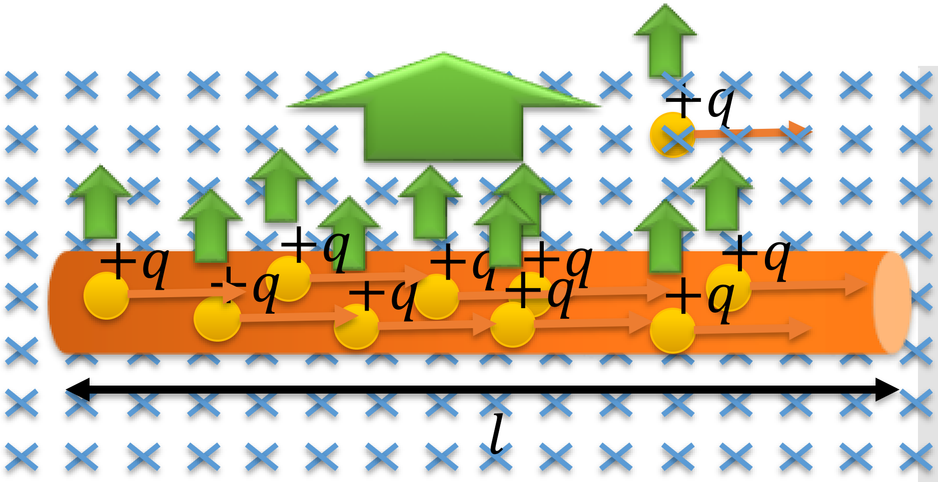
In the segment of a conductor, there are N charged particles

$N = nLA$

$\vec{F}_{net} = Nq\vec{v} \times \vec{B} = lAnq\vec{v} \times \vec{B}$

$\vec{F}_{net} = lA\vec{J} \times \vec{B} = I\vec{l} \times \vec{B}$

$d\vec{F} = I(d\vec{l}) \times \vec{B} \longrightarrow \vec{F}_{net} = I\int d\,\vec{l} \times \vec{B}$



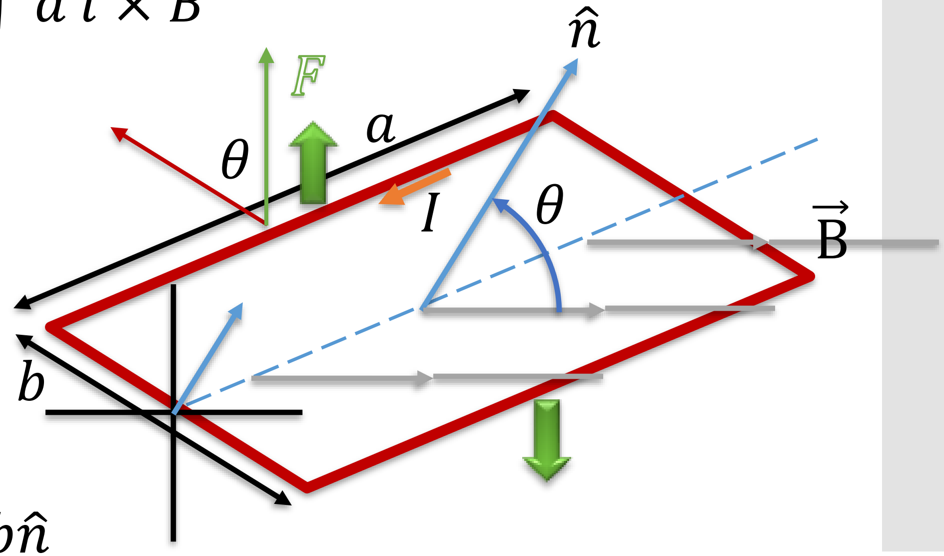
Torque on the current loop:

$F = IaB$

$\tau = \frac{b}{2}(IaB)\sin(\theta) \times 2$

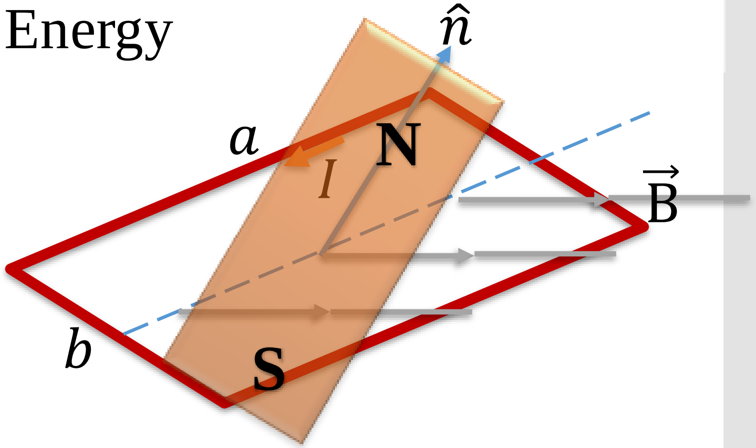
$\vec{\tau} = (Iab)B\sin(\theta) = (Iab\hat{n}) \times \vec{B}$

Define magnetic moment $\vec{m} = Iab\hat{n}$



Torque on a Current Loop

Lorentz Force on a Current Loop, Magnetic Moment, Torque, Potential Energy



$$\vec{\tau} = \vec{m} \times \vec{B}$$

Potential energy: exerted external torque

$$\tau_{ext} = -\tau = -mB\sin(\theta)$$

$$dW = (-mB\sin(\theta))d\theta$$

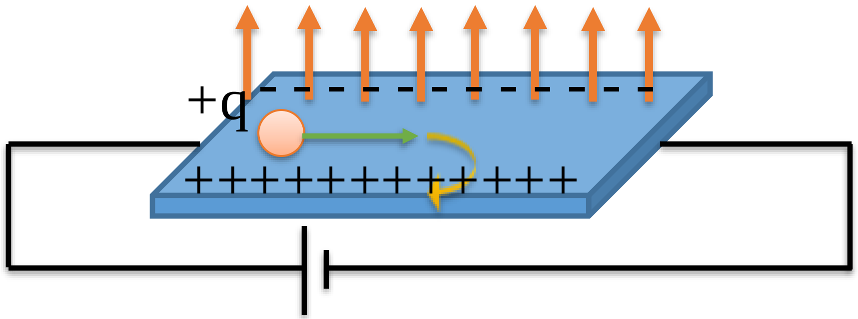
$$dU = mB\sin(\theta)\,d\theta$$

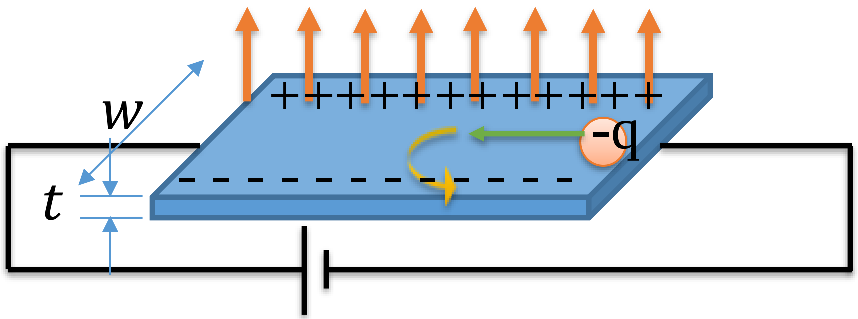
$$U = \int_{\pi/2}^{\theta} mB\sin(\theta)\,d\theta = -mB\cos(\theta)$$

$$U = -\vec{m} \cdot \vec{B}$$

Hall Effect

Determine The Polarity of Charged Particles and The Carrier Concentration





The Hall effect induced electric field:

$$F = qv_dB = qE_H \rightarrow E_H = v_dB$$

The Hall voltage is $V_H = v_dBw$

$$I = (wt)nqv_d \rightarrow v_d = I/wtnq \qquad V_H = \frac{IBw}{wtnq} = \frac{IB}{nqt}$$

$$V_H = IR \rightarrow R = \frac{B}{nqt} = \frac{1}{nq}\frac{B}{t} = R_H\frac{B}{t} \qquad R_H = \frac{1}{nq}$$

$$n = \frac{IB}{qtV_H}$$

Quantum Hall: $R = \frac{V_H}{I} \rightarrow G = \frac{I}{V_H} = \frac{nqt}{B} = m(\frac{e^2}{h})$

Ref:

Lorentz Force on a Segment of Current-Carrying Conductor

Examples

A proton of mass $m = 1.67 \times 10^{-27}$ kg and charge $q = e = 1.602 \times 10^{-19}$ C moves in a circle of radius $r = 21$ cm perpendicular to a magnetic field of $B = 4000$ G. Find (a) the period of the motion and (b) the speed of the proton.

