Chapter 26 Current & Resistance

Physics II – Part I
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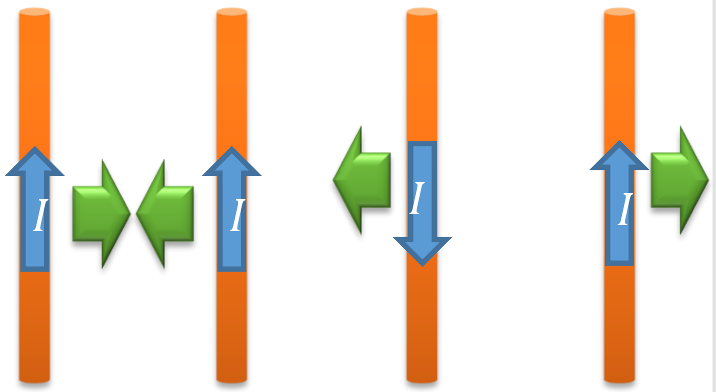
Electric Current & Electromagnetism

History

1820 AD – Hans Christian Ørsted (Danish)
discover that electric current generates magnetic field

1820 AD – André-Marie Ampère (French)
The Founder of Electromagnetism
in 1819 offered courses in philosophy and astronomy at the University of Paris, in 1824 chair in experimental physics at the Collège de France.

1900 AD – Paul Karl Ludwig Drude (German)
Drude model of electron transport



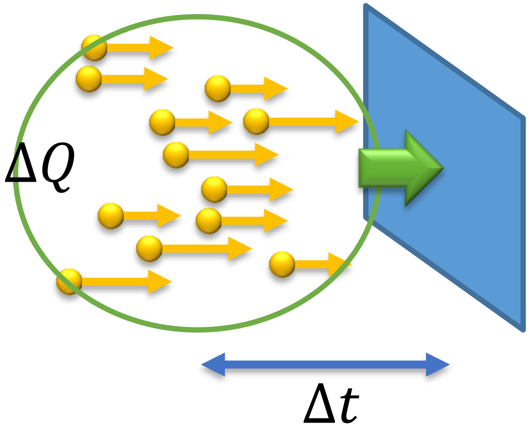
Ref: http://www.juliantrubin.com/bigten/ampereexperiments.html; https://www.britannica.com/biography/Andre-Marie-Ampere; https://en.wikipedia.org/

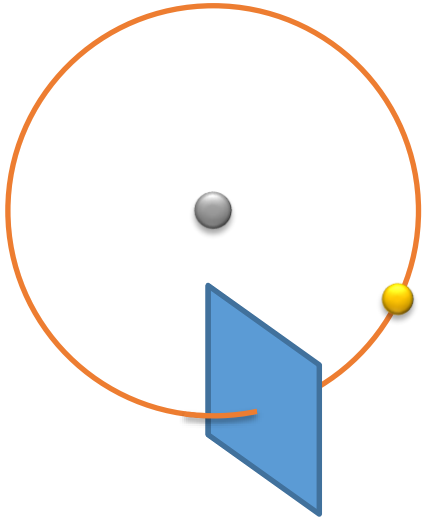
Flow Rate of Charge

Current

In a certain period Δt, current is the flow of charge, crossing an imaginary and infinite plane, divided by the period.

$$I = \frac{\Delta Q}{\Delta t}$$





Speed of the electron in a hydrogen atom:

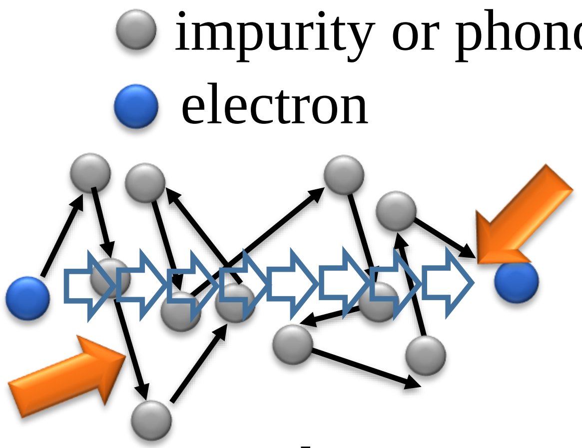
$$\hbar \cong 10^{-34}, m = 10^{-30} kg, r = 5 \times 10^{-11} m$$

$$l = mvr = \hbar \rightarrow v \cong 2 \times 10^{6}\ m/s$$

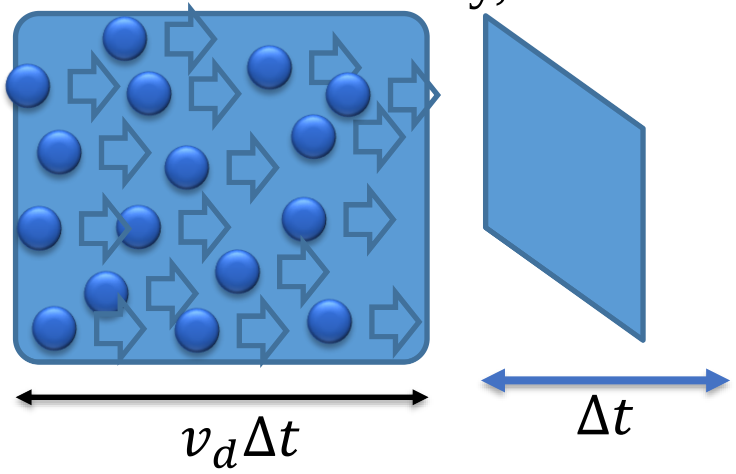
$$I = \frac{e}{T} = \frac{e}{2\pi r/v} = \frac{1.602 \times 10^{-19}}{2 * 3.14 * 5 * 10^{-11}/2 * 10^{6}} \cong 1\ mA$$

Current

Driven by Electric Fields, Suffering from Scattering, Drift Velocity



instantaneous velocity, Fermi velocity ~10^6 m/s



$$\Delta Q = qn(A v_d \Delta t)$$

$$I = \frac{\Delta Q}{\Delta t} = nqv_d A$$

$$J = \frac{I}{A} = nqv_d$$

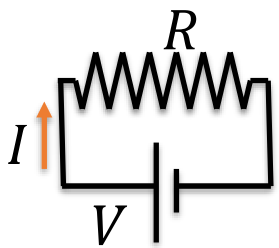
n: number of carriers per unit volume, A: cross-sectional area

Ohm's Law

Resistance

Observation of a limited current driven by a voltage

$V = IR$



A dimensionless parameter – resistivity ρ

$$R = \frac{\rho l}{A} \Rightarrow V = El, V = IR = I\frac{\rho l}{A} \rightarrow El = I\frac{\rho l}{A}$$

$$E = \frac{I}{A}\rho \quad J = \frac{I}{A} \Rightarrow E = J\rho \rightarrow J = \frac{1}{\rho}E$$

conductivity $\sigma = 1/\rho$

Ohm's law $J = \sigma E$

Resistance of a resistor

Brown	Red	Gold	Silver	None
1%	2%	5%	10%	20%

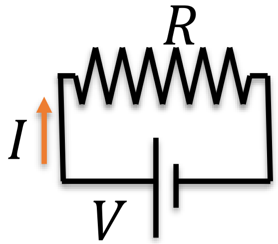
Black	Brown	Red	Orange	Yellow	Green	Blue	Violet	Gray	White
0	1	2	3	4	5	6	7	8	9

