# Physics I Lecture06-Circular Motion and Other Applications-I

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# 1. NEWTON'S 2ND LAW IN UNIFORM CIRCULAR MOTION

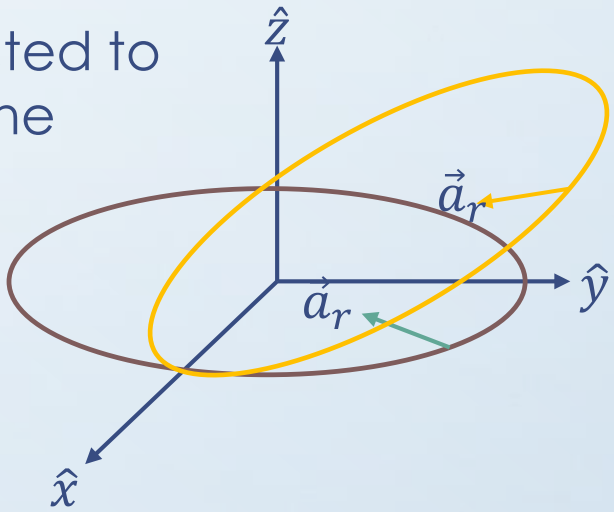
http://www.stephenhodsdenmurray.com/ 2014

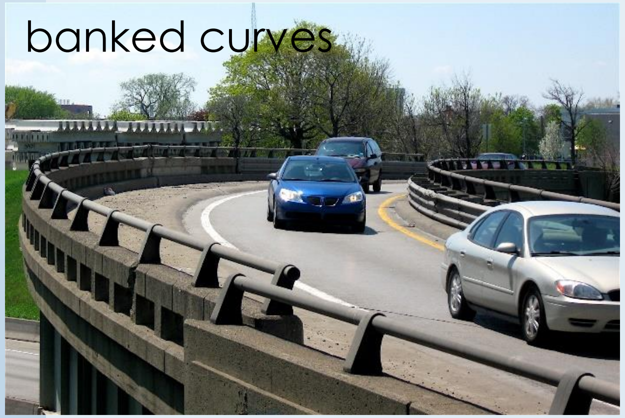
Centripetal Acceleration, Centripetal Force

1. The centripetal force is given at first, then the object can turn its moving direction.

2. The centripetal force is always directed to the center of the trajectory. Find out the plane of the circular trajectory at first.

$$\vec{a}_r = -\frac{v^2}{r}\hat{r}$$




$$\vec{F}_{net,centripetal} = \sum_{i=1}^{N} \vec{F}_i = -m\frac{v^2}{r}\hat{r}$$

# 1. NEWTON'S 2ND LAW IN UNIFORM CIRCULAR MOTION

Example: A car travels on a circular roadway of radius $r$. The roadway is flat. The car travels at a high speed $v$, such that the friction force causing the centripetal acceleration is the maximum possible value. If the same car is now driven on another flat circular roadway of radius $2r$, and the coefficient of friction between the tires and the roadway is the same as on the first roadway, what is the maximum speed of the car such that it does not slide off the roadway?

$$mg\mu_s = F_{centripetal} = m\frac{v^2}{r}$$

$$r' = 2r \qquad\qquad v' = ?$$

$$g\mu_k = \frac{v^2}{r} = \frac{v'^2}{r'} \qquad\qquad v'^2 = \frac{v^2}{r}r' = 2v^2$$

$$v' = \sqrt{2}v$$

# 1. NEWTON'S 2ND LAW IN UNIFORM CIRCULAR MOTION

Example: An object of mass $m = 0.500$ kg is attached to the end of a cord whose length is $l = 1.50$ m. The object is whirled in a horizontal circle. If the cord can withstand a maximum tension of $F = 50.0$ N, what is the maximum speed the object can have before the cord breaks?

$$F_{centripetal} = F = m\frac{v_{max}^2}{l}$$

$$v_{max} = \sqrt{\frac{Fl}{m}} = \sqrt{\frac{50.0 \times 1.50}{0.500}} = 12.2\ (m/s)$$

# 1. NEWTON'S 2ND LAW IN UNIFORM CIRCULAR MOTION

Example: A small object of mass m is suspended from a string of length L. The object revolves in a horizontal circle of radius r with constant speed v. Find (a) the speed of the object, and (b) the period of revolution.

Note that the string tension and the gravitational force give the necessary centripetal force.

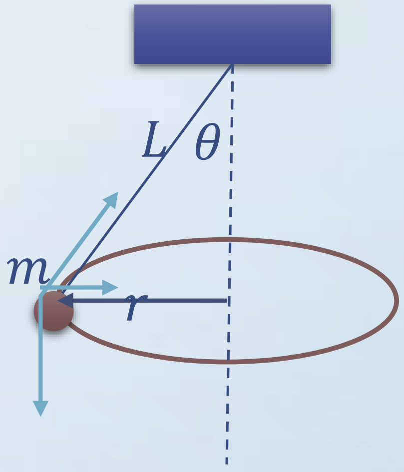
(a) $a_r = g \tan\theta \qquad \tan\theta = \dfrac{r}{\sqrt{L^2 - r^2}}$