$$qvB = m\frac{v^2}{r} \rightarrow v = \frac{qBr}{m} = \frac{(1.602 \times 10^{-19})(0.4)(0.21)}{(1.67 \times 10^{-27})}$$

$$v = 8.05 \times 10^{6} \text{ m/s}$$

$$T = \frac{2\pi r}{v} = \frac{2(3.14159)(0.21)}{8.05 \times 10^{6}} = 1.64 \times 10^{-7} \text{ s}$$

Ref:

Examples

A ^{58}Ni ion of charge +e and mass 9.62×10^{-26} kg is accelerated through a potential drop of 3 kV and deflected in a magnetic field of 0.12 T. (a) Find the radius of curvature of the orbit of the ion. (b) Find the difference in the radii of curvature of ^{58}Ni ions and ^{60}Ni ions.

(a)

$$qV = \frac{1}{2}mv^2 \;\&\; \omega = \frac{qB}{m} \rightarrow qV = \frac{1}{2}mr^2\frac{q^2B^2}{m^2}$$

$$r_{^{58}Ni} = \sqrt{2mV/qB^2} = \sqrt{\frac{2\times(9.62\times10^{-26})\times(3000)}{(1.602\times10^{-19})\times(0.12)^2}} = 0.501 \text{ m}$$

(b)

$$\frac{r_{^{60}Ni}}{r_{^{58}Ni}} = \sqrt{\frac{60}{58}} \rightarrow r_{^{60}Ni} = \sqrt{\frac{60}{58}} \times 0.501 = 0.510 \text{ m}$$

$$\Delta r = 0.009 \text{ m}$$

Ref:

The Cyclotron

Examples

A cyclotron for accelerating protons has a magnetic field of 1.5 T and a maximum radius of 0.5 m. (a) What's the cyclotron frequency? (b) What's the kinetic energy of the protons when they emerge?

(a)

$$\omega = \frac{qB}{m} = \frac{(1.602\times10^{-19})\times(1.5)}{1.67\times10^{-27}} = 1.44\times10^{8}\ \text{rad/s}$$

$$f = \frac{\omega}{2\pi} = 2.29\times10^{7}\ \text{Hz}$$

(b)

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2r^2 = 4.33\times10^{-12}\ \text{J} = 27.0\ \text{MeV}$$

Ref:

Lorentz Force on a Segment of Current-Carrying Conductor

Examples

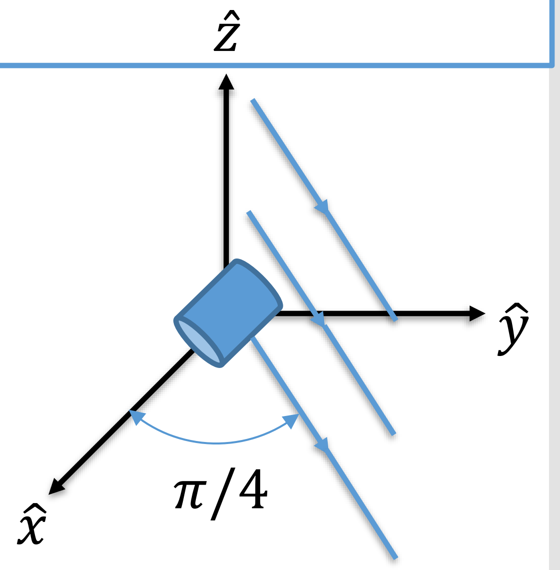
A wire segment of 1 mm in length carries a current of 1.0 A in the +x direction. If a magnetic field of 1.0 T is applied perpendicular to the z axis and making an angle of $\pi/4$ with the x-axis. What is the magnitude of the Lorentz force?

$$F = ILB\sin(\theta)$$

$$F = (1)(0.001)(1)\sin(\pi/4)$$

$$F = 7.1 \times 10^{-4}\ N$$

$$\vec{F} = 7.1 \times 10^{-4}\hat{k}\ N$$



Ref:

Lorentz Force on a Curved Wire of Current-Carrying Conductor

Examples

A wire bent into a semicircular loop of radius R lies in the xy plane and carries a current I. It is placed in a field $\vec{B} = B_0\hat{k}$. Please find the Lorentz force exerted on the wire.