Electrical Power of a Battery, Power Consumed on a Resistor

Power

Change of electric potential energy for charge ΔQ driven by a voltage V

$$\Delta U = V\Delta Q$$

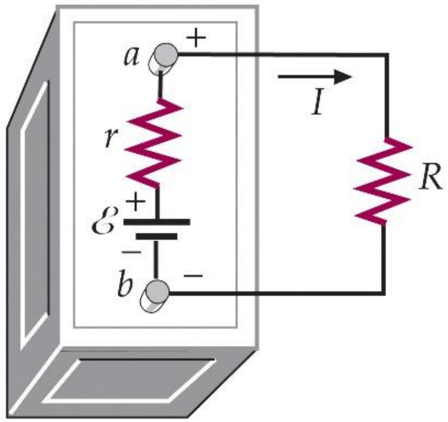


The power – energy per unit time

$$P = \frac{U}{\Delta t} = \frac{\Delta Q}{\Delta t}V = IV$$

The power consumption on the resistor

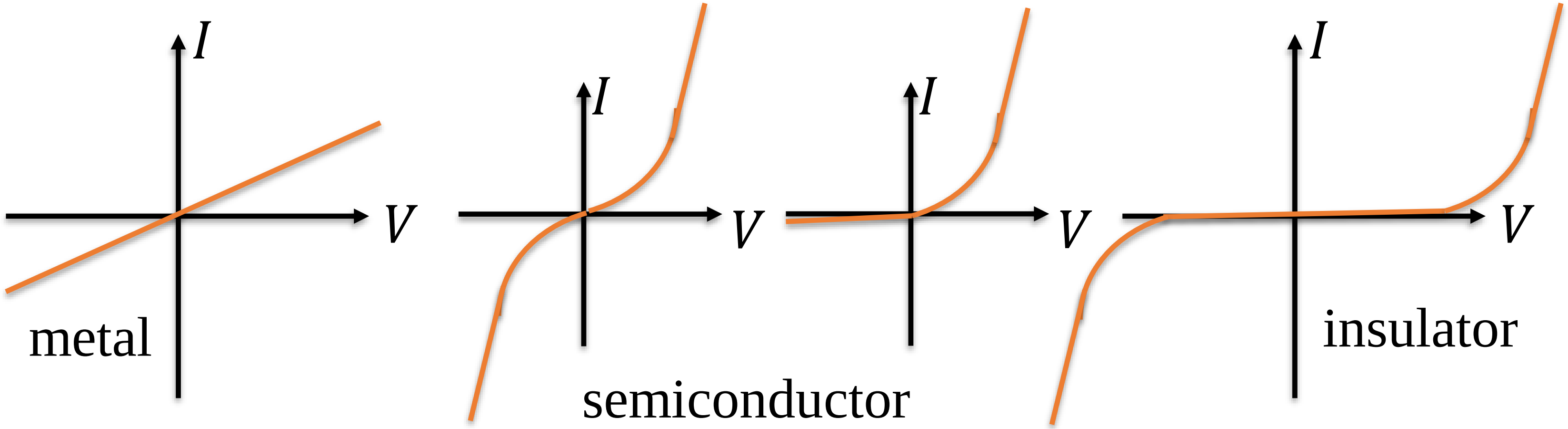
$$V = IR \rightarrow P = I^2R = \frac{V^2}{R}$$