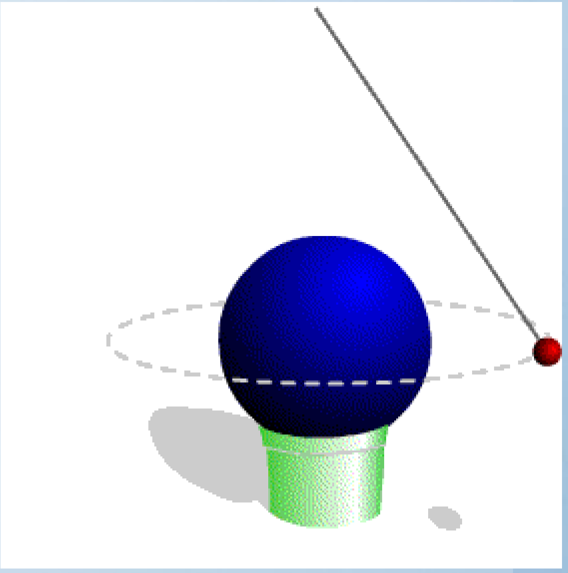
$$a_r = \frac{v^2}{r}$$

$$v = \sqrt{r a_r} = \sqrt{r g \tan\theta} = \sqrt{r^2 g / \sqrt{L^2 - r^2}}$$

(b)

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{r^2 g / \sqrt{L^2 - r^2}}} = 2\pi \sqrt{\frac{\sqrt{L^2 - r^2}}{g}}$$



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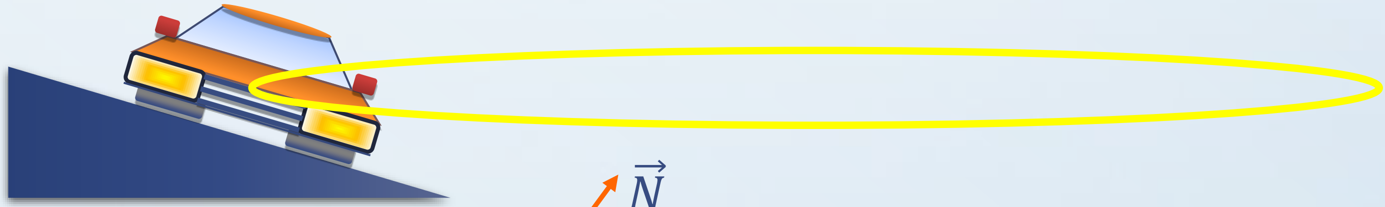
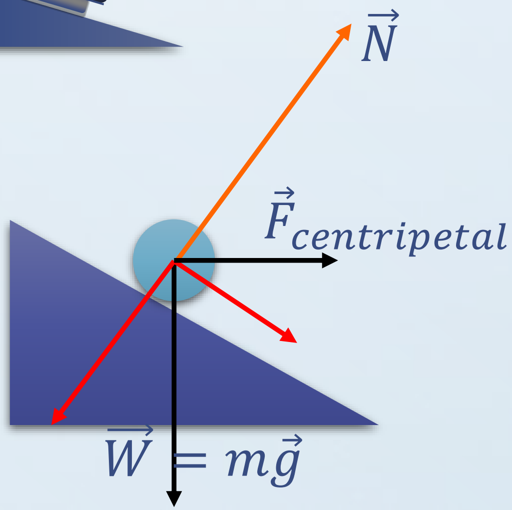
# 1. NEWTON'S 2$^{ND}$ LAW IN UNIFORM CIRCULAR MOTION

Example: A curve of radius $r = 30$ m is banked at an angle $\theta$. Find $\theta$ for which a car can round the curve at $v = 40$ km/h even if the road is covered with ice that friction is negligible.

Find out the plane of the circular motion at first,
then find out the required centripetal force.

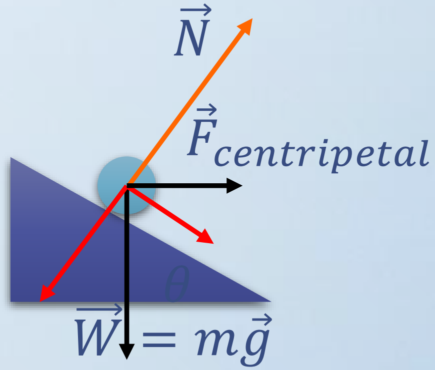





$$\vec{F}_{centripetal} = \vec{N} + \vec{W}$$

# 1. NEWTON'S 2ND LAW IN UNIFORM CIRCULAR MOTION

Example: A curve of radius $r = 30.0$ m is banked at an angle $\theta$. Find $\theta$ for which a car can round the curve at $v = 40.0$ km/h even if the road is covered with ice that friction is negligible.




$$a_r = g \tan\theta$$

$$r = 30 \text{ m} \quad v = 40\frac{km}{h} = 11.1 \text{ (m/s)}$$

$$g\tan\theta = \frac{v^2}{r} \qquad \tan\theta = \frac{v^2}{gr} \cong 0.419$$

$$\theta \cong 22.7^0$$

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# 2. NONUNIFORM CIRCULAR MOTION

