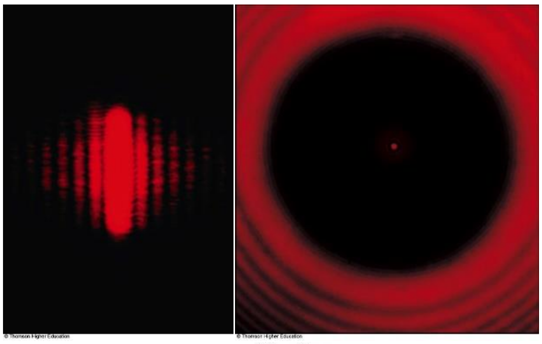
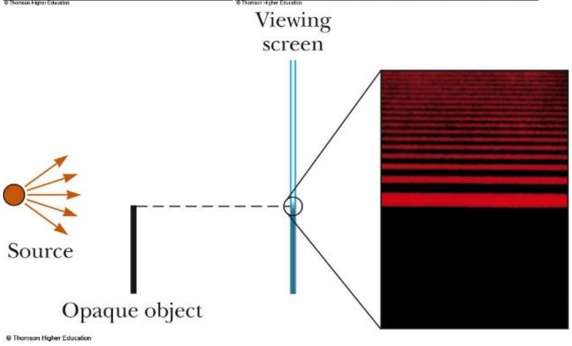
# Chapter 37 Diffraction Patterns and Polarization

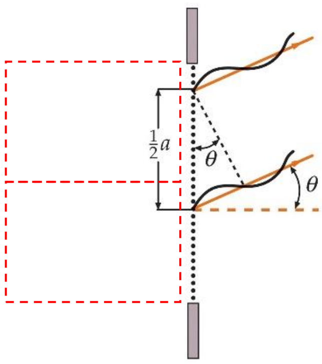
## 37.1 Introduction to Diffraction Pattern

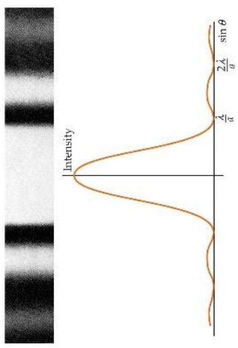
Light suffering from scattering will enhance the feature of the point-like source. For the point-like source, the divided light sources display strong interference effects.






## 37.2 Dffraction Pattern from Narrow Slits



Diffraction – one kind of interference

The first zeroes in the intensity occur at $\frac{a}{2}\sin\theta = \frac{\lambda}{2}$ (destructive interference).

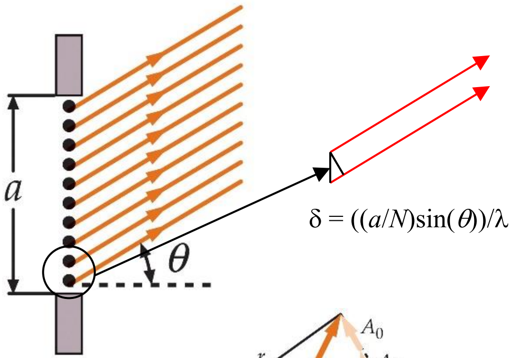
Zero intensity occurs at $a\sin\theta = m\lambda$. Note that the condition is incorrect at $m = 0$.

Example: In a lecture demonstration of single-slit diffraction, a laser beam of wavelength 700 nm passes through a vertical slit 0.2 mm wide and hits a screen 6 m away. Find the width of the central diffraction maximum on the screen.

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2} \quad \text{-->} \quad \frac{a}{2}\tan\theta \sim \frac{\lambda}{2} \quad \text{-->} \quad \frac{a}{2}\frac{y}{L} \sim \frac{\lambda}{2}$$

$$\text{-->} \quad 2y \sim 2\frac{\lambda L}{a} = 2\frac{700}{0.2\times10^{6}}6 = 0.042 \text{ m}$$

# Intensity of Single-Slit Diffraction Patterns




Assume N equally spaced sources:

$A_{max}\sin(kx - \omega t)$

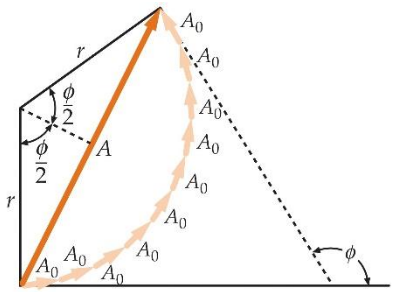
divided into N subwaves

$A_0\sin(kx - \omega t + n\delta) = A_0\sin(\alpha + n\delta)$

$A_{max} = NA_0$

Assume the phase difference between the first wave and the last wave is $\phi$

--> $\phi = N\delta$




The superposition rule -> the total amplitude is:

$A = 2r\sin(\phi/2)$ & let $A_{max} = NA_0$ → $\phi = \frac{A_{max}}{r}$

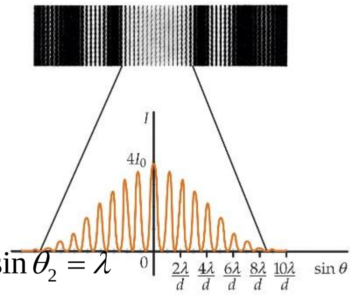
→ $r = \frac{A_{max}}{\phi}$

--> $A = 2\frac{A_{max}}{\phi}\sin(\phi/2)$

$$I = I_0\left(\frac{A}{A_{max}}\right)^2 = I_0\left(\frac{\sin(\phi/2)}{\phi/2}\right)^2$$ & $\phi$ is related to the path difference as

$$\frac{\phi}{2\pi} = \frac{a\sin\theta}{\lambda} \rightarrow I = I_0\left(\frac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda}\right)^2$$

## Interference-Diffraction Pattern of Two Slits




The separation *d* of the two slits is 10 times the width *a* of each slit -->

d produces interference at $d\sin\theta_1 = n\lambda$

a produces zero intensity at $\frac{a}{2}\sin\theta_2 = \lambda\left(m+\frac{1}{2}\right)$ → $a\sin\theta_2 = \lambda$

$d > a$ --> $\theta_1 < \theta_2$ --> $y_1 < y_2$

Example: Two slits of width a = 0.015 mm are separated by a distance d = 0.06 mm and are illuminated by light of wavelength $\lambda = 650$ nm. How many bright fringes are seen in the central diffraction maximum?

width of diffraction maximum: $2\frac{y}{L} = 2\tan\theta \sim 2\sin\theta = 2\frac{\lambda}{a}$ --> $w = 2y = 2\frac{L\lambda}{a}$

interference maximum occurs at: $\frac{\Delta y}{L} = \tan\theta \sim \sin\theta = \frac{\lambda}{d}$ --> $\Delta y = \frac{L\lambda}{d}$

number: $\frac{w}{\Delta y} - 1 = 2\frac{d}{a} - 1 = 2\frac{0.06}{0.015} - 1 = 7$

## Intensity of Two-Slit Diffraction Patterns

$$I = I_0\left(\frac{\sin(\phi/2)}{\phi/2}\right)^2 \cos^2(\delta/2)$$

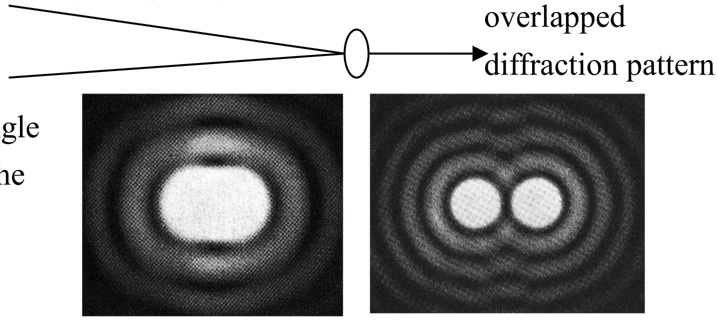
$$\frac{\phi}{2\pi} = \frac{a\sin\theta}{\lambda} \quad \& \quad \frac{\delta}{2\pi} = \frac{d\sin\theta}{\lambda}$$

# 37.3 Resolution of Single-Slit and

# Circular Aperatures

Single Slit: From diffraction pattern of a single slit, the first minimum occurs at
$a\sin\theta = \lambda$

Single Hole: The angle $\theta$ subtended by the first diffraction minimum is related to the wavelength and the diameter of the opening D by
$D\sin\theta_{\min} = 1.22\lambda$