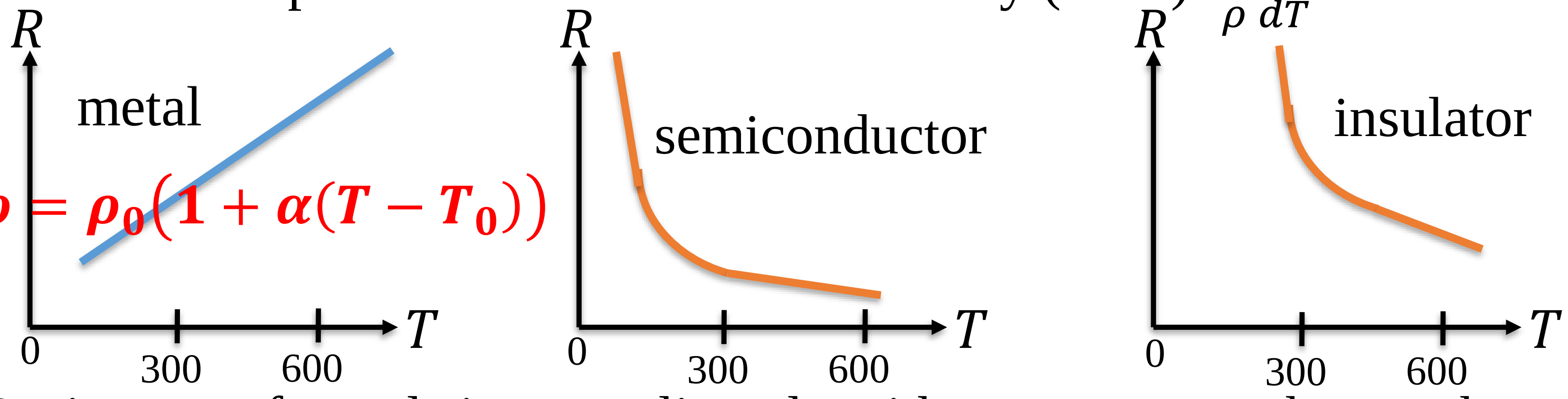
Ref:

Resistance

Resistance of Materials: Conductors, Semiconductors & Insulators



Metals show a linear I-V behavior whereas insulators show a nonlinear behavior. temperature coefficient of resistivity (TCR): $\frac{1}{\rho}\frac{d\rho}{dT}$

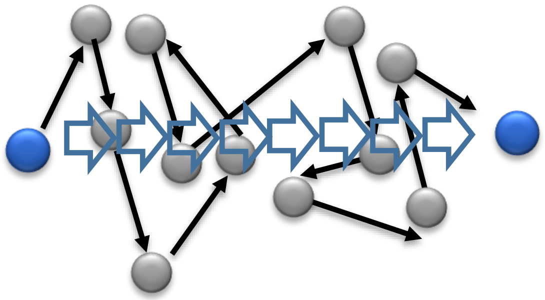


$$\boldsymbol{\rho = \rho_0\left(1 + \alpha(T - T_0)\right)}$$

Resistance of metals increase linearly with temperature whereas that of insulators decrease exponentially with temperature.

Classical, Microscopic Picture of Resistance – the Drude Model

Resistance



Electric field drives electron's motion.

Electrons suffer from phonon & impurity scatterings – like resistive force in liquid

$F = qE - bv$ Let $b = m/\tau$

$$F = qE - \frac{mv}{\tau} = ma$$

The drift velocity or terminal speed:

$$a = 0 \rightarrow qE - \frac{mv_d}{\tau} = 0$$

$$v_d = \frac{qE\tau}{m} \qquad J = nqv_d = \frac{nq^2\tau}{m}E = \sigma E$$

$$\sigma = \frac{nq^2\tau}{m}$$

for electron conduction case $\sigma = ne^2\tau/m$

Superconductor

Zero Resistance & Magnetic Levitation

Electrons for a quantum collective state that is not sensitive to impurity scatterings.

Discovered in 1911 by Heike Kamerlingh Onnes (Dutch) of Leiden University.

At temperature lower than the transition temperature of a superconductor, the resistance suddenly jumps to zero.

