# Chapter 22 Electric Fields

Physics II – Part I
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# Discovery of Charges

## History

0600 BC – Thales of Miletus (Greek)
The properties of amber are changed when rubbed.
Shows power to attract and repel straws and dry leaves.

0321 BC – Theophrastus (Greek)
0070 AD – Pliny The Elder (Greek)
1600 AD – William Gilbert (Englishman)
Dry air is better to generate electrify substances.

1745 AD – Ewald G. von Kleist (German), Prof. Pieter van Musschenbroek (Dutch)
Leyden jar was invented.


http://www.codecheck.com/cc/LeydenJar.html

Ref: https://physics.stackexchange.com/questions/17109/why-is-the-charge-naming-convention-wrong

# Discovery of Charges

## History

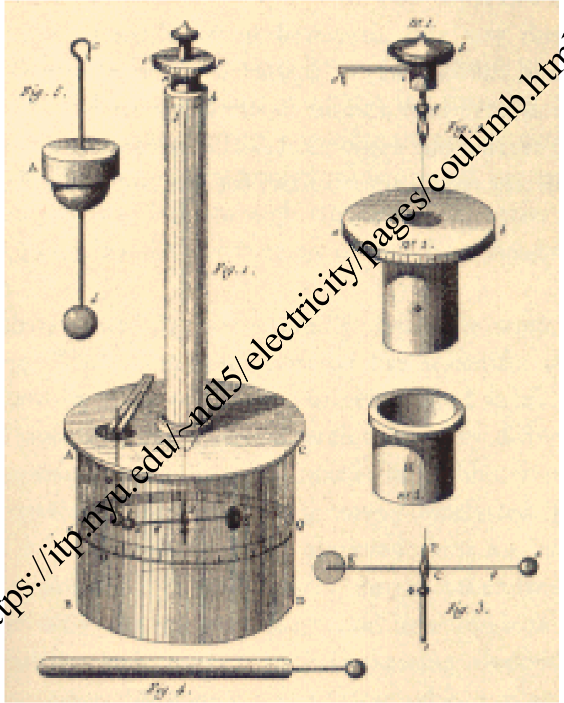
1752 AD – Benjamin Franklin (American)
Kite experiment, lightning experiment

1750 AD – Benjamin Franklin (American)
Choosing the positive electricity
https://www-spof.gsfc.nasa.gov/Education/woppos.html

1785 AD – Charles Coulomb (French)
torsion balance, published 7 papers

1798 AD – Henry Cavendish (British)
torsion balance, gravitational force
not published work and discovered by
James Maxwell in 1879

https://www.awesomestories.com/asset/view/LIGHTNING-in-a-BOTTLE



https://itp.nyu.edu/~ndl5/electricity/pages/coulumb.html

Ref: https://physics.stackexchange.com/questions/17109/why-is-the-charge-naming-convention-wrong

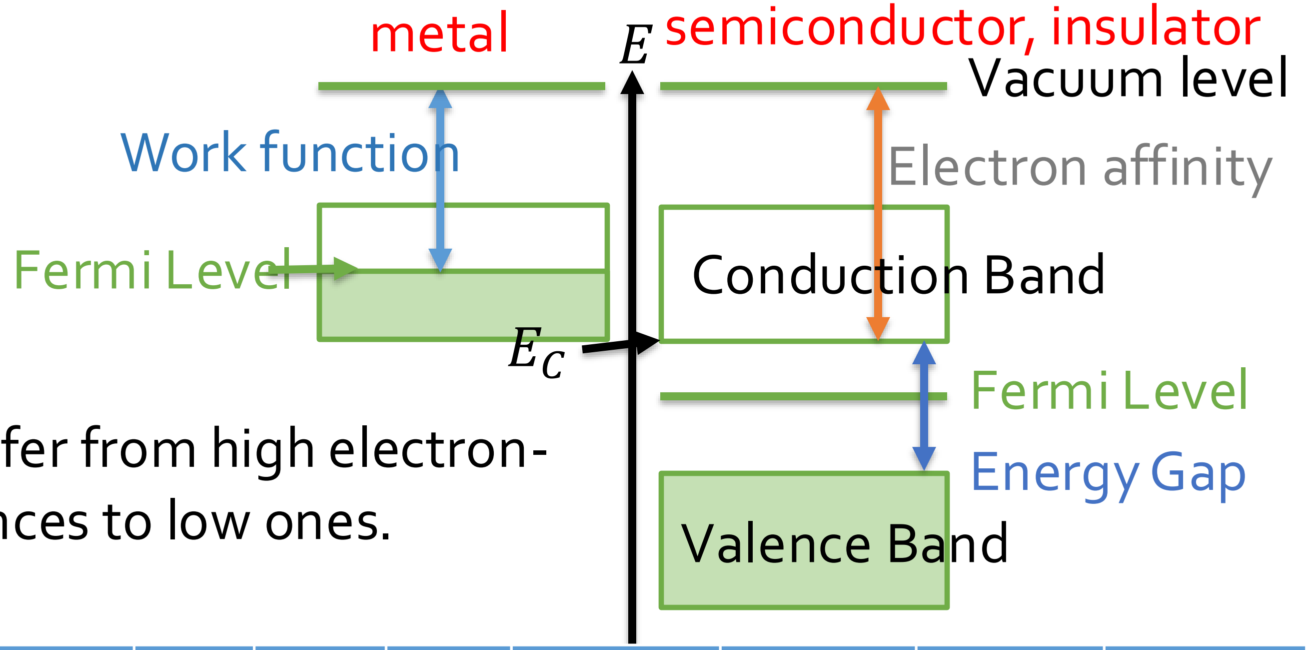
# Energy Band Diagram Introduced

Electron affinity of an atom or a molecule: $X_g + e^- \rightarrow X_g^-$.

For example, the fluorine gas atom: $F_g + e^- \xrightarrow{-328\ kJ/mol} F_g^-$.

Electron affinity of a bulk: $-(E_{Vacuum} - E_C)$

Work function of a bulk: $E_{Vacuum} - E_{Fermi\ Level}$



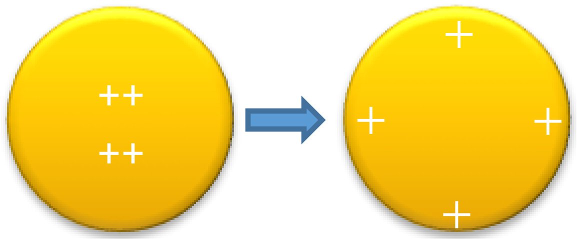

Electrons transfer from high electron-affinity substances to low ones.

| glass | nylon | wool | Pb | silk | Al | cotton | steel | brass | teflon |
|---|---|---|---|---|---|---|---|---|---|

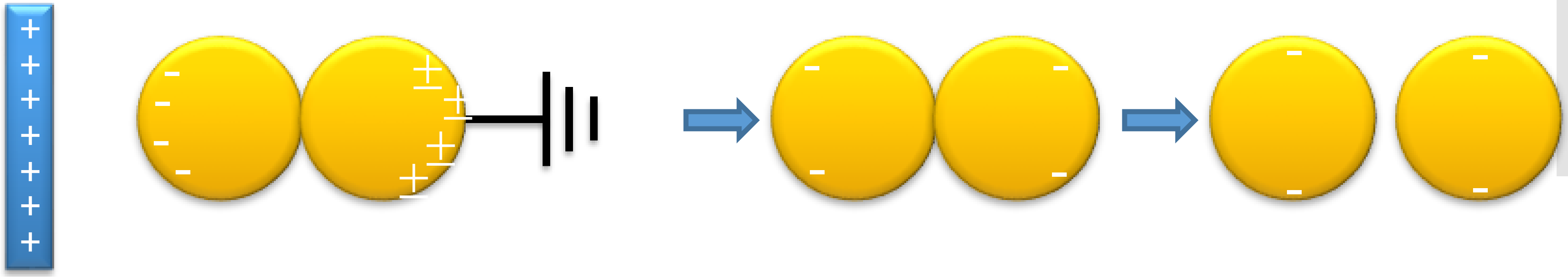
# Repulsive Forces Between Charges of Same-Polarity

The concept of grounded – connected to the Earth:


The concept of zero electric field inside the conductor:



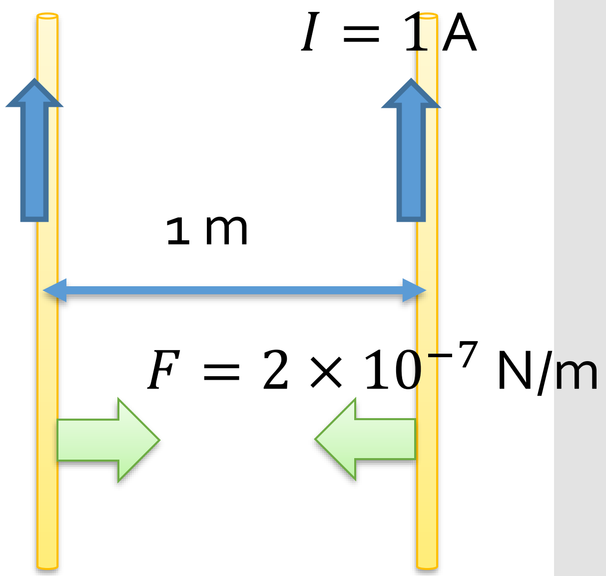
The concept of inductive electrifying:



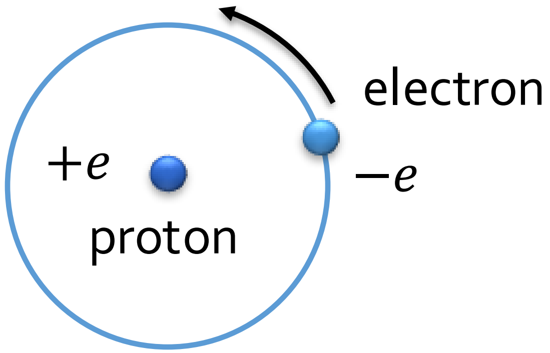
# Unit of Charges

Definition of charge unit: One Coulomb is the total charge collected from the current of one Ampere for one second.

Definition of current unit: One Ampere is the current in both of the two parallel conducting wires separated for 1 m in vacuum that generates a force per unit length of $2 \times 10^{-7}$ N/m.




Charge quantization: There is one single electron in one neutral hydrogen atom. $1\ e = 1.602 \times 10^{-19}$ C

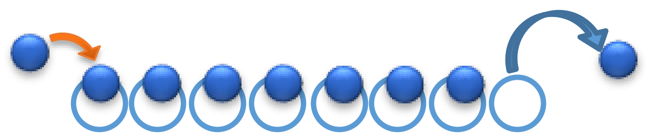


# Charge Conservation

Charge conservation: If one electron of charge $-e$ is removed from the neutral hydrogen atom, the hydrogen atom is ionized with a charge of $+e$.

$$2H^{+}+O^{2-} \rightarrow H_2O$$

Current flow model: The current flow is not the drifting process of charge carriers. It is more like information transmission. You can imagine a theme of the sitters changing their seats.

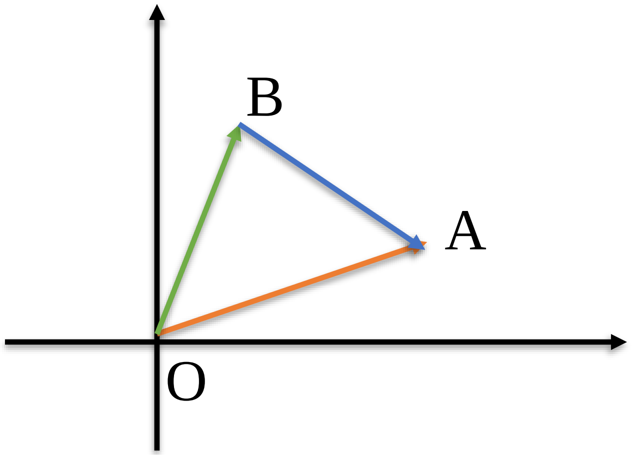


The Law for Static Electric Charges

# Coulomb's Law

Displacement vectors:

$$\vec{r}_{AB} = \vec{r}_{AO} - \vec{r}_{BO}$$

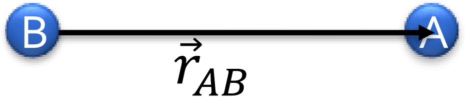



The observer sits at B to look at A.

The force exerted on B by A is $\vec{F}_{AB}$.

Coulomb's Law:

$$\vec{F}_{BA} = \frac{k q_A q_B}{r_{AB}^2} \hat{r}_{AB}$$




where $k = 1/4\pi\varepsilon_0$ is $8.99 \times 10^9$ N m$^2$ / C$^2$.

Here $\varepsilon_0$ is the permittivity of vacuum. The permittivity is a measure of a substance to resist the electric field.

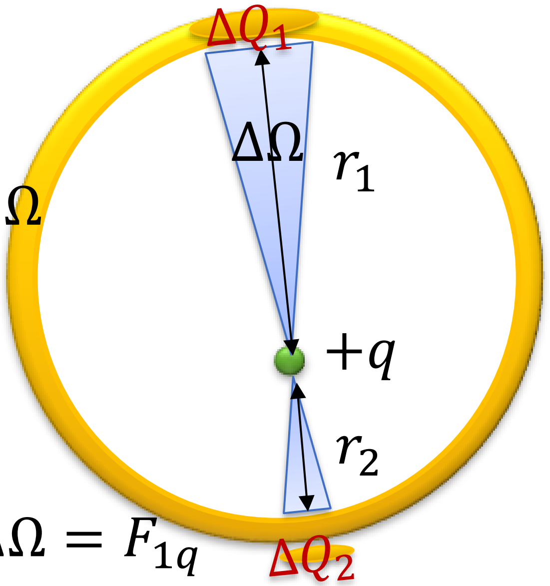
Ref: https://en.wikipedia.org/wiki/Permittivity

# The Law of Inverse Square of Distance

The inverse square law confirms zero net electric forces inside a metal box.

$$\Delta Q_1 = \sigma r_1^2 \Delta\Omega$$

$$F_{1q} = \frac{kq\Delta Q_1}{r_1^2} = kq\sigma\Delta\Omega$$




$$+Q = 4\pi R^2 \sigma$$

$$\Delta Q_2 = \sigma r_2^2 \Delta\Omega$$

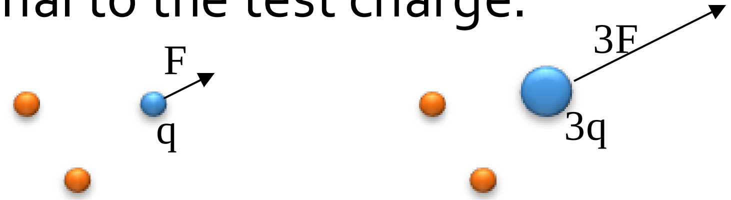
$$F_{2q} = \frac{kq\Delta Q_2}{r_2^2} = kq\sigma\Delta\Omega = F_{1q}$$

$$F_{net} = 0$$

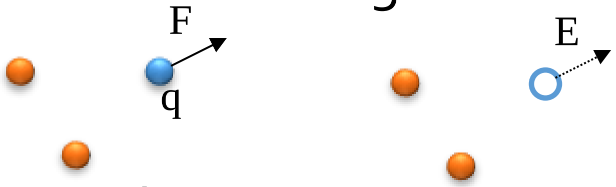
Ref:

# From Electric Force to Electric Field

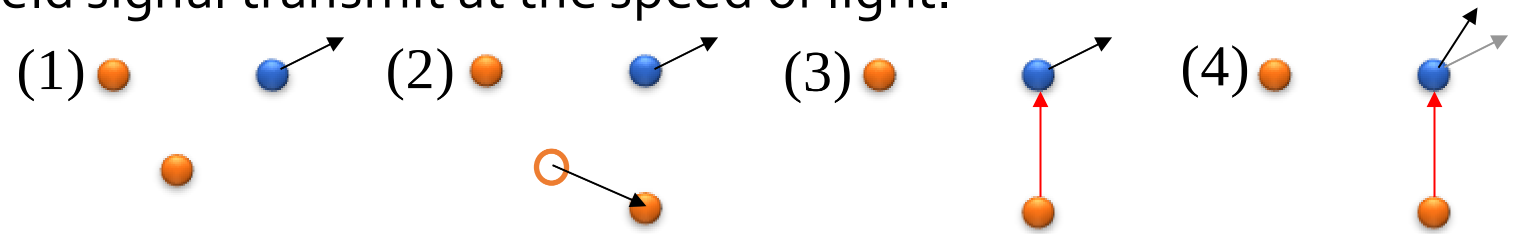
For a configuration of charges in the space, we can calculate the pulling force if we put the other charge (test charge) in the space. The force is proportional to the test charge.