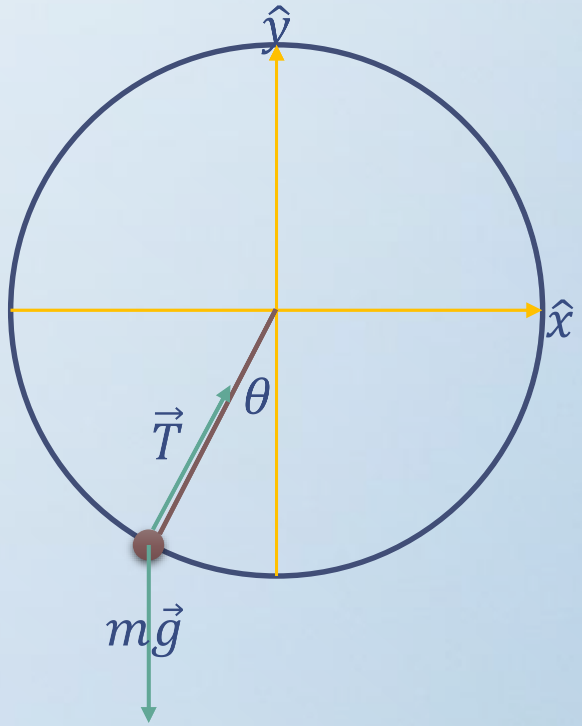
Example: A small sphere of mass m is attached to the end of a cord of length R which rotates under the influence of the gravitational force in a vertical circle about a fixed point O. Let us determine the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle theta with the vertical.

$$F_r = T = m\frac{v(\theta)^2}{R} + mg\cos\theta$$

$$F_t = -mg\sin\theta$$

To solve the problem, you need to specify the condition that, for example, the gravitation totally gives the centripetal force as the sphere is on the top.

$$\theta = \pi, T = 0, \frac{mv(\pi)^2}{R} - mg = 0, v(\pi) = \sqrt{gR}$$



# 2. NONUNIFORM CIRCULAR MOTION

Example: A small sphere of mass m is attached to the end of a cord of length R which rotates under the influence of the gravitational force in a vertical circle about a fixed point O. Let us determine the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle theta with the vertical.

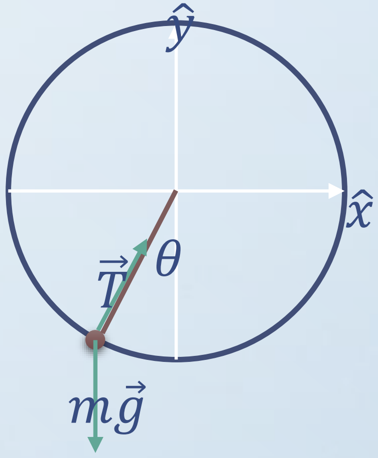
$$v(\pi) = \sqrt{gR} \qquad s = R\theta$$

$$F_t = -mg\sin\theta = ma_t = m\frac{d^2s}{dt^2} = mR\frac{d^2\theta}{dt^2}$$

$$R\frac{d^2\theta}{dt^2} = -g\sin\theta \qquad R\frac{d^2\theta}{dt^2}\frac{d\theta}{dt} = -g\sin\theta\frac{d\theta}{dt}$$

$$R\omega\frac{d\omega}{dt} = -g\sin\theta\frac{d\theta}{dt} \qquad R\frac{d\left(\omega^2/2\right)}{dt} = -g\sin\theta\frac{d\theta}{dt}$$

$$Rd\left(\omega^2/2\right) = -g\sin\theta\, d\theta$$



# 2. NONUNIFORM CIRCULAR MOTION

Example: A small sphere of mass m is attached to the end of a cord of length R which rotates under the influence of the gravitational force in a vertical circle about a fixed point O.  Let us determine the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle theta with the vertical.

$$v(\pi) = R\omega(\pi) = \sqrt{gR} \quad Rd(\omega^2/2) = -g\sin\theta\, d\theta$$

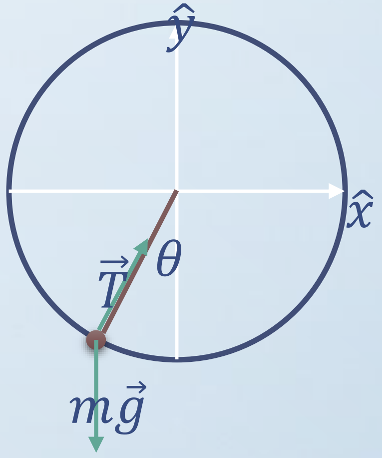
$$R\int_{\omega=\sqrt{g/R}}^{\omega(\theta)} d(\omega^2/2) = -g\int_{\pi}^{\theta} \sin\theta'\, d\theta'$$