Accompanied by Meissner effects – zero magnetic flux

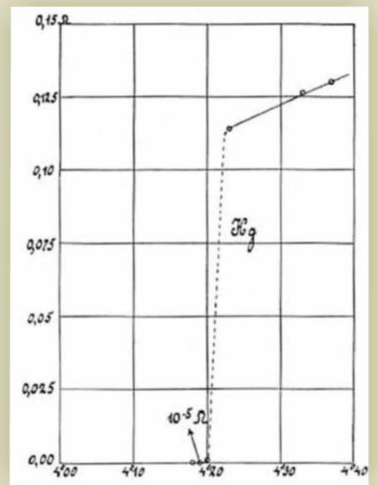
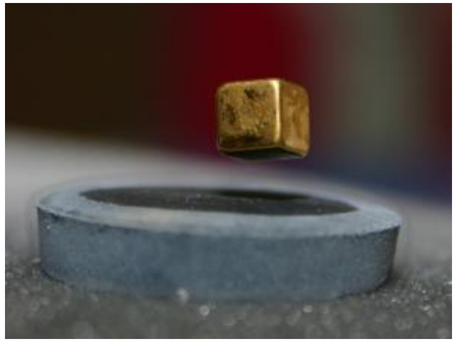
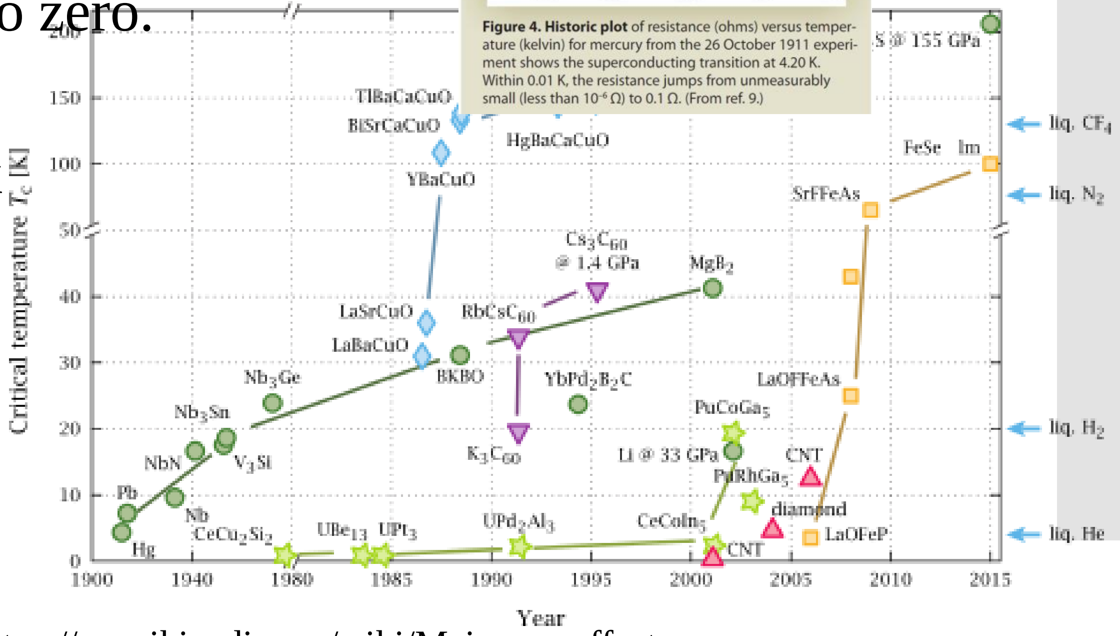


Figure 4. Historic plot of resistance (ohms) versus temperature (kelvin) for mercury from the 26 October 1911 experiment shows the superconducting transition at 4.20 K. Within 0.01 K, the resistance jumps from unmeasurably small (less than 10^{-6} Ω) to 0.1 Ω. (From ref. 9.)