Even though you do not put the test charge in the space, the "field" to produce the force on the test charge still works in the space.




If you move the source charge to a new position, the force of charges at new positions will be felt by the test charge after a time of  . The field signal transmit at the speed of light.

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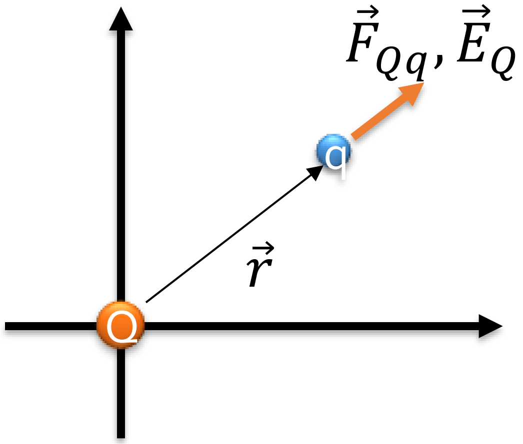
# Direction and Magnitude of Electric Field

Coulomb's force for the test charge:

$$\vec{F}_{Qq} = \frac{kqQ}{r^2}\hat{r}$$

$$\vec{E}_{Qq} = \frac{\vec{F}_{Qq}}{q} = \frac{kQ}{r^2}\hat{r}$$

$$\vec{E}_Q = \frac{kQ}{r^2}\hat{r}$$




Unit of electric field: 1 N / C = 1 V / m

Net electric field for an observer at the origin:

$$\vec{E}_{net} = \sum_{i=1}^{N} \frac{kq_i}{r_i^2}\hat{r}_{oi}$$

| Electric Fields | N/C, V/m |
|---|---|
| in conductors | $1 - 10^{-2}$ |
| bulb with tungsten wires | $10^3$ |
| in lightning bolt | $10^4$ |
| operation of transistor | $10^6$ |
| at electron in H atom | $10^{12}$ |

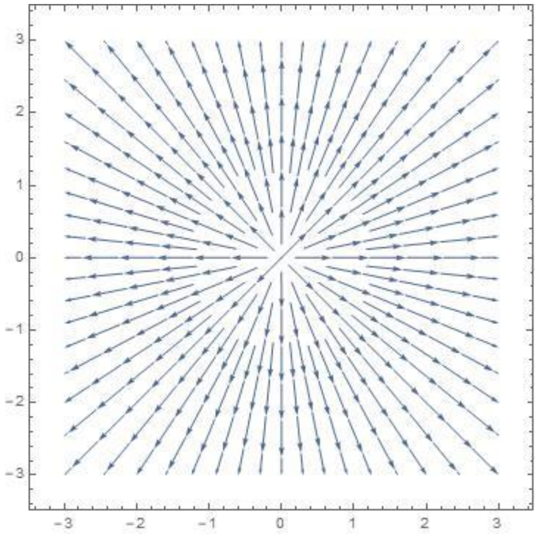
Mapping of Electric Field

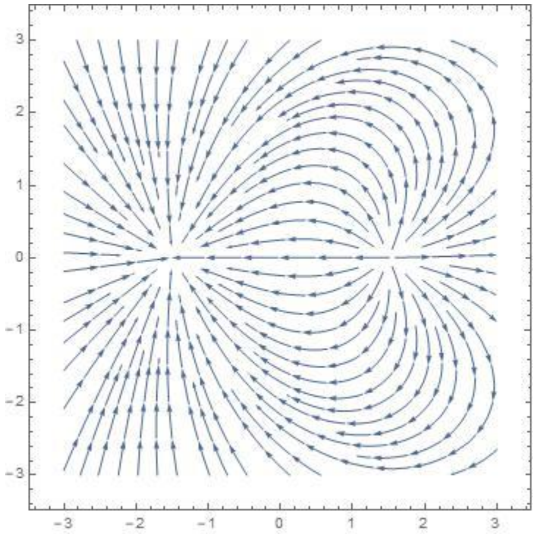
# Electric Field Lines

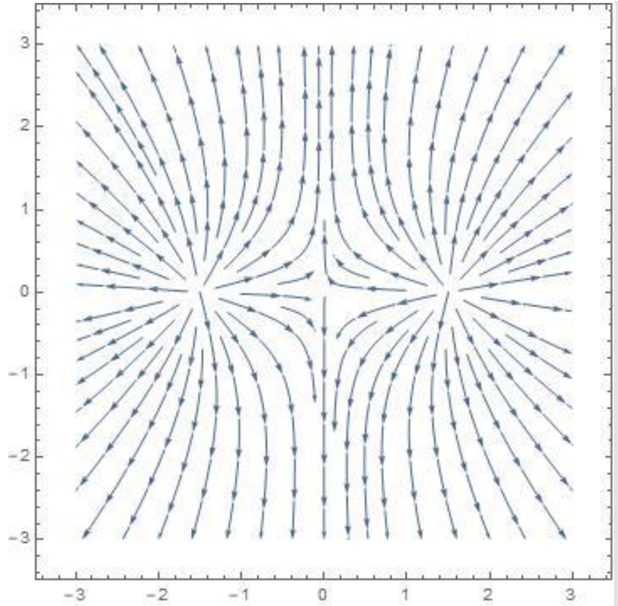
Electric field lines start from positive charges and end on negative ones.

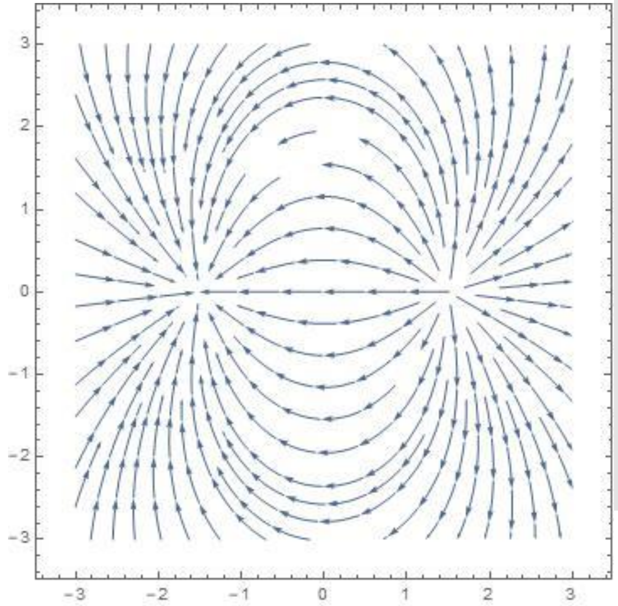
The lines are uniformly spaced entering or leaving a point charge.

The number of lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.









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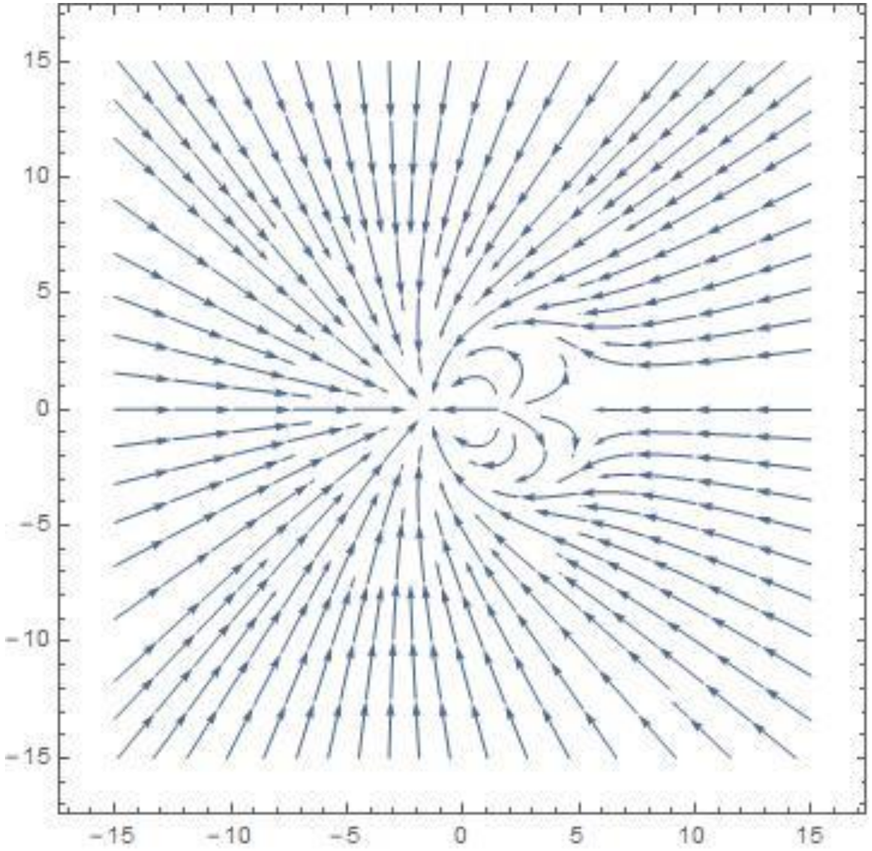
Mapping of Electric Field

# Electric Field Lines

The density of electric field lines is proportional to the magnitude of the local electric field.