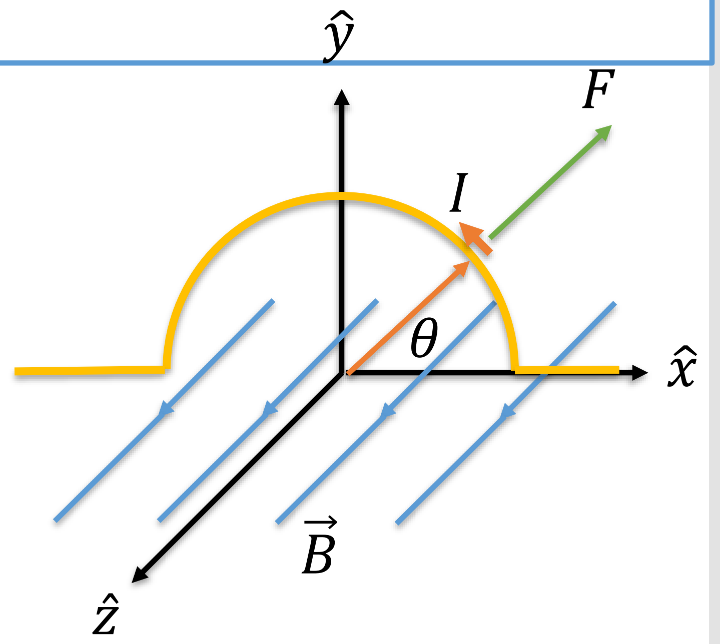
$$dF = IdlB_0$$

$$dl = Rd\theta$$

$$F_x = 0, F_y = \int dF_y$$

$$dF_y = I(Rd\theta)B_0 \sin(\theta)$$

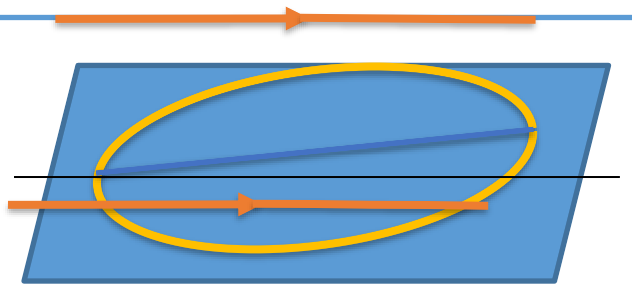
$$F_y = \int_0^{\pi} IRB_0 \sin(\theta)\, d\theta = 2IRB_0$$



Magnetic Torque on a Current Loop

Examples

A circular loop of wire with a radius R and mass m carries a current I and lies in the horizontal plane. A horizontal magnetic field B is applied in the space. How large can the current be before one edge of the loop is lifted off the plane?



$$m_B = IA = I\pi R^2$$

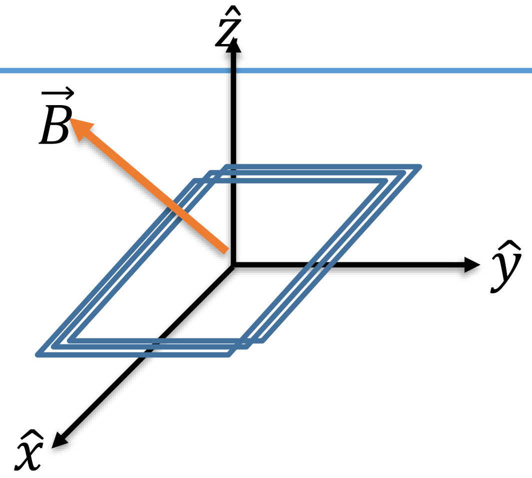
$$\tau_B < \tau_g \rightarrow I\pi R^2 B < mgR$$

$$I < \frac{mg}{\pi R B}$$

Magnetic Moment & Torque on a Current Loop

Examples

A square 12-turn coil with edge-length 40 cm carries a current of 3 A. It lies in the *xy* plane in a uniform magnetic field $\vec{B} = 0.3\hat{\imath} + 0.4\hat{k}$ (T). Find (a) the magnetic moment of the coil, (b) torque, and (c) the potential energy.



