Chapter 14 Fluids

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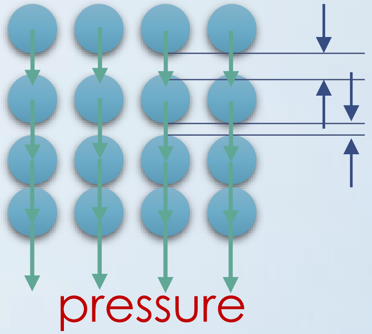
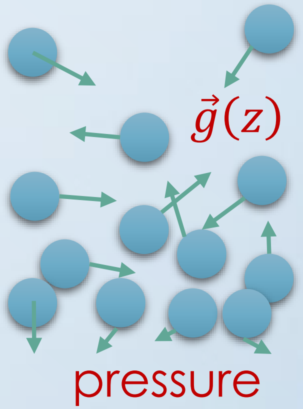
1. PRESSURE



Both liquid and gas are fluids.

The gravitational force for atoms in a solid is transferred through the bonding interactions between atoms to the atoms on the bottom area.



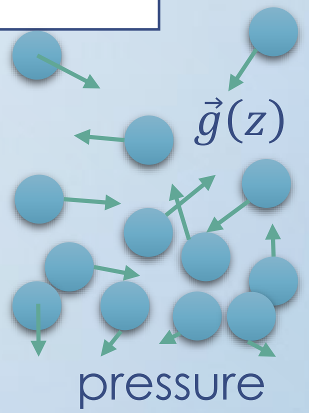
The gravitational force for atoms in gas is transferred through collisions between atoms to the atoms near the bottom.

All the interaction forces are EM forces.

Pressure: $P = F/A$, 1 atm = 1.01×10^5 Pa (N/m^2) = 760 torr = 14.7 PSI (lb/in^2) = 1 Bar = 1000 mBar

1. PRESSURE

Exmaple: A living room has floor dimensions of 3.5 m and 4.2 m and a height of 2.4 m. (a) What is the weight of the air in the room when the air pressure is 1.0 atm? (b) What is the magnitude of the atmosphere's force on the floor of the room?



The density of air ρ is ~1.23 kg/m³.

(a)

$$W = \rho V g = 1.23 \times (3.5 \times 4.2 \times 2.4) \times 9.8 = 430\ (N)$$

(b)

$$P = 1.0\ atm = 1.01 \times 10^5 (N/m^2)$$

$$F = PA = 1.01 \times 10^5 \times (3.5 \times 4.2) = 1.5 \times 10^6$$

2. VARIATION OF PRESSURE WITH DEPTH

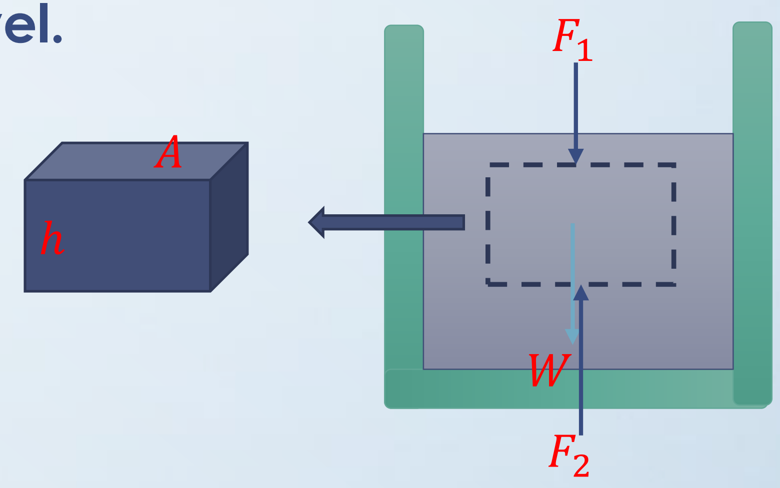
The variation of pressure for a depth h below a level.

$$F_1 + W = F_2$$

$$P_1 A + \rho_{liquid} V g = P_2 A$$

$$P_2 = P_1 + \rho_{liquid} g h$$

$$\Delta P = \rho g h$$



The atmosphere pressure at a height h above a level at which the pressure is P_0, its pressure is