At large distances from a system of non-zero total charges, the field lines look as if they came from a single point charge.

Electric field lines do not cross each other.

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# Examples

## Number of "Free" Electrons in Metal

Assume that one Al atom contributes three free charges in the solid. The density and atomic weight of Al bulk are 2.70 g/cm$^3$. and 27.0 g/mol. What is the total free charge in an Al lump with a volume of 1 cm$^3$?

The mass of the Al lump is:

$$m = Vd = 2.7 \text{ g}$$

The number of electrons is:

$$3 \times \frac{2.7}{27} \times 6.02 \times 10^{23} = 1.8 \times 10^{23}$$

The total charge is:

$$1.602 \times 10^{-19} \times 1.8 \times 10^{23} = 2.9 \times 10^{4} \text{ C}$$

# Examples

## The Magnitude of Coulomb's Force in Atoms

An electron is orbiting around a proton with a radius of 0.53 Å in a hydrogen atom. Please evaluate the Coulomb force between the electron and the proton.

Coulomb's force gives us

$$F = \frac{kq_1q_2}{r^2} = -8.99 \times 10^9 \frac{(1.602 \times 10^{-19})^2}{(0.53 \times 10^{-10})^2} = -8.21 \times 10^{-8}\ \mathrm{N}$$

# The Ratio Between Coulomb’s and Gravitational Forces

## Examples

Please calculate the ratio between Coulomb’s and gravitational forces of an electron in an hydrogen atom. The orbiting radius is 0.53 Å.

The ratio is

$$\frac{F_C}{F_G} = \frac{\frac{kq_1q_2}{r^2}}{\frac{Gm_1m_2}{r^2}} = \frac{kq_1q_2}{Gm_1m_2}$$

$$\frac{F_C}{F_G} = \frac{8.99 \times 10^9(1.602 \times 10^{-19})^2}{(6.67 \times 10^{-11})(1.67 \times 10^{-27})(9.11 \times 10^{-31})}$$

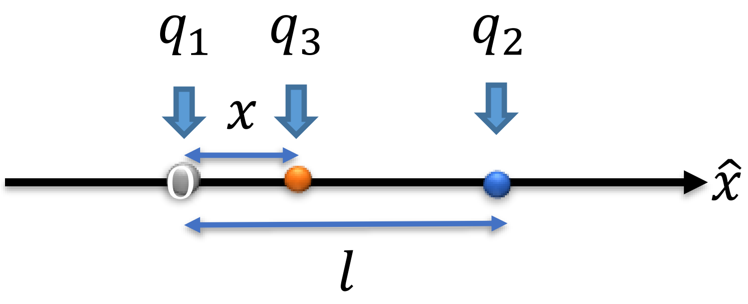
$$\frac{F_C}{F_G} = 2.27 \times 10^{39}$$

## The Balance Point

# Examples

Three charged particles are placed on the $x$-axis. One particle of charge $q_1$ is placed at the origin and another particle of charge $q_2$ is placed at $x = l$. Where can the other particle of charge $q_3$ be placed with a zero net force between the first and the second particles?

Let the particle of charge $q_3$ be placed at a distance $x$ away from the origin.




$$F_{13} = F_{23} \rightarrow \frac{kq_1q_3}{x^2} = \frac{kq_2q_3}{(l-x)^2}$$

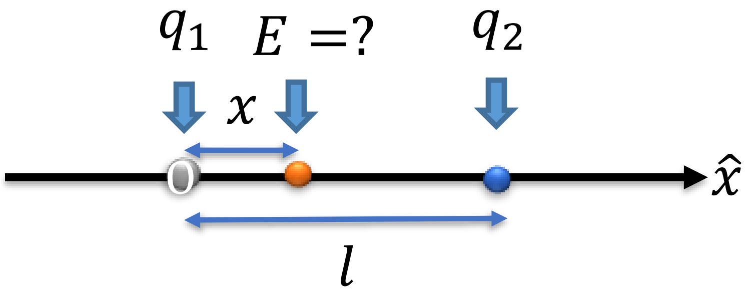
$$q_1(l-x)^2 = q_2x^2 \rightarrow (q_1 - q_2)x^2 - 2q_1lx + q_1l^2 = 0$$

$$x = \frac{q_1 \pm \sqrt{q_1q_2}}{(q_1 - q_2)} l \rightarrow x_1 = \frac{\sqrt{q_1}}{\sqrt{q_1} + \sqrt{q_2}} l, x_2 = \frac{\sqrt{q_1}}{\sqrt{q_1} - \sqrt{q_2}} l$$

# Examples

Three charged particles are placed on the $x$-axis. One particle of charge $q_1$ is placed at the origin and the other particle of charge $q_2$ is placed at $x = l$. Please estimate the electric field between the two particles at a distance $x$ away from the origin

$$\vec{E} = \frac{kq_1}{x^2}\hat{\imath} - \frac{kq_2}{(l-x)^2}\hat{\imath}$$



# Experience The "Vector" Feature of The Electric Field

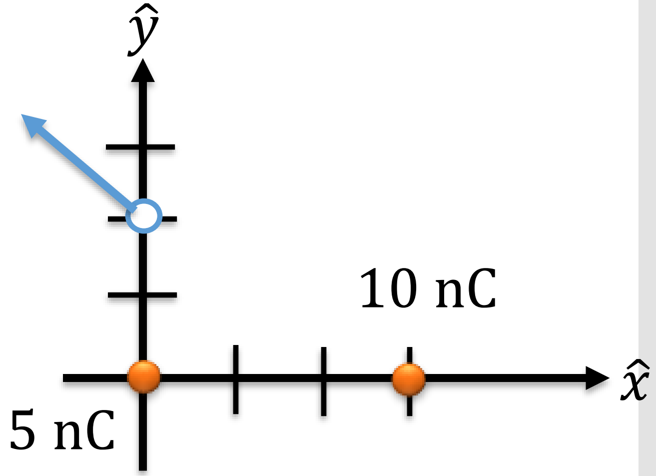
## Examples

Two particles of charges 5nC and 10 nC is placed at $x = 0$ and $x = 3$ m, respectively on the $x$-axis. What is the electric field observed at $y = 2$ m on the $y$-axis?

$$\vec{E} = 9 \times 10^9 \frac{10 \times 10^{-9}}{(2^2 + 3^2)} \left( -\frac{3}{\sqrt{2^2 + 3^2}} \hat{\imath} + \frac{2}{\sqrt{2^2 + 3^2}} \hat{\jmath} \right)$$

$$+9 \times 10^9 \frac{5 \times 10^{-9}}{2^2} \hat{\jmath}$$

$$\vec{E} = -5.76\hat{\imath} + 15.1\hat{\jmath} \ (\text{N/C})$$



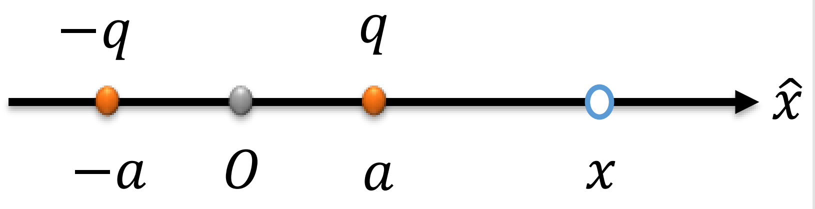
Debut of "Electric Dipole"

# Examples

A charge $+q$ is placed at $x = a$ on the $x$-axis and a second charge $-q$ is placed at $x = -a$. (a) Find the electric field on the $x$-axis at an arbitrary point $x$, where $x > a$. (b) What is the limit result of $x \gg a$?

