Chapter 12
Static Equilibrium & Elasticity-I

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1. The Rigid Body in Equilibrium
2. The Center of Gravity
3. Examples of Rigid Body in Equilibrium
4. Elastic Properties of Solids

1. THE RIGID BODY IN EQUILIBRIUM

The Required Conditions for Equilibrium:

1. The net external force acting on the body is zero.

$$\sum \vec{F}_i = 0$$

2. The net external torque about any point is zero.

$$\sum \vec{\tau}_i = 0$$

The momentum is conserved.

$$\vec{F}_{net} = 0 \rightarrow \frac{d\vec{p}}{dt} = 0 \rightarrow \vec{p} = const$$

The angular momentum is conserved.

$$\vec{\tau}_{net} = 0 \rightarrow \frac{d\vec{L}}{dt} = 0 \rightarrow \vec{L} = const$$

2. THE CENTER OF GRAVITY

The Center of Gravity

$$\vec{r}_{COM} = \frac{\sum m_i \vec{r}_i}{\sum m_i} \qquad \vec{r}_{COG} = \frac{\sum m_i g_i \vec{r}_i}{\sum m_i g_i}$$

If the gravity acceleration is the same for all elements in the body, the COG of the body is coincident with its COM.

$$\vec{r}_{COG} = \frac{\sum m_i g_i \vec{r}_i}{\sum m_i g_i} = \frac{\sum m_i g \vec{r}_i}{\sum m_i g} = \frac{\sum m_i \vec{r}_i}{\sum m_i} = \vec{r}_{COM}$$

$$z_{COG} = \frac{\sum m_i g_i z_i}{\sum m_i g_i} = \frac{\int z g dm}{\int g dm}$$

2. THE CENTER OF GRAVITY

Example: The gravity acceleration at a height z on the Earth surface is $g(z) = \left(\frac{R_0}{R_0+z}\right)^2 g_0$, where R_0 and g_0 are the radius of the Earth and the gravitational acceleration on the Earth surface. Please calculate the center of gravity for a long uniform rod, which has a length of R_0 and a mass per unit length of λ, standing vertically on the ground.

$$dm = \lambda dz \quad z_{COG} = \frac{\int_0^{R_0} \left(\frac{R_0}{R_0+z}\right)^2 g_0 z \lambda dz}{\int_0^{R_0} \left(\frac{R_0}{R_0+z}\right)^2 g_0 \lambda dz}$$

$$\int_0^{R_0} \left(\frac{R_0}{R_0+z}\right)^2 g_0 \lambda dz = R_0^2 g_0 \lambda \left[-\frac{1}{R_0+z}\right]_0^{R_0} = R_0^2 g_0 \lambda \left(-\frac{1}{2R_0} + \frac{1}{R_0}\right) = \frac{R_0 g_0 \lambda}{2}$$

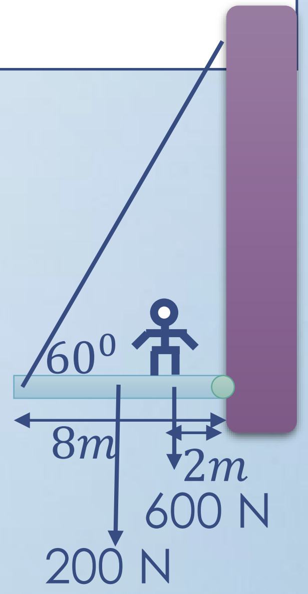
$$\int_0^{R_0} \left(\frac{R_0}{R_0+z}\right)^2 z g_0 \lambda dz = R_0^2 g_0 \lambda \int_0^{R_0} \left(\frac{1}{z+R_0} - \frac{R_0}{(z+R_0)^2}\right) dz = R_0^2 g_0 \lambda (\ln 2 - 1/2)$$

$$z_{COG} = 0.386 R_0 \neq R_0/2$$

3. EXAMPLES OF RIGID BODY IN EQUILIBRIUM

Static Equilibrium, At Rest, $\sum \vec{F}_i = 0$, $\sum \vec{\tau}_i = 0$ about any point

Example: A uniform horizontal beam of length 8 m and weight 200 N is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle of 60° with the horizontal. If a 600-N man stands 2 m from the wall, find the tension in the cable and the force exerted by the wall on the beam.



zero net torque: $8T\sin 60^0 = 4\times 200 + 2\times 600$

$T = 289\ (N)$

$T\cos 60^0 + F_x = 0 \quad F_x = -145\ (N)$

$F_y + 289\sin 60^0 - 200 - 600 = 0$

$F_y = 550\ (N)$

3. EXAMPLES OF RIGID BODY IN EQUILIBRIUM

Example: A uniform ladder of length l and mass m rests against a smooth, vertical wall. If the coefficient of static friction between ladder and the ground is $\mu_s = 0.4$, find the minimum angle θ_{min} such that the ladder does not slip.