Ref: https://en.wikipedia.org/wiki/Superconductivity, Physics Today 2010 September p38. https://en.wikipedia.org/wiki/Meissner_effect

Drift Velocity in Copper Wires

Examples

A copper wire of diameter 2.91 mm carries a maximum current of 19 Amperes for power transmission. Please calculate the drift speed of electrons in the wire. The density and atomic weight of copper is 8.95 g/cm^3 and 63.5 g, respectively.

$$J = \frac{I}{A} = \frac{19}{\pi\left(\frac{0.00291}{2}\right)^2} = 2.9 \times 10^6\ A/m^2$$

$$J = nev_d \qquad n = \frac{8.95 \times 10^6}{63.5} \times 6.02 \times 10^{23} = 8.48 \times 10^{28}\ m^{-3}$$

$$v_d = \frac{J}{ne} = \frac{2.9 \times 10^6}{8.48 \times 10^{28} \times 1.602 \times 10^{-19}} = 2.1 \times 10^{-4}\ m/s$$

The Number Density of Charged Particle Beam

Examples

In a certain particle accelerator, a current of 0.5 mA is carried by a 5-MeV proton beam that has a radius of 1.5 mm. (a) Find the number density of protons in the beam.

$$K = 5\ \text{MeV} = 5 \times 10^{6} \times 1.602 \times 10^{-19}\ \text{J} = 8.01 \times 10^{-13}\ \text{J}$$

$$\frac{1}{2} m_p v^2 = K \rightarrow \frac{1}{2}(1.6 \times 10^{-27}) v^2 = 8.01 \times 10^{-13}$$

$$v = 3.1 \times 10^{7}\ \text{m/s}$$

$$I = nqvA \rightarrow n = \frac{I}{qvA} = \frac{0.5 \times 10^{-3}}{(1.602 \times 10^{-19})(3.1 \times 10^{7})(\pi \times 2.25 \times 10^{-6})}$$

$$n = 1.42 \times 10^{13}\ \text{m}^{-3}$$

How Many Electron Per Meter Cubic?

Examples

In scanning electron microscope, a current of 10 pA is carried by 300 eV electron beam within a circular spot of 1 micrometer in diameter. Find the number density of electrons in the beam.

$$K = 300\ eV = 300 \times 1.602 \times 10^{-19} = 4.8 \times 10^{-17}$$

$$K = \frac{1}{2} m_e v^2 \rightarrow v = \sqrt{2K/m} = 1.0 \times 10^7\ m/s$$

$$J = nev$$

$$n = \frac{J}{ev} = \frac{I}{Aev}$$

$$n = \frac{10 \times 10^{-12}}{\pi \times \left(\frac{10^{-6}}{2}\right)^2 \times 1.602 \times 10^{-19} \times 1.0 \times 10^7} = 7.9 \times 10^{12}\ m^{-3}$$

The Electric Field That Drives The Current

Examples

A 14-gauge copper means its wire diameter D of 1.628 mm. Find the electric field strength E in the 14-gauge copper wire when the wire is carrying a current of 1.3 A and has a resistivity of $1.7 \times 10^{-8}\ \Omega\text{m}$.

The electric field strength $E = \frac{V}{l} = \frac{IR}{l} = \frac{I}{l}\varrho\frac{l}{A} = \frac{I\varrho}{A}$

$$E = \frac{I\rho}{A} = \frac{(1.3)(1.7 \times 10^{-8})}{\pi(8.14 \times 10^{-4})^2} = 1.06 \times 10^{-2}\ \text{V/m}$$

Temperature Dependent Behavior of Resistance in Metal

Examples

A more general definition of the temperature coefficient of resistivity is $\alpha = \frac{1}{\rho}\frac{d\rho}{dT}$, where ρ is the resistivity at temperature T. (a) Assuming α is a constant, show that $\rho(T) = \rho_0 e^{\alpha(T-T_0)}$, where ρ_0 is the resistivity at T_0. (b) Using the series expansion, show that $\rho(T) \cong \rho_0\big(1 + \alpha(T - T_0)\big)$ for $\alpha(T - T_0) \ll 1$.

$$\alpha = \frac{1}{\rho(T)}\frac{d\rho(T)}{dT}$$

Here ρ and T are two independent variables. Others are constants.