$$R\left(\frac{\omega^2}{2} - \frac{g}{2R}\right) = g(\cos\theta + 1)$$

$$\frac{d\theta}{dt} = \omega = \sqrt{\frac{g}{R}(2\cos\theta + 3)}$$

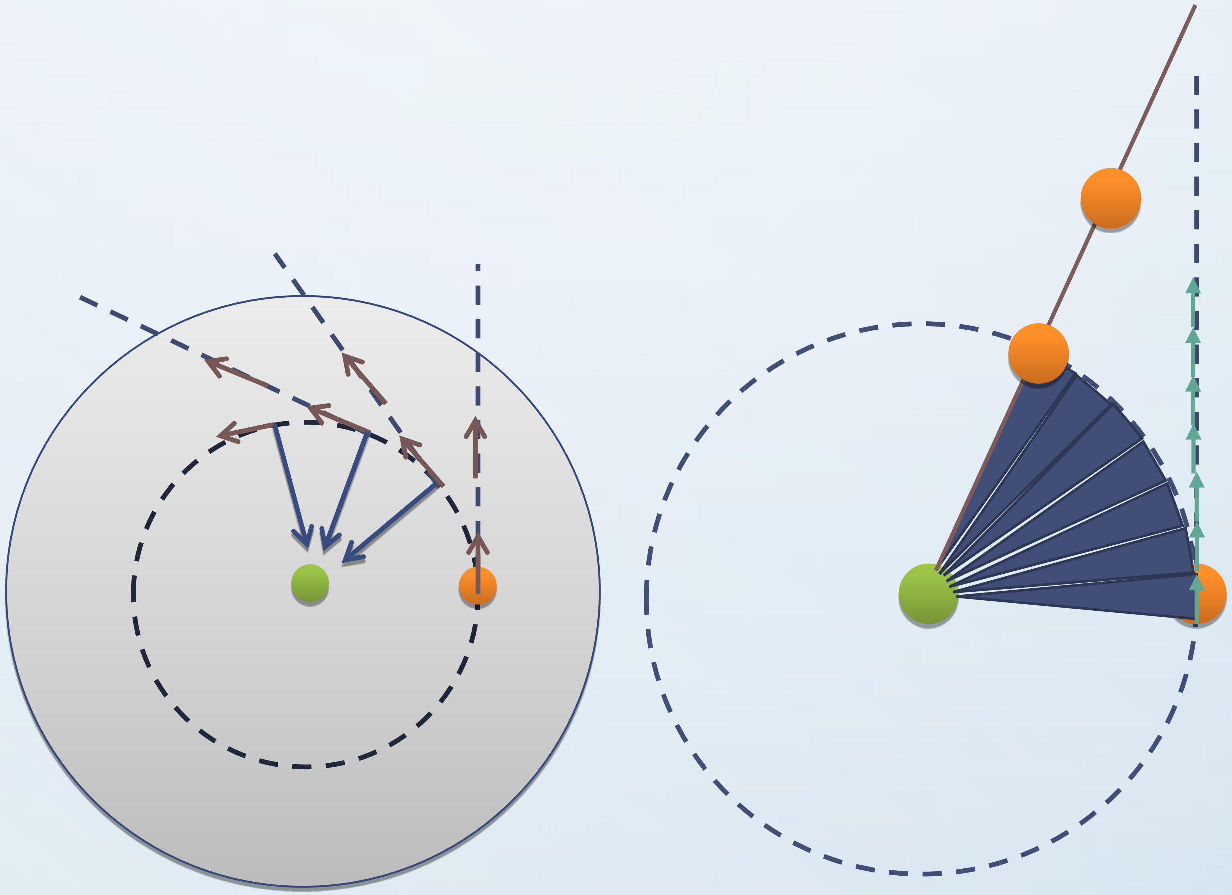
$$\omega(\theta = 0) = \sqrt{5\frac{g}{R}}$$

$$\omega(\theta = \pi/2) = \sqrt{3\frac{g}{R}}$$



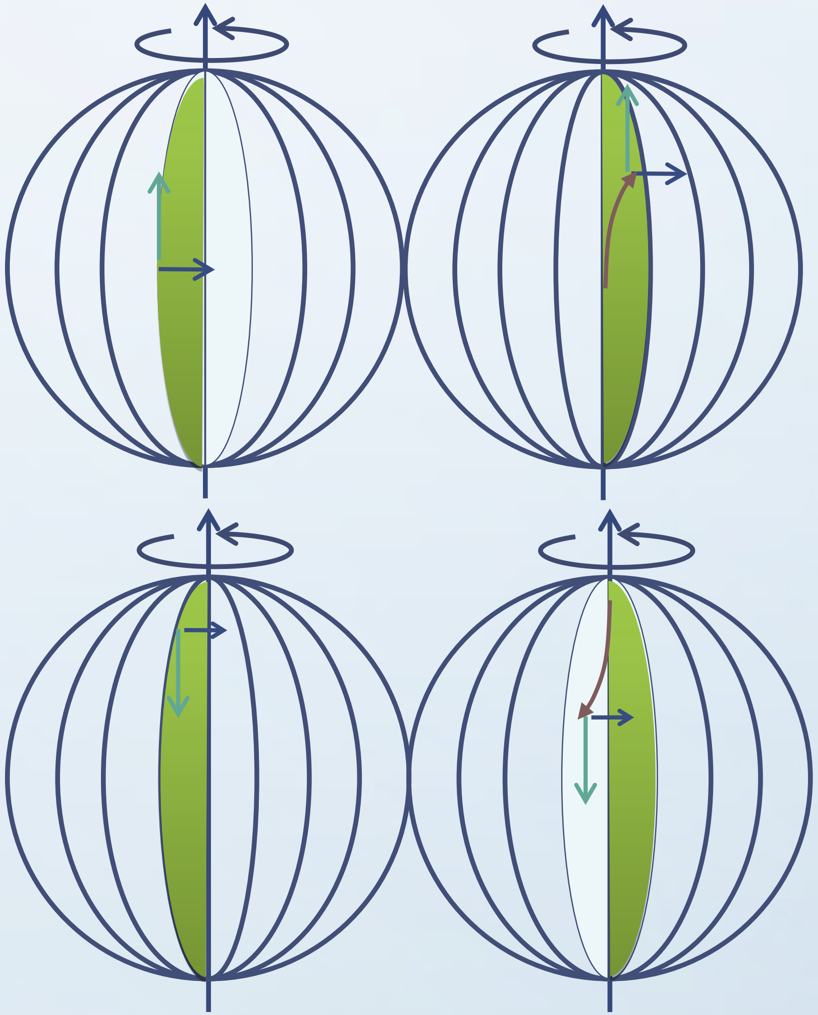
# 3. MOTION IN ACCELERATED FRAMES

Centripetal or Centrifugal Forces? What's The Mechanism of Spin Dryer?