Two point sources subtended an angle $\alpha$ at a circular aperture far from the sources:
Rayleigh's criterion for resolution:

$$\alpha_c = \theta_{\min} \sim 1.22\frac{\lambda}{D}$$

Example: Light of wavelength 500 nm, near the center of the visible spectrum, enters a human eye. Although pupil diameter varies from person to person, estimate a daytime diameter of 2 mm. (a) Estimate the limiting angle of resolution for this eye, assumes its resolution is limited by diffraction.

$$D\sin\theta_{\min} = 1.22\lambda \rightarrow \theta_{\min} \approx 1.22\frac{\lambda}{D} = 1.22\frac{500}{2000000} = 3\times10^{-4}\text{ rad}$$

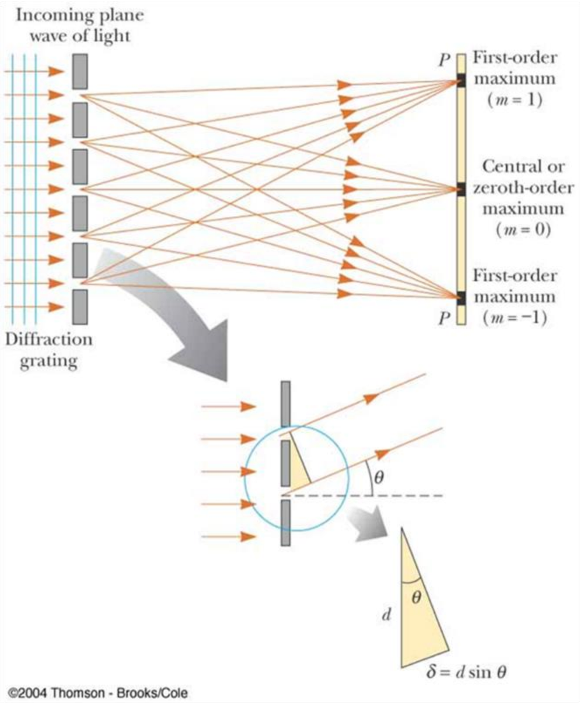
(b) Determine the minimum separation distance d between two point sources that the eye can distinguish if the point sources are a distance L = 25 cm from the observer.

$$\theta \approx \frac{d}{L} \rightarrow d = L\theta = 25\times3\times10^{-4} = 8\times10^{-3}\ \text{cm}$$

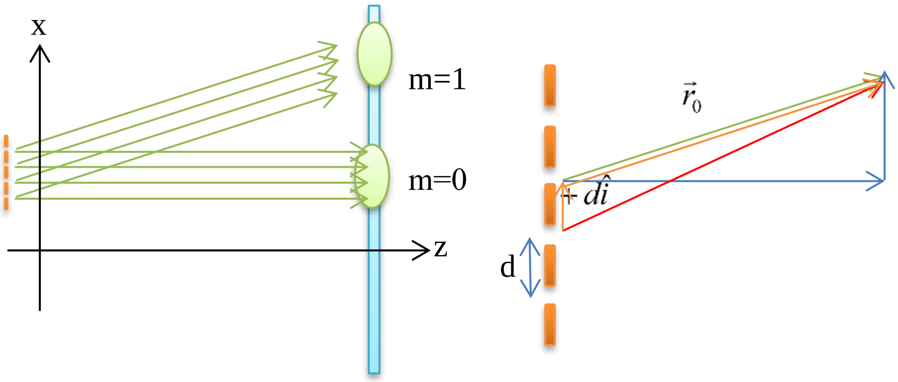
Example: The Keck telescope at Mauna Kea, Hawaii, has an effective diameter of 10 m. What is its limiting angle of resolution for 600-nm light?

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\frac{6\times10^{-7}}{10} = 7.3\times10^{-8}\ \text{(rad)}$$

# <u>37.4 The Diffraction Grating</u>




$$d\sin\theta_{bright} = m\lambda$$

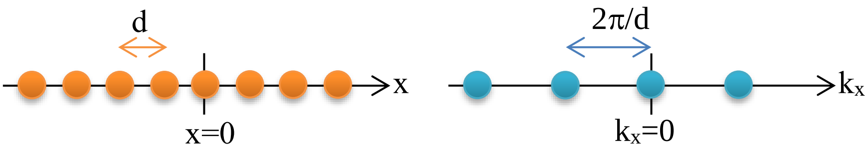



Propagation vector: $\vec{k} = k_x\hat{i} + k_z\hat{k}$, Path – displacement vector: $\vec{r} = d\hat{i} + \vec{r}_0$