$$N_g = mg$$

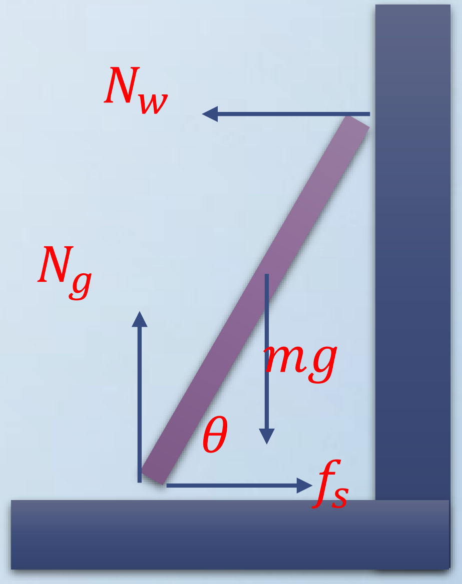
$$N_w = f_s = N_g \mu_s = 0.4mg$$

$$l N_w \sin\theta \geq \frac{l}{2} mg \sin(90^0 - \theta)$$

$$l(0.4mg)\sin\theta \geq \frac{l}{2} mg \cos\theta$$

$$\tan\theta \geq 1.25$$

$$\theta \geq \theta_{min} = 51^0$$

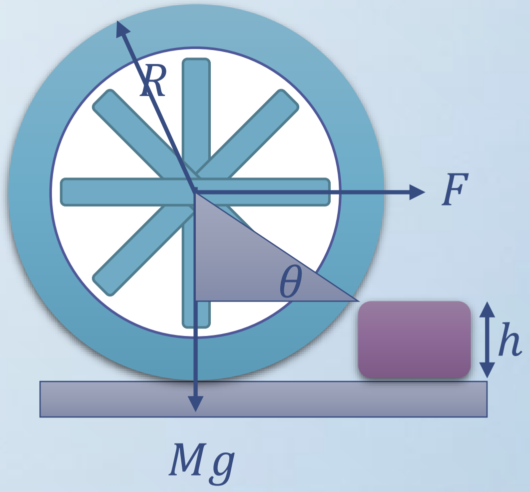


3. EXAMPLES OF RIGID BODY IN EQUILIBRIUM

Example: A wheel of mass M and radius R rests on a horizontal surface against a step of height h (h < R). The wheel is to be raised over the step by a horizontal force F applied to the axle of the wheel as shown. Find the minimum force F_{min} necessary to raise the wheel over the step.

$$F(R-h) \geq Mg\left(\sqrt{R^2-(R-h)^2}\right)$$

$$F \geq F_{min} = \frac{Mg\left(\sqrt{R^2-(R-h)^2}\right)}{R-h}$$



3. EXAMPLES OF RIGID BODY IN EQUILIBRIUM

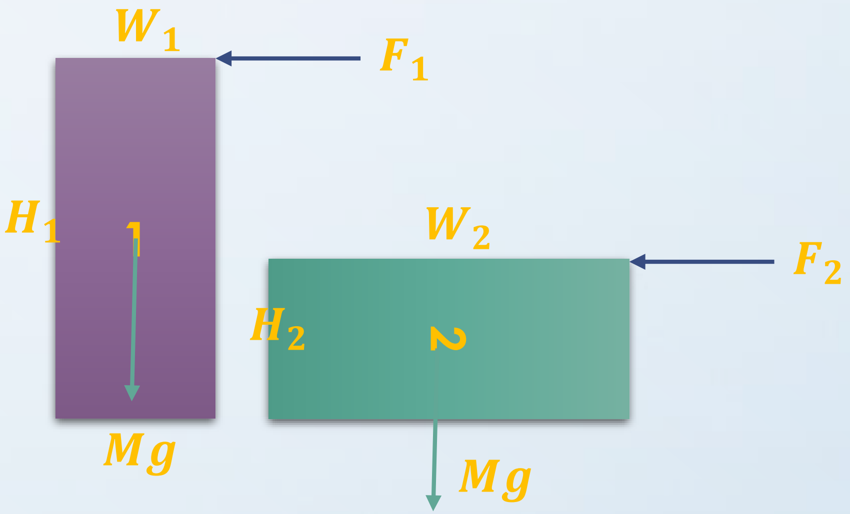
Stability

$$H_1 F_1 \geq \frac{W_1}{2} Mg$$

$$F_{1,min} = \frac{W_1 Mg}{2H_1}$$

$$F_{2,min} = \frac{W_2 Mg}{2H_2} > F_{1,min}$$

Block 2 is more stable.