# 3. MOTION IN ACCELERATED FRAMES

The Coriolis Force:



http://nqpl.ep.nctu.edu.tw/simul-physik/part-3/

搜尋...

動力學與轉向運動 – Simul-...

檔案(F) 編輯(E) 檢視(V) 我的最愛(A) 工具(T) 說明(H)

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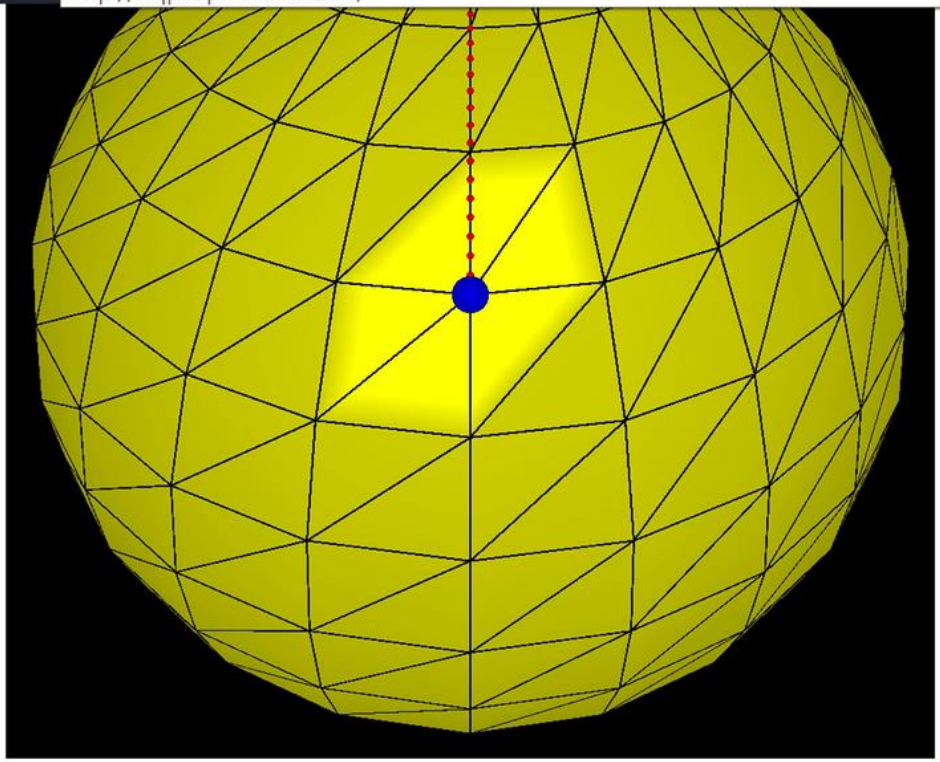
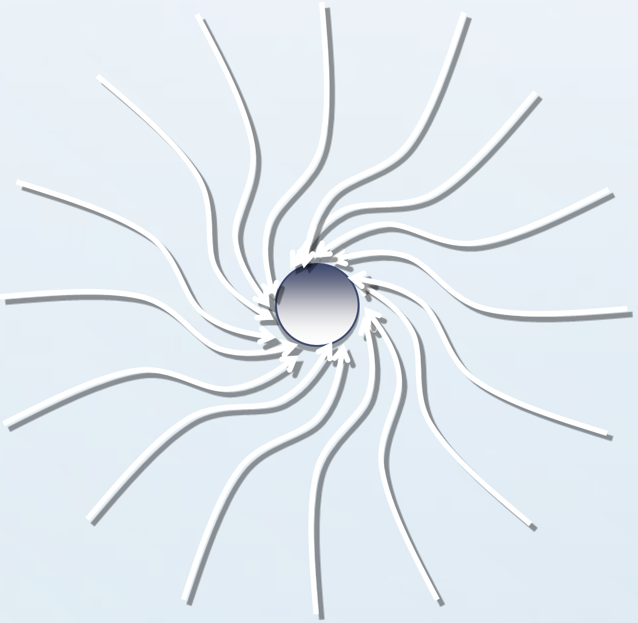
你好，wbjian



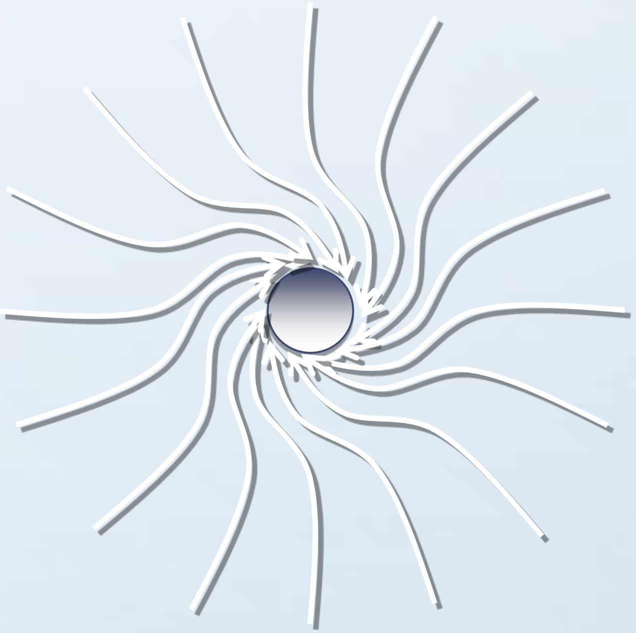
柯氏力(Coriolis force)的產生主要是因為我們以地球為座標系統，而地球是非慣性座標系統，在此座標系統下觀察直線慣性運動，會因座標轉換而產生一個相對的作用力-柯氏力。下圖中黃色球體模擬地球，藍色球體為物體在地球上的起始位置，紅色虛線表示預期的運動軌跡，當按下Start鍵啟動後，綠色的球是球體拋出後的運動，起始速度有地球