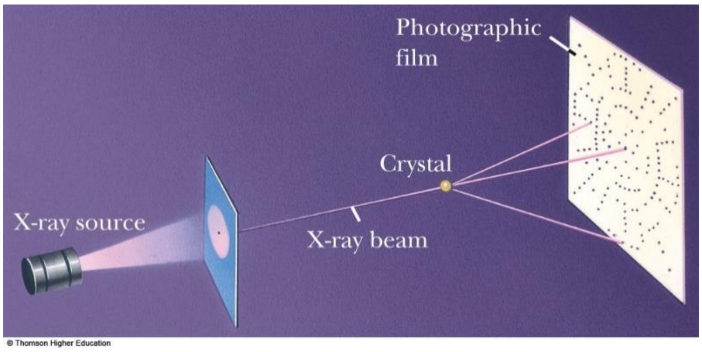
The phase difference: $\delta = \vec{k}\cdot(md\hat{i}) = mk_x d = 2\pi m$ for constructive interference

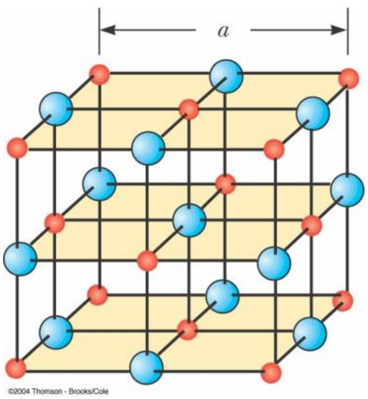
The condition of $k_x = \dfrac{2\pi}{d}$ satisfies the requirement.

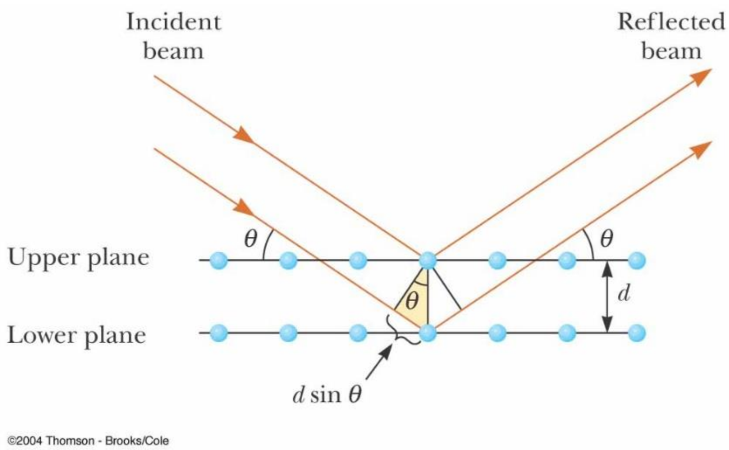
Thus the diffraction pattern gives the image of k-space for a one-dimensional lattice system.




# 37.5 Diffraction of X-Rays by Crystals




1. The Bragg Law.

$2d\sin(\theta)=n\lambda$

2. Reciprocal Lattice Vectors.

We know that when we talk about a wave, we may mention about is wavelength or other wave related parameters. Moreover, we often use wave vector *k* which is the reciprocal of the wavelength rather than wavelength to describe the wave.

The wave vector *k* is related to the length according to the relation $k = 2\pi / \lambda$. Or we may say $k = 2\pi / length$.

Here we find that the reciprocals of the length or the wave vector could be important parameters.
In our previous descriptions, we mention about the translational vector of the lattice.

$\vec{r}' = \vec{r}_0 + u_1 a_1 \hat{i} + u_2 a_2 \hat{j} + u_3 a_3 \hat{k}$ This is the real space or length vector.

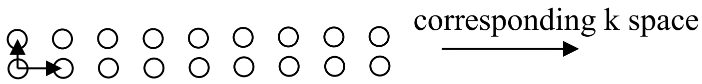
From it, we may obtain the special wave vector in *k* (wave vector or momentum) space.

$\vec{b}_1 = \frac{2\pi}{a_1}\hat{i}$, $\vec{b}_2 = \frac{2\pi}{a_2}\hat{j}$, $\vec{b}_3 = \frac{2\pi}{a_3}\hat{k}$

The three vectors are in k space and are called reciprocal lattice vectors.