Indeterminate Equilibrium for Some Structures

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4. ELASTIC PROPERTIES OF SOLIDS

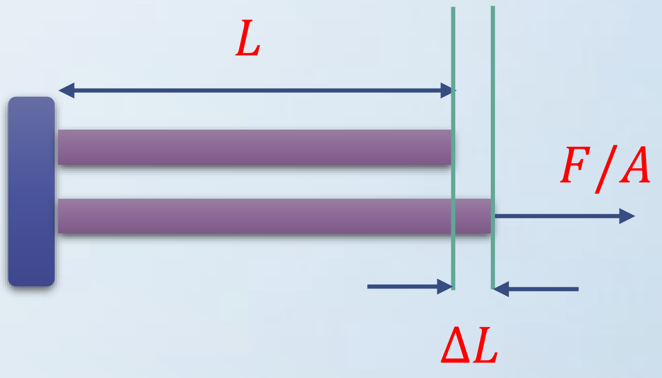
Stress: Deforming force per unit area, stress= F/A

Strain: Unit Deformation, strain= $\Delta L/L$

Elastic Modulus: stress / strain

Young's Modulus – Elasticity in Length

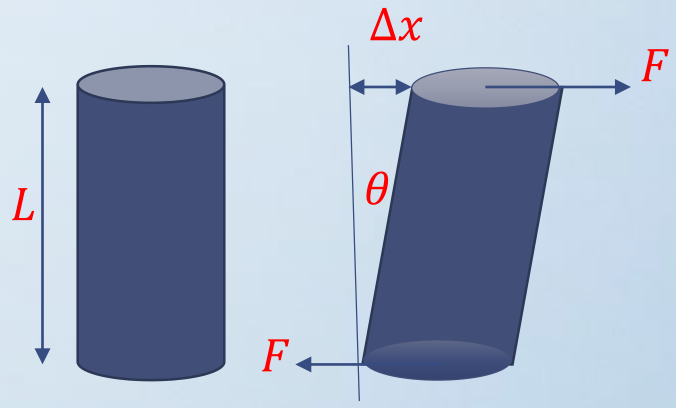
$$Y = \frac{tensile\ stress}{tensile\ strain} = \frac{F/A}{\Delta L/L}$$



Shear Modulus – Elasticity of Shape

$$S = \frac{shear\ stress}{shear\ strain} = \frac{F/A}{\Delta x/L}$$

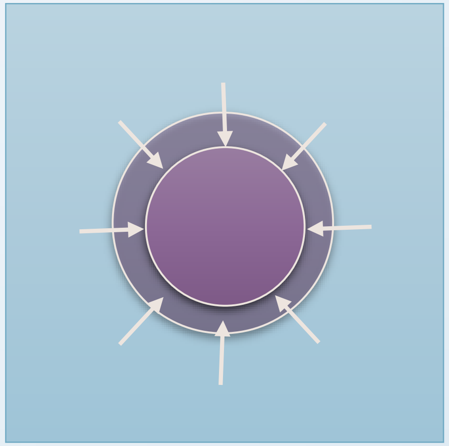
shear strain: $\frac{\Delta x}{L} = \tan\theta$



4. ELASTIC PROPERTIES OF SOLIDS

Bulk Modulus – Volume Elasticity

$$B = \frac{volume\ stress}{volume\ strain} = \frac{\Delta F/A}{\Delta V/V}$$



Material	Young's Modulus (N/m²)	Shear Modulus (N/m²)	Bulk Modulus (N/m²)
Steel	$20X10^{10}$	$8.4X10^{10}$	$6X10^{10}$
Copper	$11X10^{10}$	$4.2X10^{10}$	$14X10^{10}$
Aluminum	$7.0X10^{10}$	$2.5X10^{10}$	$5.0X10^{10}$
Glass	$6.5X10^{10}$	$2.6X10^{10}$	$5.0X10^{10}$
Concrete	$1.7X10^{10}$	$2.1X10^{10}$	

4. ELASTIC PROPERTIES OF SOLIDS

Example: A structural steel rod has a radius R of 9.5 mm and a length L of 81 cm. A 62 kN force stretches it along its length. What are the stress on the rod and the elongation and strain of the rod?

stress: $\frac{F}{A} = \frac{62000}{\pi(0.0095)^2} = 2.2 \times 10^8\ (N/m^2)$

$$Y_{steel} = 20 \times 10^{10} = \frac{stress}{strain}$$

$$strain = \frac{\Delta L}{L} = \frac{2.2 \times 10^8}{20 \times 10^{10}} = 1.1 \times 10^{-3}$$

$$\Delta L = 1.1 \times 10^{-3} L = 0.089\ (cm)$$

4. ELASTIC PROPERTIES OF SOLIDS

Example: A solid copper sphere is initially surrounded by air, and the air pressure exerted on it is 1.0 X 10^5 N/m^2 (normal atmosphere pressure). The sphere is lowered into the ocean to a depth where pressure is 2.0 X 10^7 N/m^2. The volume of the sphere in air is 0.50 m^3. By how much does this volume change once the sphere is submerged?

$$B_{Cu} = 14 \times 10^{10} = \frac{\left(2.0 \times 10^7 - 1.0 \times 10^5\right)}{\Delta V/0.5}$$

$$\Delta V = 7.1 \times 10^{-5}\ (m^3)$$

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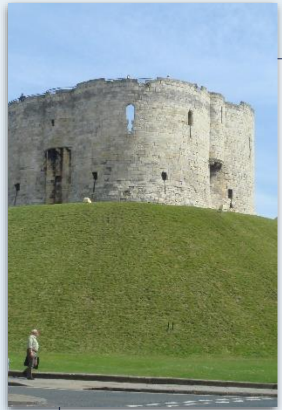

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