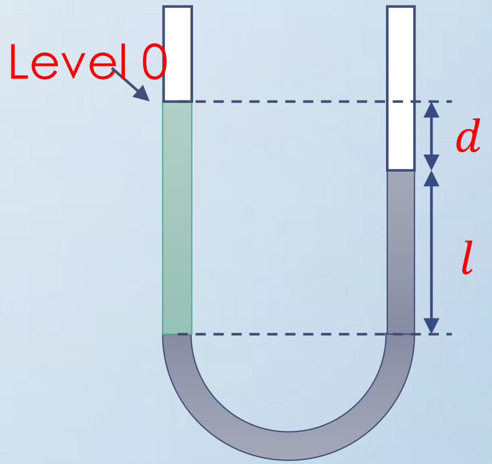
Above the level: $P(h) = P_0 - \rho_{air} g h$

Below the level: $P(h) = P_0 + \rho_{air} g h$

2. VARIATION OF PRESSURE WITH DEPTH

Example: The U-tube in the figure contains two liquids in static equilibrium: Water of density (ρ_w= 998 kg/m^3) is in the right arm and oil of unknown density is in the left. Measurement gives l = 135 mm and d = 12.3 mm. What is the density of the oil?

$$\rho_{air} = 1.23\frac{kg}{m^3} \ll \rho_w$$



$$P_0 + \rho_x g(d+l) = P_0 + \rho_{air} g d + \rho_w g l$$

$$\rho_x = \frac{\rho_{air} d + \rho_w l}{d+l} = 915\left(\frac{kg}{m^3}\right) = 0.915\left(\frac{g}{cm^3}\right)$$

2. VARIATION OF PRESSURE WITH DEPTH

Example: A novice scuba diver practicing in a swimming pool takes enough air from his tank to fully expand his lungs before abandoning the tank at depth L and swimming to the surface. He ignores instructions and fails to exhale during his ascent. When he reaches the surface, the difference between the external pressure on him and the air pressure in his lungs is 9.3 kPa. From what depth does he start?

$$P_L = P_0 + \rho g L \rightarrow \Delta P = \rho g L$$

$$\rho_w = 998\ (kg/m^3)$$

$$9300 = 998 \times 9.8 \times L$$

$L = 0.95$ m

https://www.deeperblue.com/4-tips-year-scuba-diver/

2. VARIATION OF PRESSURE WITH DEPTH

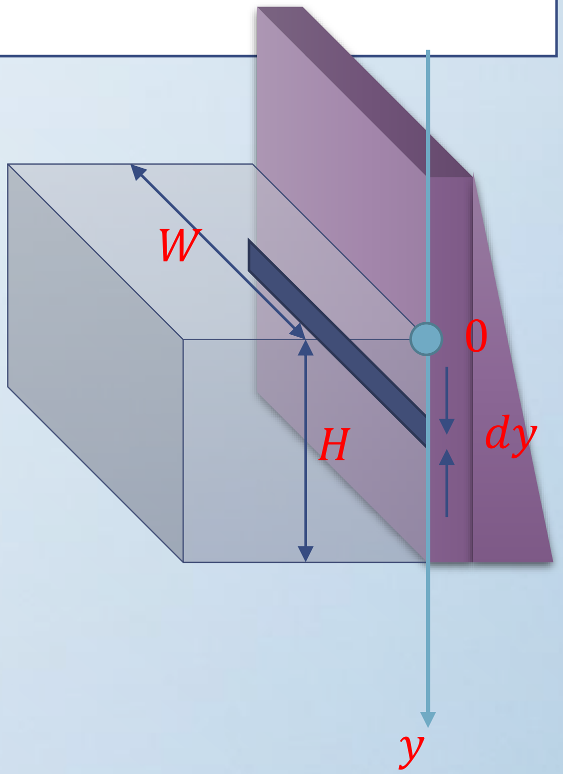
Example: A dam has a water level at a height of H. Assume that the water density is ρ_w. If the width of the dam is W, please determine the resultant force on the wall of the dam.