(a)

$$E = -\frac{kq}{(x+a)^2} + \frac{kq}{(x-a)^2}$$




(b)

$$E = kq\frac{-(x-a)^2 + (x+a)^2}{(x^2-a^2)^2}$$

$$E = kq\frac{4xa}{(x^2-a^2)^2}$$

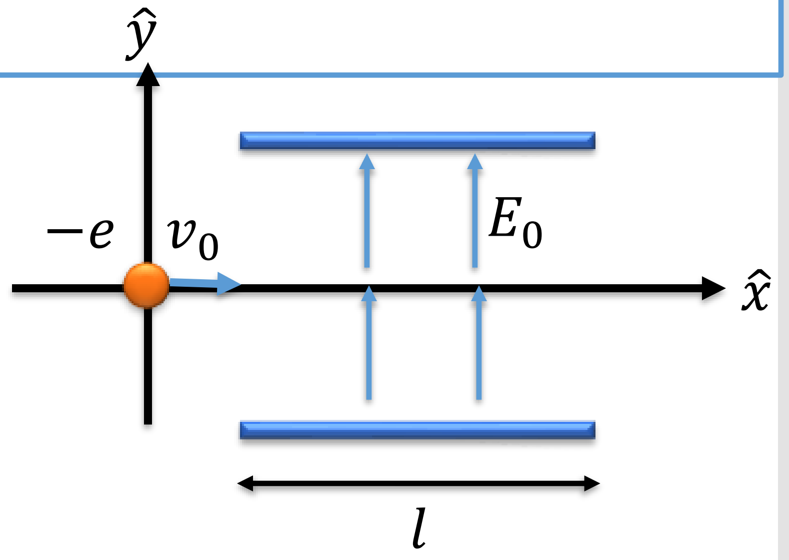
$$x \gg a \rightarrow E \cong kq\frac{4xa}{x^4} = 2k\frac{2qa}{x^3}$$

dipole moment $p = 2qa$

Cathode Ray Tube

# Examples

An electron of charge $-e$ and mass $m_e$ is moving on the central axis with a constant velocity $v_0$. When it is exerted by a perpendicular, uniform electric field $E_0$ for a distance $l$ along the x-axis, what is its deflection distance along the y-axis?




Estimate the time to travel through the region of uniform electric field:

$$t_{travel} = \frac{l}{v_0}$$

$$\vec{a} = -\frac{eE_0}{m_e}\hat{j}$$

$$y_{deflection} = -\frac{1}{2}\frac{eE_0}{m_e}\left(\frac{l}{v_0}\right)^2$$

# Mathematical Calculation

Total charge of $Q$ (C) is uniformly distributed on 1. a wire, 2. a plane, or 3. in a volume.

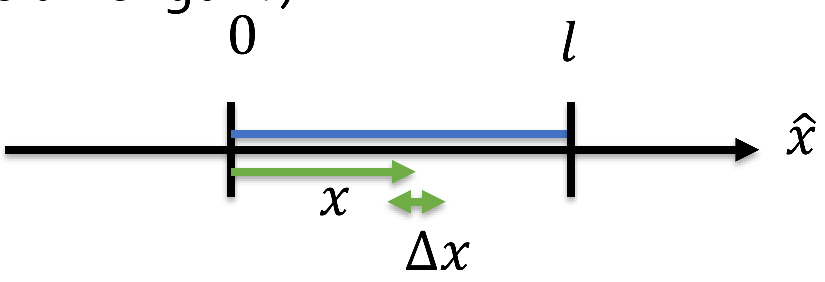
The charge per unit length is defined as $\lambda$, $\lambda = Q/l$, where $l$ is the length of the wire.

The charge per unit area is defined as $\sigma$, $\sigma = Q/A$, where $A$ is the area of the plane.

The charge per unit volume is defined as $\varrho$, $\varrho = Q/V$, where $V$ is the volume of the object.

# Mathematical Calculation

Using of integration (for example, to get the total charge back from a wire of length $l$)




$$dq = \lambda dx, \int dq = \int_0^l \lambda dx$$

$$= \frac{Q}{l}\int_0^l dx = \frac{Q}{l} l = Q$$

Using integration to get the total charge back from a rectangle of an area $a \times b$.

$$dq = \sigma da = \sigma dxdy$$

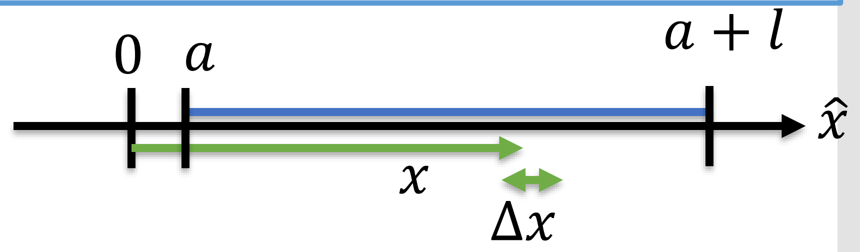
$$\int dq = \int_0^b \int_0^a \sigma \, dxdy$$

$$= \frac{Q}{ab}\int_0^b \int_0^a dxdy = \frac{Q}{ab} ab = Q$$

# Total Electric Field of Line Charges

## Examples

A rod of length $l$ is charged with $Q$ and placed on the axis as shown in the figure. Please calculate the electric field at the origin ($x = 0$).




$$\lambda = \frac{Q}{l}, dq = \lambda dx, d\vec{E} = -\hat{x}\frac{kdq}{r^2}$$

$$\vec{E} = -\hat{x}\int k\frac{dq}{r^2} = -\hat{x}k\int_a^{a+l}\frac{\lambda dx}{x^2}$$

$$\vec{E} = -\hat{x}k\lambda\left[-\frac{1}{x}\right]_{x=a}^{x=a+l} = -\hat{x}k\lambda\left(-\frac{1}{a+l}+\frac{1}{a}\right)$$

$$= -\hat{x}k\lambda\frac{l}{a(a+l)} = -\hat{x}\frac{kQ}{a(a+l)}$$

Charged Ring

# Examples

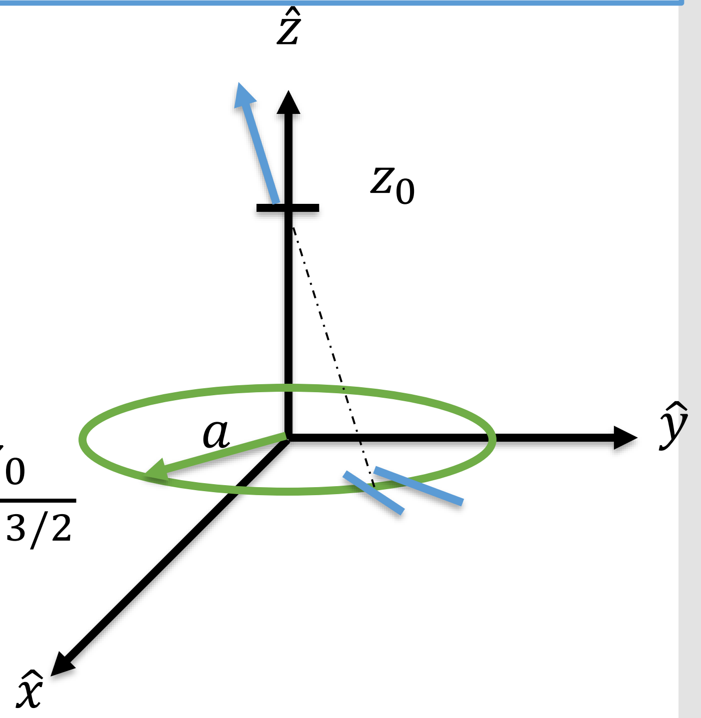
A ring of radius $a$ is charged with $Q$ and placed on the xy plane with its axis coincident with the z-axis as shown in the figure. Please calculate the electric field on the z-axis with a distance $z_0$ above the origin.