# 3. MOTION IN ACCELERATED FRAMES

Typhoon is counterclockwise in the northern hemisphere of the Earth.



Northern Hemisphere



Southern Hemisphere

https://giphy.com/gifs/south-korea-10mG6dlMM4nWxi

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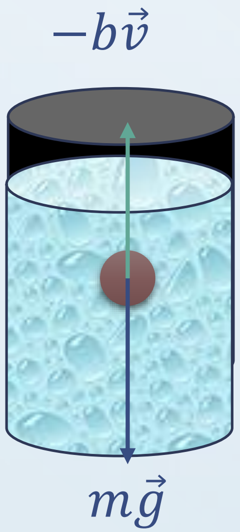
# 4. MOTION IN THE PRESENCE OF RESISTIVE FORCE

Two Types of Resistive Force:

1. Objects in Liquid. The resistive force is proportional to the velocity.

$$\vec{F}_{res} = -b\vec{v}$$



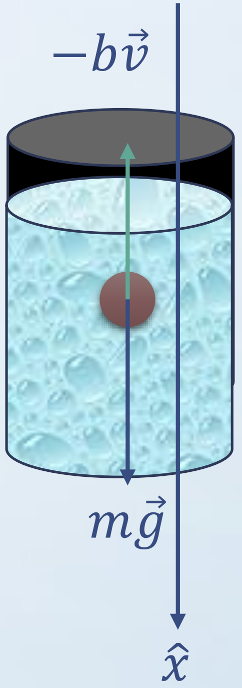

2. Objects in Gas. The resistive force is proportional to the square of the speed.

$$\vec{F}_{res} = -\frac{D\rho A}{2} v^2 \hat{v}$$

$D$: drag coefficient (~0.6),
$\rho$: density of gas,
$A$: cross-sectional area

https://giphy.com/gifs/LKKzKm9jMdoli

# 4. MOTION IN THE PRESENCE OF RESISTIVE FORCE




Object in Liquid. Start falling from rest.

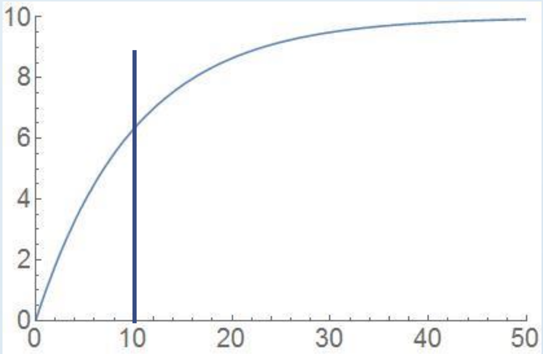
$$F = mg - bv = ma \quad \Rightarrow \quad v_t = \frac{mg}{b}$$

$$m\frac{dv}{dt} = mg - bv$$

$$\int_0^v \frac{m}{mg - bv} dv = \int_0^t dt$$

$$-\frac{m}{b}\left(\ln\left(\frac{mg - bv}{mg}\right)\right) = t \quad \Rightarrow \quad v(t) = \frac{mg}{b}\left(1 - e^{-\frac{bt}{m}}\right)$$

Time Constant: $\tau = m/b$



# 4. MOTION IN THE PRESENCE OF RESISTIVE FORCE

Example: A Sphere Falling in Oil.
A small sphere of mass 2.00 g is released from rest in a large vessel filled with oil. The sphere approaches a terminal speed of 5.00 cm/s. Determine (a) the time constant $\tau$ and (b) the time it takes the sphere to reach 90% of its terminal speed.

(a) $$v_t = 0.05(m/s) = \frac{mg}{b}$$

$$b = \frac{0.002 \times 9.8}{0.05} = 0.392(kg/s)$$

$$\tau = \frac{m}{b} = 5.1 \times 10^{-3}(s)$$

(b) $$v(t) = \frac{mg}{b}\left(1 - e^{-\frac{bt}{m}}\right)$$

$$t = -\tau \ln\left(\frac{mg - b \times \frac{0.9mg}{b}}{mg}\right) = 11.7 \times 10^{-3}(s)$$

# 4. MOTION IN THE PRESENCE OF RESISTIVE FORCE

For objects dropping in air, terminal speed is:

$$F = mg - \frac{D\rho A}{2} v^2 = ma$$

The condition to reach the terminal speed is $a = 0$.

$$\Rightarrow mg - \frac{D\rho A}{2} v_t^2 = 0 \Rightarrow v_t = \sqrt{\frac{2mg}{D\rho A}}$$