$$P_0 \; at \; y = 0$$

$$\Delta P = \rho_w g y$$

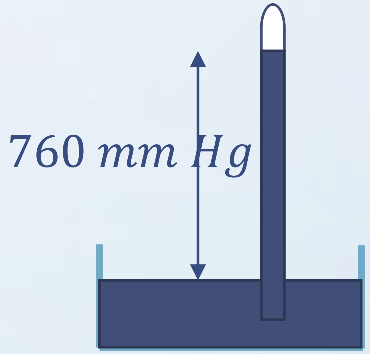
$$dF = \Delta P A = \Delta P(W dy) = \rho_w g W y dy$$

$$F_{net} = \int dF = \int_0^H \rho_w g W y dy = \frac{1}{2}\rho_w g W H^2$$



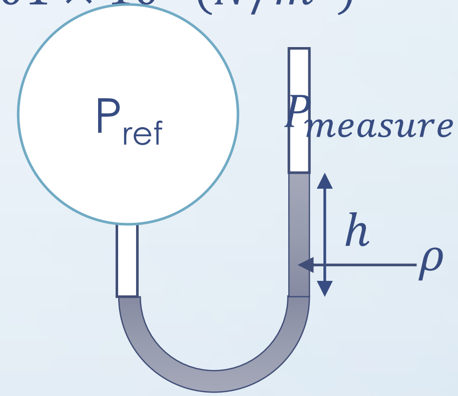
3. PRESSURE MEASUREMENTS

The mercury barometer:



$$P_{air} = 0.760 \times \left(13.6 \times 10^3\right) \times 9.8 = 1.01 \times 10^5\ (N/m^2)$$

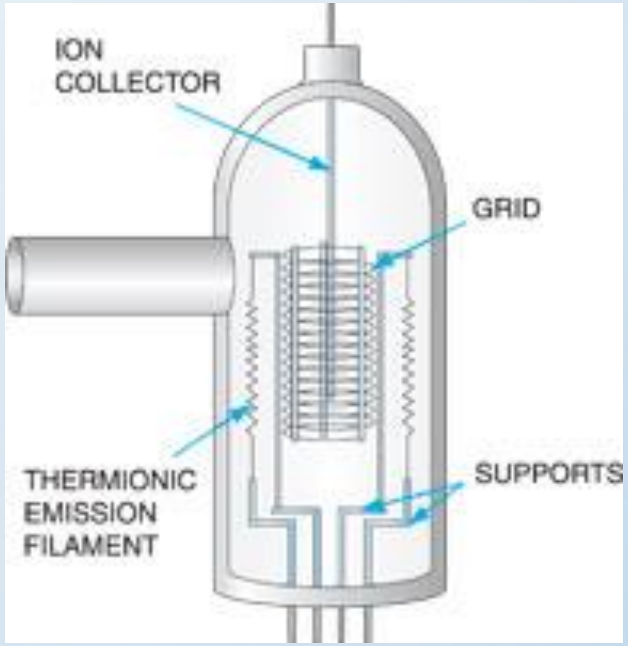
The open-tube manometer:



$$P_{measure} + \rho g h = P_{ref}$$

The vacuum gauge:

1. Electrons are generated by thermionic emission and accelerated by a high voltage from the grid.
2. Residue gas molecules are ionized to have positive charges by the high energy electrons.
3. The ionized gas molecules are collected by the ion collector and the current through the collector gives the number of residue gas molecules.



http://www.lesker.com

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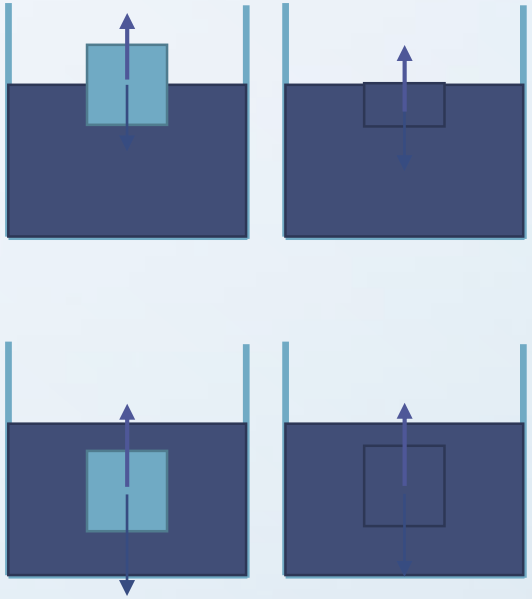
4. BUOYANCY FORCES AND ARCHIMEDE'S PRINCIPLE



Buoyancy force

$$B = \rho_{fluid} g V$$

Floating of a body

$$\rho_b < \rho_{fluid}$$

Apparent weight in a fluid

$$W_{app} = W - B = W - \rho_f g V_{in\ fluid}$$

https://jp.techcrunch.com/2017/07/12/20170711buoyant-raises-10-5m-series-a-round-as-it-looks-to-bring-service-meshes-to-the-masses/

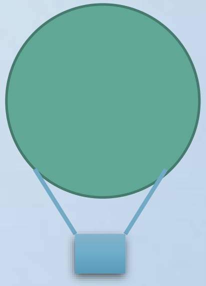
4. BUOYANCY FORCES AND ARCHIMEDE'S PRINCIPLE

Example: What fraction of the volume of an iceberg floating in seawater is visible?