According to symmetry, $E_x = E_y = 0$

$$d\vec{E} = \hat{z}\frac{k\lambda}{a^2+z_0^2}ds\frac{z_0}{\sqrt{a^2+z_0^2}} \qquad ds = ad\theta$$

$$\vec{E} = \hat{z}\int_0^{2\pi}\frac{k\lambda z_0}{(a^2+z_0^2)^{3/2}}ad\theta = \hat{z}\frac{k2\pi a\lambda z_0}{(a^2+z_0^2)^{3/2}}$$

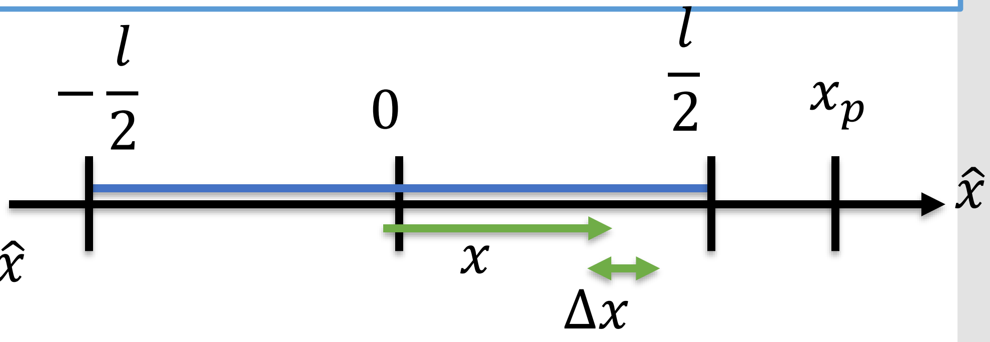
$$= \hat{z}\frac{kQz_0}{(a^2+z_0^2)^{3/2}}$$



## Line Charge, Observer on The Axis of The Line Charge

# Examples

A rod of length $l$ is uniformly charged with $Q$ and placed on the axis as shown in the figure. Please calculate the electric field at $x_p$.




$$\lambda = \frac{Q}{l}, dq = \lambda dx = \frac{Q}{l} dx$$

$$d\vec{E} = \frac{kdq}{\left(x_p - x\right)^2}\hat{x} = \frac{k\lambda dx}{\left(x_p - x\right)^2}\hat{x}$$

$$\vec{E} = \hat{x}\int_{-l/2}^{l/2} \frac{k\lambda dx}{\left(x_p - x\right)^2} = \hat{x}k\lambda \int_{-l/2}^{l/2} \frac{d\left(x - x_p\right)}{\left(x - x_p\right)^2}$$

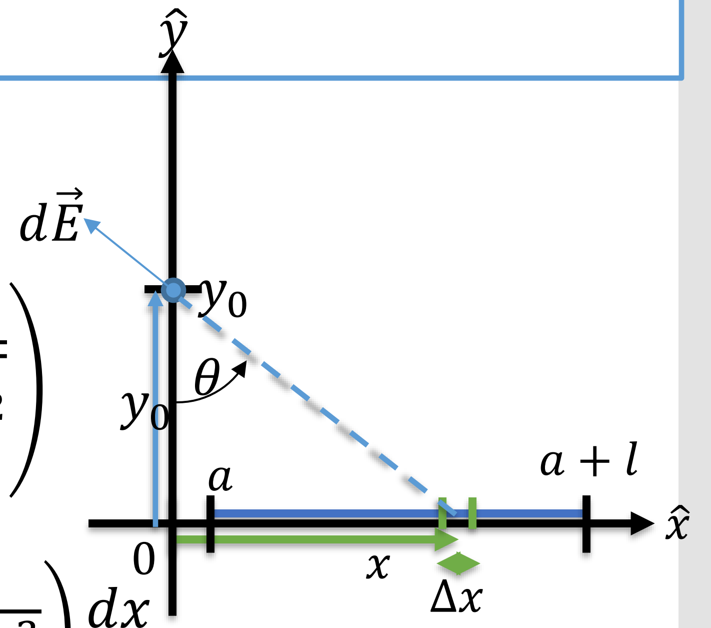
$$= \hat{x}k\lambda \left[-\frac{1}{x - x_p}\right]_{x=-l/2}^{x=l/2} = \hat{x}k\lambda\left(-\frac{1}{\frac{l}{2} - x_p} + \frac{1}{-\frac{l}{2} - x_p}\right)$$

$$= \hat{x}k\lambda\left(\frac{1}{x_p - \frac{l}{2}} - \frac{1}{x_p + \frac{l}{2}}\right) = \frac{\hat{x}k\lambda l}{x_p^2 - \frac{l^2}{4}}$$

# Line Charge, Off The Axis of The Line Charge

# Examples

A rod of length $l$ is charged with $Q$ and placed on the x-axis as shown in the figure. Please calculate the electric field at the y-axis with a distance of $y_0$ away from the origin.




$$d\vec{E} = \frac{kdq}{r^2}\hat{r}$$

$$= \frac{k\lambda dx}{x^2 + y_0^2}\left(-\hat{x}\frac{x}{\sqrt{x^2+y_0^2}} + \hat{y}\frac{y_0}{\sqrt{x^2+y_0^2}}\right)$$

$$\vec{E} = k\lambda\int_a^{a+l}\left(-\hat{x}\frac{x}{(x^2+y_0^2)^{\frac{3}{2}}} + \hat{y}\frac{y_0}{(x^2+y_0^2)^{\frac{3}{2}}}\right)dx$$

Let $\frac{x}{y_0} = \tan\theta$ and $\frac{a}{y_0} = \tan(\theta_1)$, $\frac{a+l}{y_0} = \tan(\theta_2)$

# Line Charge, Off The Axis of The Line Charge

## Examples

A rod of length $l$ is charged with $Q$ and placed on the x-axis as shown in the figure. Please calculate the electric field at the y-axis with a distance of $y_0$ away from the origin.

$$x = y_0 \tan\theta \rightarrow dx = y_0 \sec^2\theta \, d\theta$$

$$\vec{E} = k\lambda \int_{\theta_1}^{\theta_2} \left( -\hat{x} \frac{y_0 \tan\theta}{y_0^3 \sec^3\theta} + \hat{y} \frac{y_0}{y_0^3 \sec^3\theta} \right) y_0 \sec^2\theta \, d\theta$$

$$\vec{E} = \frac{k\lambda}{y_0} \int_{\theta_1}^{\theta_2} (-\hat{x} \sin\theta + \hat{y} \cos\theta) d\theta$$

$$= \frac{k\lambda}{y_0} [\hat{x} \cos\theta + \hat{y} \sin\theta]_{\theta=\theta_1}^{\theta=\theta_2}$$

$$= \frac{k\lambda}{y_0} \left( \hat{x}(\cos\theta_2 - \cos\theta_1) + \hat{y}(\sin\theta_2 - \sin\theta_1) \right)$$

# Examples

## Infinitely Long Line of Charge

A rod of length $l$ is charged with $Q$ and placed on the x-axis as shown in the figure. Please calculate the electric field at the y-axis with a distance of $y_0$ away from the origin.

$$\vec{E} = \frac{k\lambda}{y_0}\left(\hat{x}(\cos\theta_2 - \cos\theta_1) + \hat{y}(\sin\theta_2 - \sin\theta_1)\right)$$

$$\tan\theta_1 = -\frac{\infty}{y_0} \to \theta_1 = -\frac{\pi}{2}$$

$$\tan\theta_2 = \frac{\infty}{y_0} \to \theta_2 = \frac{\pi}{2}$$

$$\vec{E} = \frac{k\lambda}{y_0}\left(\hat{x}\left(\cos\left(\frac{\pi}{2}\right) - \cos\left(-\frac{\pi}{2}\right)\right) + \hat{y}\left(\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right)\right)$$

$$\vec{E} = 2\frac{k\lambda}{y_0}\hat{y}$$

Symmetrically Displaced Line Charge, Off The Axis

# Examples

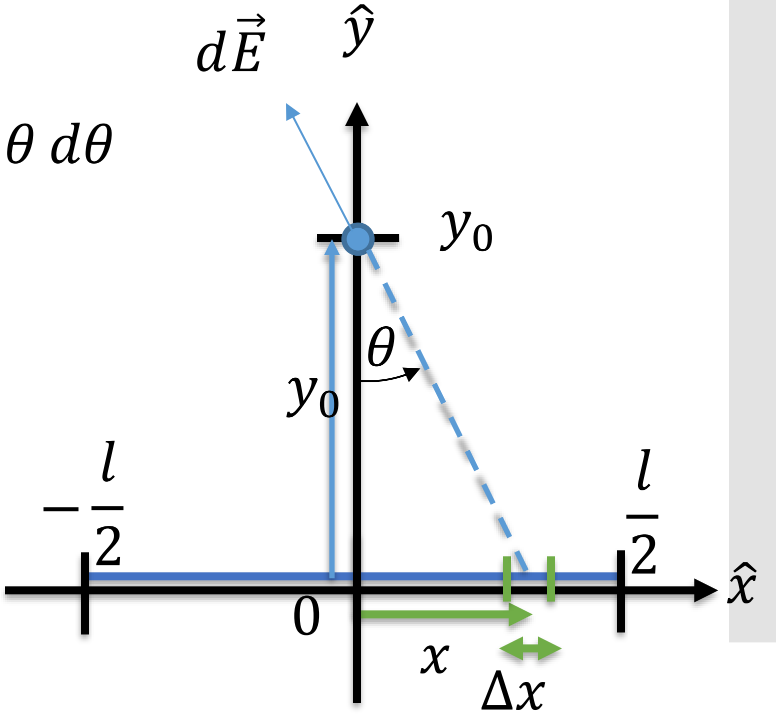
A rod of length $l$ is charged with $Q$ and placed on the x-axis as shown in the figure. Please calculate the electric field at the y-axis with a distance of $y_0$ away from the origin.