For a human body in the free fall motion in air, the terminal speed is:

$$v_t = \sqrt{\frac{2 \times 60 \times 9.8}{0.6 \times (0.028/0.0224) \times 1}} \cong 39.6\left(\frac{m}{s}\right) = 143\left(\frac{km}{h}\right)$$

| Object | Mass (kg) | Cross-Section ($m^2$) | $V_t$ (m/s) |
|---|---|---|---|
| **Sky diver** | 75 | 0.70 | **60** |
| **Bassball** | 0.145 | $4.2\text{x}10^{-3}$ | **43** |
| **Golfball** | 0.046 | $1.4\text{x}10^{-3}$ | **44** |
| **Hailstone** | $4.8\text{x}10^{-4}$ | $7.9\text{x}10^{-5}$ | **14** |
| **Raindrop** | $3.4\text{x}10^{-5}$ | $1.3\text{x}10^{-5}$ | **9.0** |

https://giphy.com/gifs/3ohs7U04OERhUtefja

# 4. MOTION IN THE PRESENCE OF RESISTIVE FORCE

Object in Air:

Example: If a falling cat reaches a first terminal speed of 97 km/h while it is tucked in and then stretches out, doubling A, how fast is it falling when it reaches a new terminal speed?

$$v_t = \sqrt{\frac{2mg}{D\rho A}} \qquad v_{t1} = 97(km/h)$$

$$\frac{v_{t2}}{v_{t1}} = \sqrt{\frac{A_1}{A_2}} \qquad v_{t2} = \frac{97}{\sqrt{2}} = 69(km/h)$$

# 4. MOTION IN THE PRESENCE OF RESISTIVE FORCE

Object in Air:

Example: A raindrop with radius R = 1.5 mm falls from a cloud that is at height h = 1200 m above the ground. The drag coefficient D for the drop is 0.60. Assume that the drop is spherical throughout its fall. The density of water $\rho_w$ is 1000 kg/m$^3$, and the density of air $\rho_a$ is 1.2 kg/m$^3$.

$$A = \pi R^2 = \pi(0.0015)^2 = 7.1 \times 10^{-6}(m^2)$$

$$m = 1000 \times 4\pi(0.0015)^3/3 = 1.4 \times 10^{-5}(kg)$$

$$v_t = \sqrt{\frac{2mg}{D\rho A}} = \sqrt{\frac{2 \times 1.4 \times 10^{-5} \times 9.8}{0.6 \times 1.2 \times 7.1 \times 10^{-6}}} = 7.3\left(\frac{m}{s}\right) = 26(\frac{km}{h})$$

# 5. NUMERICAL INTEGRATION – EULER'S METHODS

If you cannot solve the exact solutions of x(t), you need to express it numerically.  In the real world you may always need the numerical representation of motion.

Example: Consider the initial value problem $\frac{dy}{dx} = 0.1\sqrt{y} + 0.4x^2$, $y(2) = 4$. Use Euler's method to obtain an approximation of $y(2.5)$ using $\Delta x = 0.1$ and $\Delta x = 0.05$.

$\Delta x = 0.1$

| | x | y | y' |
|---|---|---|---|
| 0 | 2 | 4 | 1.8 |
| 1 | 2.1 | 4.18 | 1.9685 |
| 2 | 2.2 | 4.3768 | 2.1452 |
| 3 | 2.3 | 4.5914 | 2.3303 |
| 4 | 2.4 | 4.8244 | 2.5236 |

B3=B2+0.1

C3=C2+0.1*D2

D3=0.1*SQRT(C3)+0.4*B3*B3

# 5. NUMERICAL INTEGRATION – EULER'S METHODS

Example: Consider the initial value problem $\frac{dy}{dx} = 0.1\sqrt{y} + 0.4x^2$, $y(2) = 4$. Use Euler's method to obtain an approximation of $y(2.5)$ using $\Delta x = 0.1$ and $\Delta x = 0.05$.