$\rho_{ice} = 920\ (kg/m^3),\ \rho_{sea} = 1025\ (kg/m^3)$

$\rho_{ice} V_{ice} = \rho_{sea} V_{under} \qquad \frac{V_{visible}}{V_{ice}} = 1 - \frac{V_{uder}}{V_{ice}} = 1 - \frac{\rho_{ice}}{\rho_{sea}} = 0.102 = 10.2\%$

Example: A spherical, helium-filled balloon has a radius R of 12.0 m. The balloon, support cables, and basket have a mass m of 196 kg. What maximum load M can the balloon support while it floats at an altitude at which the helium density is 0.160 kg/m^3 and the air density is 1.25 kg/m^3?



$$\rho_{air} \frac{4}{3} \pi R^3 g = \rho_{He} \frac{4}{3} \pi R^3 g + 196g + Mg$$

$$M = (1.25 - 0.16) \times 7238 - 196 = 7690\ kg$$

5. FLUID DYNAMICS

Ideal fluid in motion:

1. **Stead flow:** $\vec{v}(\vec{r}, t) = \vec{v}(\vec{r})$
2. **Incompressible flow:** $\rho(\vec{r}, t) = const$
3. **Nonviscous flow: no resistive force for the fluid to flow**
4. **Irrotational flow: no water vortex**

Streamlines and the continuity equation for fluids:

In a short time Δt, a small volume of water is $\Delta V = A_1 v_1 \Delta t$. After traveling a distance, the same volume of water is reshaped to have different cross-section $\Delta V = A_2 v_2 \Delta t$.

volume flow rate: $A_1 v_1 = A_2 v_2$

5. FLUID DYNAMICS

Example: The cross-sectional area of the aorta (the major blood vessel emerging from the heart) of a person is 3.0 cm^2, and the speed of the blood through it is 30 cm/s. A typical capillary (diameter of 6.0 μm) has a flow speed of 0.050 cm/s. How many capillaries does this person have?

$$A_{aorta} = 3.0\ cm^2,\ A_{capillary} = \pi(3.0 \times 10^{-4})^2 = 2.8 \times 10^{-7}\ cm^2$$

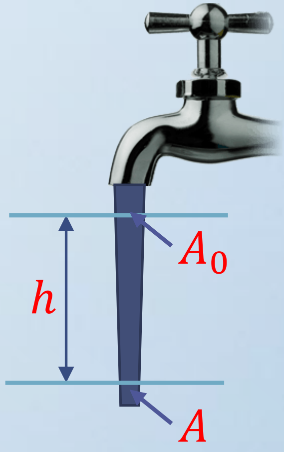
$$A_{aorta} v_{aorta} = N A_{capillary} v_{capillary}$$

$$3.0 \times 30 = N \times (2.8 \times 10^{-7}) \times (0.050)$$

$$N = 6.4 \times 10^9$$

5. FLUID DYNAMICS

Example: The stream of water emerging from a faucet "necks down" as it falls. The indicated cross-sectional areas are A_0 = 1.2 cm^2 and A = 0.30 cm^2. The two levels are separated by a vertical distance h = 45 mm. What is the volume flow rate from the tap?



$Av = A_0 v_0$, v_0, v unknown variables

$$v = \frac{A_0}{A} v_0 = \frac{1.2}{0.30} v_0 = 4.0 v_0$$

$$v^2 = v_0^2 + 2gh$$

$$15 v_0^2 = 2gh = 2 \times 9.8 \times \left(45 \times 10^{-3}\right)$$

$$v_0 = 0.24\, m/s$$

$$R_V = A_0 v_0 = Av = (1.2 \times 10^{-4})(0.24) = 2.9 \times 10^{-5} \frac{m^3}{s} = 29 \frac{cm^3}{s}$$