$$\alpha dT = \frac{d\rho}{\rho} \qquad \int_{T_0}^{T} \alpha dT' = \int_{\rho_0}^{\rho} \frac{d\rho'}{\rho'} \qquad \alpha(T - T_0) = \ln\left(\frac{\rho}{\rho_0}\right)$$

$$\frac{\rho}{\rho_0} = e^{\alpha(T-T_0)} \qquad \rho(T) = \rho_0 e^{\alpha(T-T_0)}$$

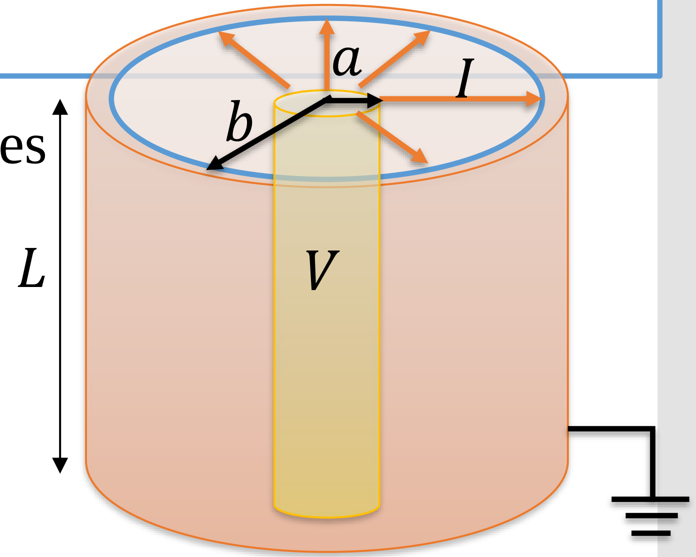
for $\alpha(T - T_0) \ll 1$ $\qquad \rho(T) = \rho_0 e^{\alpha(T-T_0)} \cong \rho_0\left(1 + \frac{\alpha(T - T_0)}{1!}\right)$

$$\rho(T) \cong \rho_0\big(1 + \alpha(T - T_0)\big)$$

Connected in Series

Examples

A coaxial cable consists of two cylindrical conductors of diameters a and b and length L. The material M_A of resistivity ρ is filled into the region between the two cylindrical conductors. Please calculate the resistance between the two conductors.



Check that the resistors are connected in series

$$R = \frac{\rho l}{A}$$

A shell of the material M_A with a diameter r, thickness dr, and area $2\pi rL$ gives resistance

$$dR = \rho \frac{dr}{2\pi rL}$$

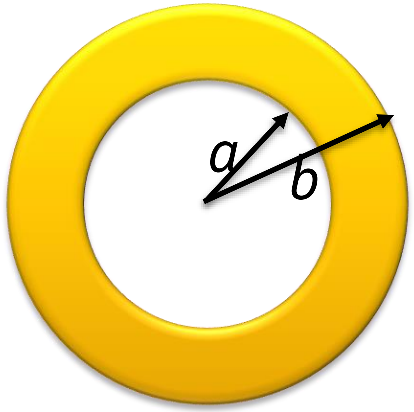
Total resistance is

$$R = \int dR = \int_a^b \rho \frac{dr}{2\pi rL} = \frac{\rho}{2\pi L} \ln(b/a)$$

Connected in Series

Examples

Please calculate the total resistance between the inner and the outer surface of a spherical shell where the inner and the outer radii are a and b, respectively, and the resistivity of the spherical shell is ρ.