$$E_x = 0 \rightarrow \lambda = \frac{Q}{l}, d\vec{E} = \hat{y}\frac{k\lambda y_0 dx}{(x^2+y_0^2)^{3/2}}, x = y_0 \tan\theta$$

$$\vec{E} = \hat{y}2k\lambda \int_0^{\tan^{-1}(l/2y_0)} \frac{y_0}{y_0^3 \sec^3\theta} y_0 \sec^2\theta \, d\theta$$

$$\vec{E} = \hat{y}\frac{2k\lambda}{y_0}\int_0^{\tan^{-1}(l/2y_0)} \cos\theta \, d\theta$$

$$\vec{E} = \hat{y}\frac{2k\lambda}{y_0}\frac{\frac{l}{2}}{\sqrt{y_0^2+\frac{l^2}{4}}}$$



## Charge on The Semicircular Arc

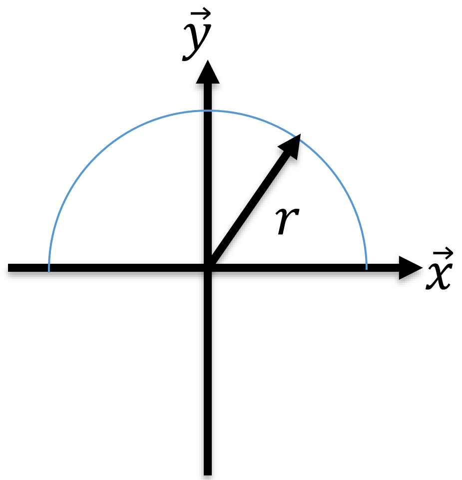
# Examples

A uniformly charged insulating rod of length $L$ is bent into the shape of a semicircle. The rod has a total charge of Q. Find the electric field at the center of the semicircle.

$$L = \pi r \rightarrow r = \frac{L}{\pi}$$

$$d\vec{E} = -\frac{kdq}{r^2}(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$$

$$dq = \lambda ds = \lambda(rd\theta)\ \&\ \lambda = \frac{Q}{L}$$




$$\vec{E} = -\int_0^{\pi}\frac{k}{r^2}(\cos\theta\,\hat{x} + \sin\theta\,\hat{y})\lambda r d\theta = -2\frac{k\lambda}{r}\hat{y} = -\frac{2k\left(\frac{Q}{L}\right)}{\left(\frac{L}{\pi}\right)}\hat{y} = -\frac{2\pi kQ}{L^2}\hat{y}$$

Charged Disc

# Examples

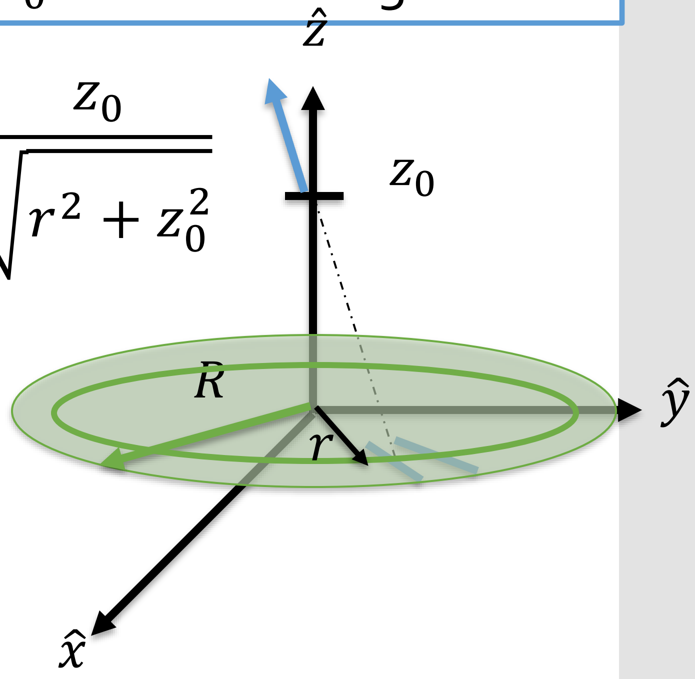
A disk of radius $R$ is charged with $Q$ and placed on the xy plane with its axis coincident with the z-axis as shown in the figure. Please calculate the electric field on the z-axis with a distance $z_0$ above the origin.

$$\sigma = \frac{Q}{\pi R^2} \quad E_x = E_y = 0 \quad d\vec{E} = \hat{z}\frac{k(2\pi r\sigma dr)}{r^2+z_0^2}\frac{z_0}{\sqrt{r^2+z_0^2}}$$

$$\vec{E} = \hat{z}2\pi k\sigma z_0 \int_0^R \frac{rdr}{(r^2+z_0^2)^{3/2}}$$

$$= \hat{z}2\pi k\sigma z_0 \left[-\frac{1}{\sqrt{r^2+z_0^2}}\right]_{r=0}^{r=R}$$

$$= \hat{z}2\pi k\sigma z_0 \left(\frac{1}{z_0} - \frac{1}{\sqrt{R^2+z_0^2}}\right)$$



Charged Disc

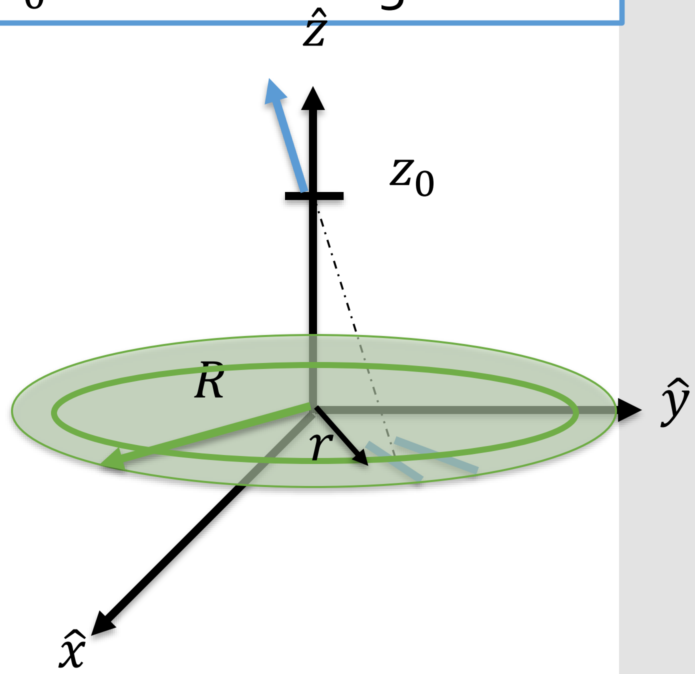
# Examples

A disk of radius $R$ is charged with $Q$ and placed on the xy plane with its axis coincident with the z-axis as shown in the figure. Please calculate the electric field on the z-axis with a distance $z_0$ above the origin.

$$\vec{E} = \hat{z}2\pi k\sigma z_0\left(\frac{1}{z_0} - \frac{1}{\sqrt{R^2 + z_0^2}}\right)$$

$$\lim_{R\to\infty} \vec{E}(R) = \lim_{R\to\infty} \hat{z}2\pi k\sigma z_0\left(\frac{1}{z_0} - \frac{1}{\sqrt{R^2 + z_0^2}}\right)$$