(a)

$$\vec{m} = 12 \times 3 \times (0.4)^2 \hat{k} = 5.76\hat{k}\ (\mathrm{Am}^2)$$

(b)

$$\vec{\tau} = \vec{m} \times \vec{B} = \left(5.76\hat{k}\right) \times \left(0.3\hat{\imath} + 0.4\hat{k}\right)$$

$$\vec{\tau} = 1.73\hat{\jmath}$$

(c)

$$U = -\vec{m} \cdot \vec{B} = -\left(5.76\hat{k}\right) \cdot \left(0.3\hat{\imath} + 0.4\hat{k}\right) = 2.30\ \mathrm{J}$$

Calculation of Magnetic Moment

Examples

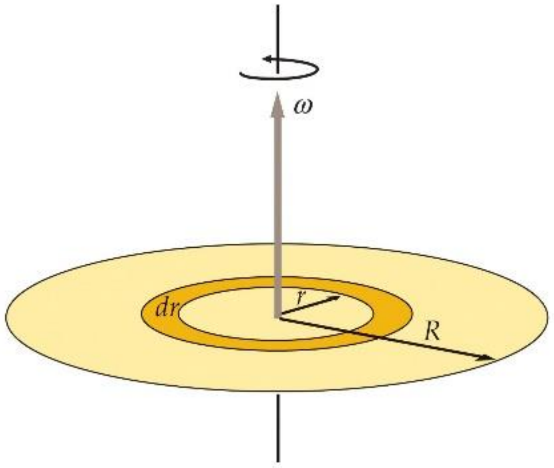
A thin **nonconducting** disk of mass m and radius R has a uniform surface charge per unit area σ and rotates with angular velocity ω about the axis. Find the magnetic moment.

$$\mu = IA$$

$$d\mu = AdI \rightarrow A = \pi r^2, dI = \frac{dQ}{T} = \frac{\sigma 2\pi r dr}{2\pi/\omega}$$

$$d\mu = (\pi r^2)(\sigma\omega r dr)$$

$$\mu = \int_0^R \sigma\omega\pi r^3 dr = \frac{\sigma\omega\pi}{4}R^4$$



Calculation of Torque

Examples

A non-conducting sphere has mass m and radius r. A flat, compact coil of wire with N turns is wrapped tightly around it, with each turn concentric with the sphere. The sphere is placed on an inclined plane that slopes downward to the left, making an angle θ with the horizontal so that the coil is parallel to the inclined plane. A uniform magnetic field B vertically upward exists in the region of the sphere. (a) What current in the coil will enable the sphere to rest in equilibrium on the inclined plane? (b) Explain why the results does not depend on the value θ?

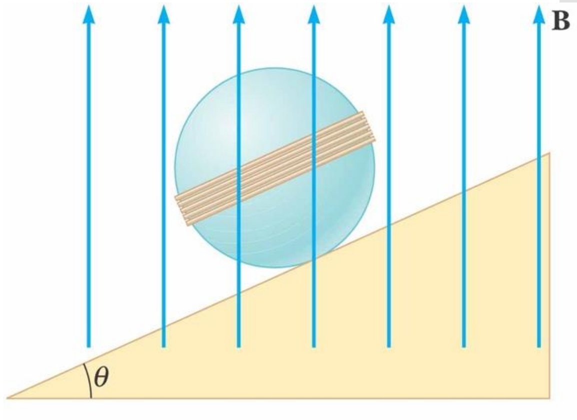
$$\tau_g = rmg\sin(\theta)$$

$$m = NI\pi r^2$$

$$\tau_B = mB\sin(\theta) = NI\pi r^2 B\sin(\theta)$$

$$\tau_B \geq \tau_g \rightarrow NI\pi r^2 B\sin(\theta) \geq mgr\sin(\theta)$$

$$I \geq \frac{mg}{N\pi rB}$$



Calculation of Torque

Examples

The upper portion of the circuit is fixed. The horizontal wire at the bottom has a mass of m and a length of l. The wire hangs in the gravitational field with an acceleration of g of the earth from identical light spring connected to the upper portion of the circuit. The springs stretch Δl_1 under the weight of the wire, and the total resistance of the circuit is R. A magnetic field is turned on and is directed out of the page. The springs then stretch an additional length of Δl_2. Assume that only the horizontal wire at the bottom is in the magnetic field. What is the magnitude of the magnetic field?

$$F = 2k(\Delta l_1) = mg$$

$$k = \frac{mg}{2\Delta l_1}$$

$$F = 2k(\Delta l_2) = IlB$$

$$\frac{mg(\Delta l_2)}{\Delta l_1} = \frac{V}{R} lB$$

$$B = \frac{mgR(\Delta l_2)}{Vl(\Delta l_1)}$$