Check that the resistors are connected in series

$$R = \frac{\rho l}{A}$$

A shell of resistivity of ρ, a diameter r, a thickness dr, and an area $4\pi r^2$ has a resistance dR:

$$dR = \rho \frac{dr}{4\pi r^2}$$

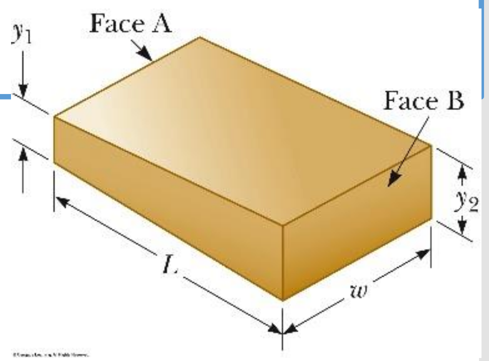
Total resistance is

$$R = \int dR = \int_a^b \rho \frac{dr}{4\pi r^2} = \frac{\rho}{4\pi}\left[-\frac{1}{r}\right]_{r=a}^{r=b} = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Connected in Series

Examples

Material with uniform resistivity ρ is formed into a wedge as shown in the right figure. Show that the resistance between face A and B of the wedge is $R = \rho \frac{l}{w(y_2 - y_1)} \ln\left(\frac{y_2}{y_1}\right)$.



Give a function to describe gradual variation of the edge from y_1 to y_2.

$$y(x) = y_1 + \frac{x}{l}(y_2 - y_1)$$

$$R = \int_{x=0}^{l} \rho \frac{dx}{w(y_1 + x(y_2 - y_1)/l)}$$

$$= \frac{\rho l}{w(y_2 - y_1)} \int_{x=0}^{l} \frac{d(x(y_2 - y_1)/l)}{(y_1 + x(y_2 - y_1)/l)}$$

$$= \frac{\rho l}{w(y_2 - y_1)} \int_{x=0}^{l} \frac{d(y_1 + x(y_2 - y_1)/l)}{(y_1 + x(y_2 - y_1)/l)}$$

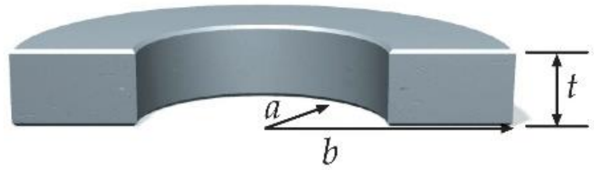
$$= \frac{\rho l}{w(y_2 - y_1)} [\ln(y_1 + x(y_2 - y_1)/l)]_{x=0}^{x=l} \qquad = \frac{\rho l}{w(y_2 - y_1)} \ln\left(\frac{y_2}{y_1}\right)$$

Examples

Please find the total resistance for current flowing through the curving semicircular disc. The resistivity, inner radius, outer radius and thickness of the disc are $\rho, a, b,$ and t, respectively.

For resistors connected in series, $R = \frac{\rho l}{A}$

For resistors connected in parallel, $G = \frac{A}{\rho l}$



$$dG = \frac{1}{\rho}\frac{tdr}{\pi r} \rightarrow G = \frac{t}{\pi\rho}\int_a^b \frac{dr}{r} = \frac{t}{\pi\rho}\ln\left(\frac{b}{a}\right)$$

$$R = \frac{\pi\rho}{t\ln(b/a)}$$

The Electrical Power

Power

An electric heater is constructed by applying a potential difference of 120 V across a Nicrome wire that has a total resistance of 8.00 Ω. Find the current carried by the wire and the power rating of the heater.

$$I = \frac{V}{R} = \frac{120}{8.00} = 15.0\ \text{A} \qquad P = I^2 R = (15.0)^2 \times (8.00) = 1800\ \text{W}$$

An immersion heater must increase the temperature of 1.50 kg of water from 10°C to 90°C in 10.0 min while operating at 110 V. What is the required resistance of the heater?

$$P = \frac{mc\Delta T \times 4.18}{\Delta t} = \frac{1500 \times 1 \times 80 \times 4.18}{10 \times 60} = 836\ \text{J/s}$$

$$\frac{V^2}{R} = 836 \rightarrow R = \frac{V^2}{836} = 14.5\ \Omega$$