6. BERNOULLI'S EQUATION

The work done by external pressure:

$$W = F\Delta x = PA\Delta x = P\Delta V$$

$$\Delta W = W_1 - W_2 = (P_1 - P_2)\Delta V$$

$$\Delta W = \Delta K + \Delta U = \frac{1}{2}(\rho\Delta V)\left(v_2^2 - v_1^2\right) + (\rho\Delta V)g(y_2 - y_1)$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g y_2 - \rho g y_1$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

6. BERNOULLI'S EQUATION

Example: Ethanol of density 791 kg/m^3 flows smoothly through a horizontal pipe that tapers in cross-sectional area from A_1 = 1.20X10^{-3} m^2 to $A_2 = A_1 / 2$. The pressure difference between the wide and narrow sections of pipe is 4120 Pa. What is the volume flow rate of the ethanol?

$$A_1 v_1 = A_2 v_2$$

$$v_2 = 2v_1$$

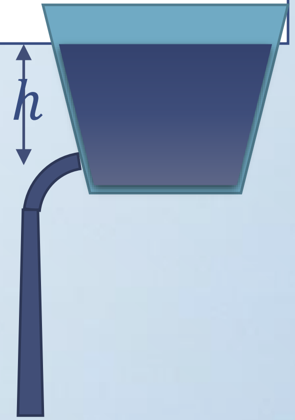
$$\Delta P = \frac{\rho}{2}\left(v_2^2 - v_1^2\right) \rightarrow 4120 = \frac{791}{2} 3v_1^2$$

$$v_1 = 1.86\, m/s$$

$$R_V = A_1 v_1 = 1.20 \times 10^{-3} \times 1.86 = 2.24 \times 10^{-3}\, m^3/s$$

6. BERNOULLI'S EQUATION

Example: A gunman fires a bullet into an open water tank, creating a hole a distance h below the water surface. What is the speed v of the water emerging from the hole? Assume that the density of water is ρ.



$$P_{air} + \rho g h + 0 = P_{air} + 0 + \frac{1}{2}\rho v^2$$

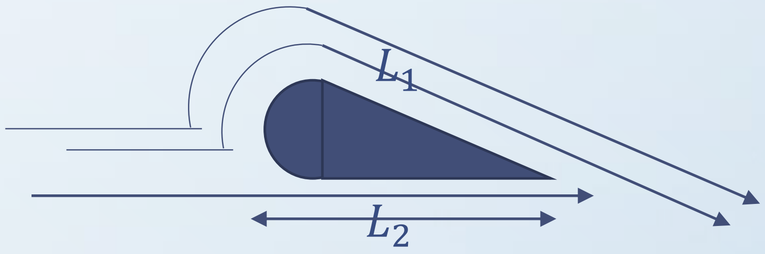
$$v = \sqrt{2gh}$$

7. OTHER APPLICATIONS

Lift of an airplane

Assume the same time period T for gas traveling through different paths.

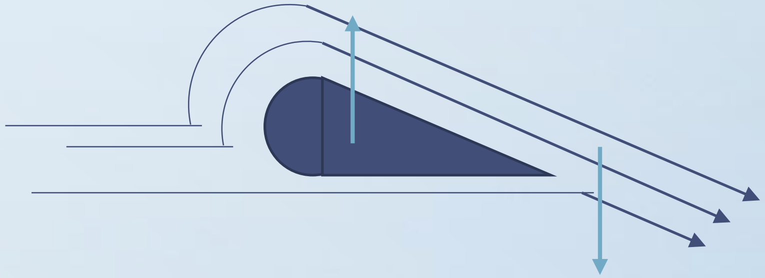
$$v_1 = \frac{L_1}{T}, v_2 = \frac{L_2}{T}, v_1 > v_2$$



Bernoulli's equation:

$$P_1 + \frac{\rho v_1^2}{2} = P_2 + \frac{\rho v_2^2}{2} \rightarrow P_1 < P_2 \rightarrow lift\ force$$

Newton's 3rd Law: action and reaction force



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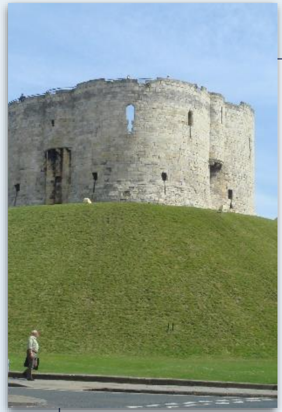

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