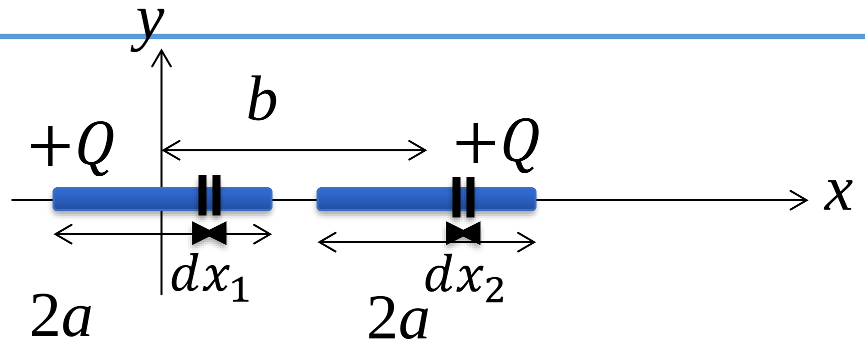
$$\lim_{R\to\infty} \vec{E}(R) = \hat{z}2\pi k\sigma$$



# Line Charge, Observer on The Axis of The Line Charge

## Challenge Problems

Identical thin rods of length $2a$ carry equal charges $+Q$ uniformly distributed along their lengths. The rods lie along the $x$-axis with their centers separated by a distance $b$ ( $b > a$ ). Please calculate the magnitude of the force exerted by the left rod on the right one.




two variables of infinitesimal charges $dq_1$ and $dq_2$

$$dF = k\frac{dq_1 dq_2}{r^2}$$

$$dF = k\frac{\lambda dx_1 \lambda dx_2}{(x_1 - x_2)^2} \qquad F = k\int_{b-a}^{b+a}\int_{-a}^{a}\frac{\lambda dx_1 \lambda dx_2}{(x_1 - x_2)^2}$$

$$F = k\lambda^2\int_{b-a}^{b+a}\left[\int_{-a}^{a}\frac{d(x_1 - x_2)}{(x_1 - x_2)^2}\right]dx_2$$

$$F = k\lambda^2\int_{b-a}^{b+a}\left[\left(-\frac{1}{x_1 - x_2}\right)_{x_1=-a}^{x_1=a}\right]dx_2 = k\lambda^2\int_{b-a}^{b+a}\left(-\frac{1}{a - x_2} + \frac{1}{-a - x_2}\right)dx_2$$

# Challenge Problems

## Line Charge, Observer on The Axis of The Line Charge

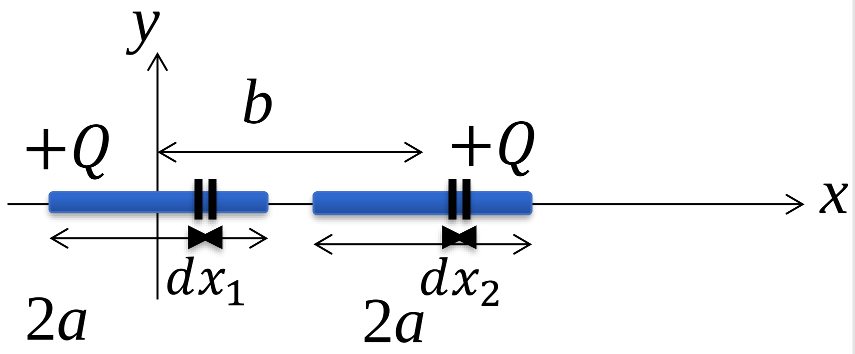
Identical thin rods of length 2*a* carry equal charges +*Q* uniformly distributed along their lengths. The rods lie along the *x*-axis with their centers separated by a distance *b* ( *b* > *a* ). Please calculate the magnitude of the force exerted by the left rod on the right one.

$$F = k\lambda^2 \int_{b-a}^{b+a} \left( \frac{1}{x_2 - a} - \frac{1}{x_2 + a} \right) dx_2$$

$$= k\lambda^2 \left[ \ln\left( \frac{x_2 - a}{x_2 + a} \right) \right]_{b-a}^{b+a}$$

$$= k\lambda^2 \left( \ln\left( \frac{b}{b+2a} \right) - \ln\left( \frac{b-2a}{b} \right) \right)$$

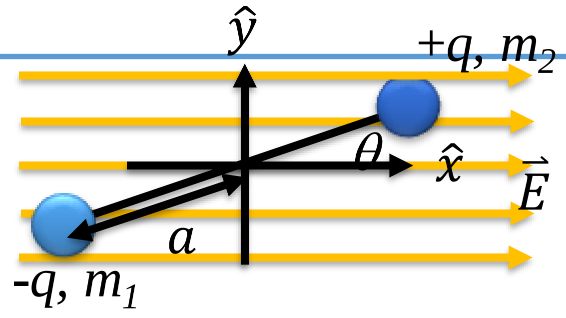
$$= k\lambda^2 \left( \ln\left( \frac{b}{b+2a} \frac{b}{b-2a} \right) \right) = k\lambda^2 \ln\left( \frac{b^2}{b^2 - 4a^2} \right)$$



Electric Force to Torque

# Challenge Problems

An electric dipole in a uniform horizontal electric field is displaced slightly from its equilibrium position for a small angle $\theta$. The separation of the charges (+$q$ and -$q$) is 2$a$, and each of the two particles has mass $m$. (a) Assume the dipole is released from the non-equilibrium position with a small angle $\theta$, please calculate the frequency of the simple harmonic motion along its angular orientation. (b) What will it be if the masses of the two particles, $m_1$ and $m_2$ , are different?




$$\vec{F}_1 = -qE\hat{x} \rightarrow \vec{\tau}_1 = -qEa\sin\theta\,\hat{z}$$

$$\vec{F}_2 = qE\hat{x} \rightarrow \vec{\tau}_2 = -qEa\sin\theta\,\hat{z}$$

$$\vec{\tau} = -2qEa\sin\theta\,\hat{z} \qquad I = ma^2 + ma^2$$

$$I\alpha = -2qEa\sin\theta \rightarrow I\frac{d^2\theta}{dt^2} + 2qEa\sin\theta = 0$$

small angle assumption for $\theta$ → $I\dfrac{d^2\theta}{dt^2} + 2qEa\theta = 0$

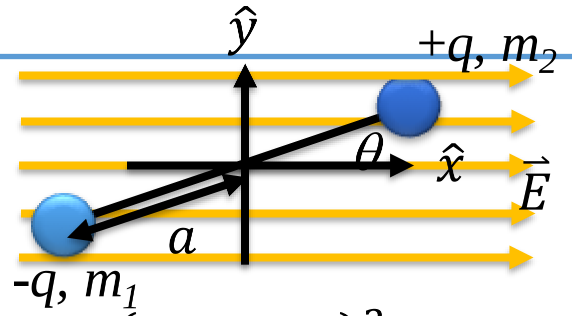
compare with the differential equation $m\dfrac{d^2x}{dt^2} + kx = 0$ with the solution $\omega = \sqrt{\dfrac{k}{m}}$

$$\omega = \sqrt{\frac{2qEa}{2ma^2}} = \sqrt{\frac{qE}{ma}} \rightarrow f = \frac{1}{2\pi}\sqrt{\frac{qE}{ma}}$$

Electric Force to Torque

# Challenge Problems

An electric dipole in a uniform horizontal electric field is displaced slightly from its equilibrium position for a small angle $\theta$. The separation of the charges (+$q$ and -$q$) is 2$a$, and each of the two particles has mass $m$. (a) Assume the dipole is released from the non-equilibrium position with a small angle $\theta$, please calculate the frequency of the simple harmonic motion along its angular orientation. (b) What will it be if the masses of the two particles, $m_1$ and $m_2$ , are different?




$$\vec{\tau} = 2qEa\sin\theta\,\hat{z}$$

$$I_O = m_1a^2 + m_2a^2, x_{CM} = \frac{m_2a - m_1a}{m_1 + m_2}$$

$$I_O = I_{CM} + (m_1 + m_2)x_{CM}^2 \to I_{CM} = (m_1 + m_2)a^2 - \frac{(m_1 - m_2)^2}{m_1 + m_2}a^2$$

$$I_{CM} = \frac{4m_1m_2}{m_1 + m_2}a^2 \qquad I\frac{d^2\theta}{dt^2} + 2qEa\theta = 0$$

$$\omega = \sqrt{\frac{2qEa}{\frac{4m_1m_2}{m_1 + m_2}a^2}} = \sqrt{\frac{qE(m_1 + m_2)}{2m_1m_2a}} \to f = \frac{1}{2\pi}\sqrt{\frac{qE(m_1 + m_2)}{2m_1m_2a}}$$