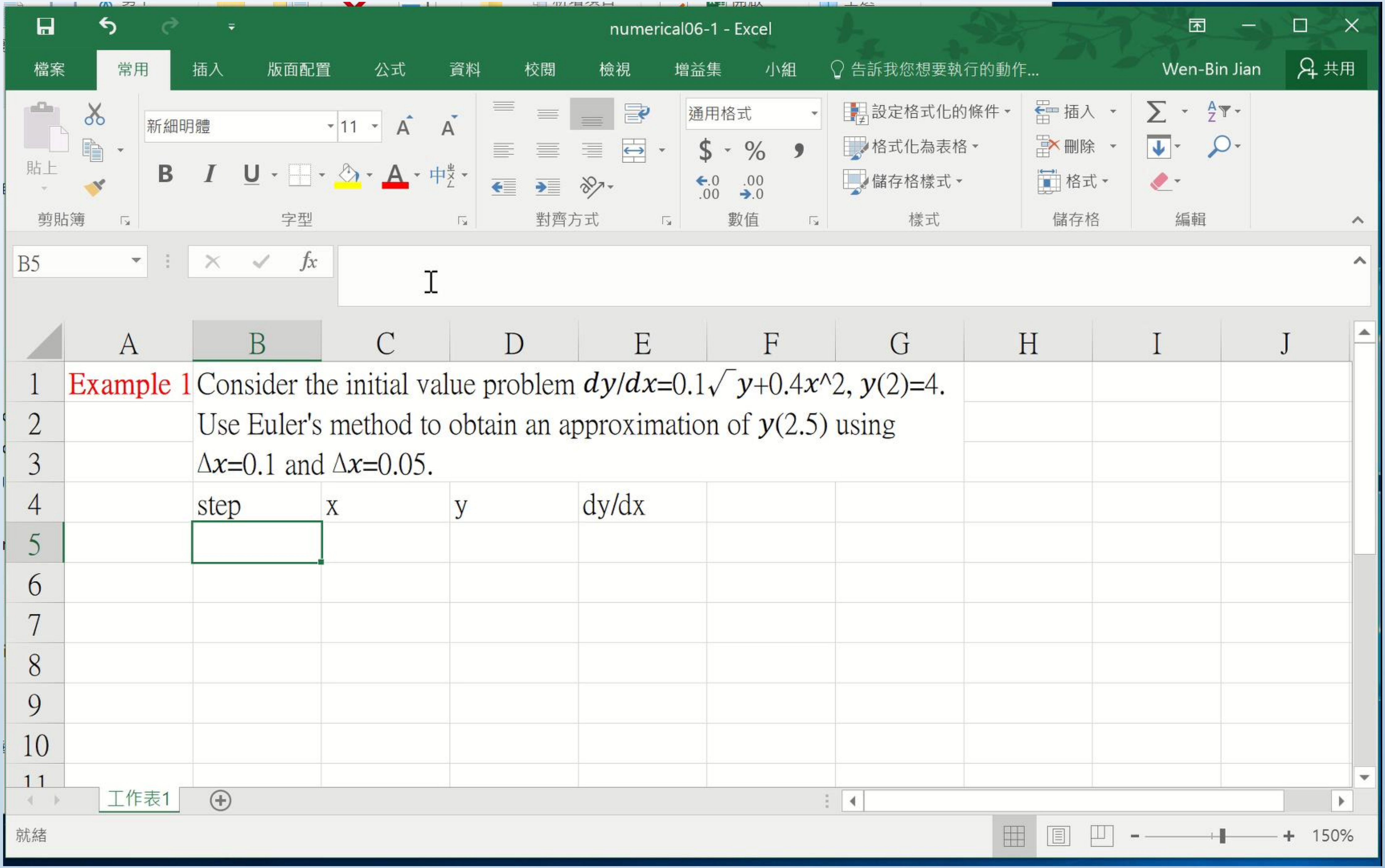
$\Delta x = 0.05$

| | x | y | y' |
|---|---|---|---|
| 0 | 2 | 4 | 1.8 |
| 1 | 2.05 | 4.09 | 1.883237 |
| 2 | 2.1 | 4.184162 | 1.968552 |
| 3 | 2.15 | 4.282589 | 2.055944 |
| 4 | 2.2 | 4.385387 | 2.145413 |
| 5 | 2.25 | 4.492657 | 2.236959 |
| 6 | 2.3 | 4.604505 | 2.330581 |
| 7 | 2.35 | 4.721034 | 2.426279 |
| 8 | 2.4 | 4.842348 | 2.524053 |
| 9 | 2.45 | 4.968551 | 2.623902 |
| 10 | 2.5 | 5.099746 | 2.725826 |

B3=B2+0.05

C3=C2+0.05*D2

D3=0.1*SQRT(C3)+0.4*B3*B3


numerical06-1 - Excel
檔案
常用
插入
版面配置
公式
資料
校閱
檢視
增益集
小組
告訴我您想要執行的動作...
Wen-Bin Jian
共用
貼上
剪貼簿
新細明體
11
字型
對齊方式
通用格式
數值
設定格式化的條件
格式化為表格
儲存格樣式
樣式
插入
刪除
格式
儲存格
編輯
B5
Example 1
Consider the initial value problem dy/dx=0.1√y+0.4x^2, y(2)=4.
Use Euler's method to obtain an approximation of y(2.5) using
Δx=0.1 and Δx=0.05.
step
x
y
dy/dx
工作表1
就緒
150%

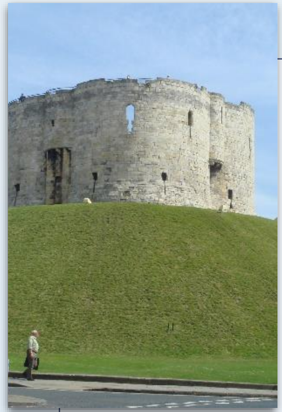
# 5. NUMERICAL INTEGRATION – EULER'S METHODS

Example: Compare the numerical results with the integrated function for the constant acceleration of $a = 2.0$ (m/s$^2$), $v(0) = 0$ (m/s), $x(0) = 0$ (m).

| a = 2.0 m/s$^2$, $\Delta t = 0.1$ s, v(0) = 0, x(0) = 0 - | | | | | v(t) = at, x(t)=1/2*at$^2$ | |
|---|---|---|---|---|---|---|
| **Step** | t | v | x | | v(t) | **x(t)** |
| **0** | 0. | 0 | 0 | | 0 | **0** |
| **1** | 0.1 | 0.2 | 0 | | 0.2 | **0.01** |
| **2** | 0.2 | 0.4 | 0.02 | | 0.4 | **0.04** |
| **3** | 0.3 | 0.6 | 0.06 | | 0.6 | **0.09** |
| **4** | **0.4** | **0.8** | **0.12** | | **0.8** | **0.16** |

# ACKNOWLEDGEMENT

國立交通大學理學院
自主愛學習計畫



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