$\vec{G} \equiv h\vec{b}_1 + k\vec{b}_2 + l\vec{b}_3$

Given a wave traveling in the x-direction of the 2D lattice




The wave function of a wave may be expressed as $A\sin(kx - \omega t) = A\sin(\vec{k}\cdot\vec{r} - \omega t)$

$\vec{r} = x\hat{i}$ → $\vec{k} = kx\hat{i}$

The wave vector $\vec{k}$ points to the traveling direction of the wave.
For the special wave having the same spatial period as the atomic position, its wave vector must be $\vec{k} = \frac{2\pi}{a}\hat{i}$, where $a$ is the spatially periodic distance.

The wave traveling in y-direction may have a wave vector $\vec{k} = k_y\hat{j}$. The wave traveling in any directions can be described by $\vec{k} = k_x\hat{i} + k_y\hat{j}$.

The special wave having the same spatial period as atomic position will have the wave vector of $\vec{k} = m\frac{2\pi}{a}\hat{i} + n\frac{2\pi}{b}\hat{j}$, where $m, n$ are integers.

Why do we talk about the special wave which consists of an atomic periodicity?

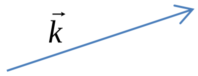
3. X-Ray Diffraction and Fourier Transform.

Why do we need the reciprocal lattice vectors in the k or momentum (or the reciprocal of the wavelength) space. The origin comes from electromagnetic waves or X-Ray diffraction.

For a plane wave, its wave function can be expressed as

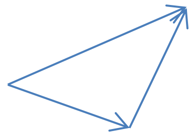
$$\vec{E}(\vec{r},t)=\vec{E}_0 \sin(\vec{k}\cdot\vec{r}-\omega t)$$

What's the direction of its wave vector k? It's just the direction of this EM wave.




It means that you can produce any specified k direction by adjusting the EM wave direction.

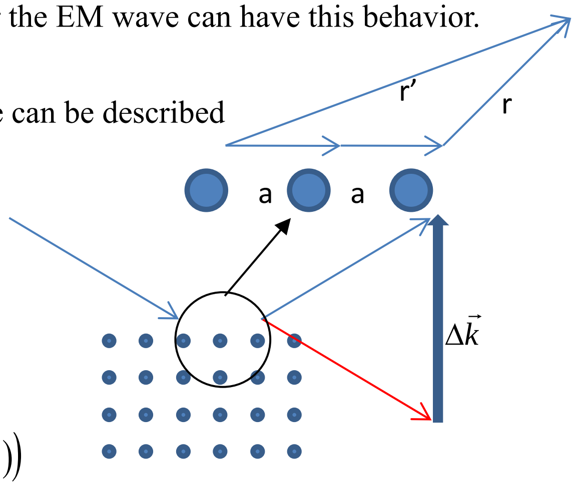
Now we know the wave vector of an EM wave. Moreover, the vector can be added or subtracted:



$$\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}$$

Then we may imagine that the wave vector or the EM wave can have this behavior.

If one atom reflect the EM to r', the EM wave can be described by r+2a




Now we can calculate the X-Ray diffraction:

The reflected EM wave function

, from the superposition rule, is:

$$\phi=\sum_{u1,u2,u3} A_{u1,u2,u3}\exp\left(\Delta\vec{k}\cdot(\vec{r}_0+u_1\vec{a}_1+u_2\vec{a}_2+u_3\vec{a}_3)\right)$$

For the constructive interference, the wave vector change $\Delta\vec{k}$ shall satisfy a simple condition of $\Delta\vec{k}\cdot(u_1\vec{a}_1+u_2\vec{a}_2+u_3\vec{a}_3)=n(2\pi)$.

Here we define another translational vector of the reciprocal lattice as

$\vec{G}=v_1\vec{b}_1+v_2\vec{b}_2+v_3\vec{b}_3$ → we find the coincidence of

$$\vec{G}\cdot(u_1\vec{a}_1+u_2\vec{a}_2+u_3\vec{a}_3)=(v_1u_1+v_2u_2+v_3u_3)(2\pi)$$

We then conclude that the change of the EM wave vector can be expressed as the translational vector of the reciprocal lattice.

$\Delta\vec{k}=\vec{G}$ → using EM wave, we can see the wave vector space

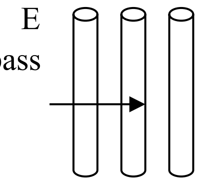
# 37.6 Polarization of Light Waves

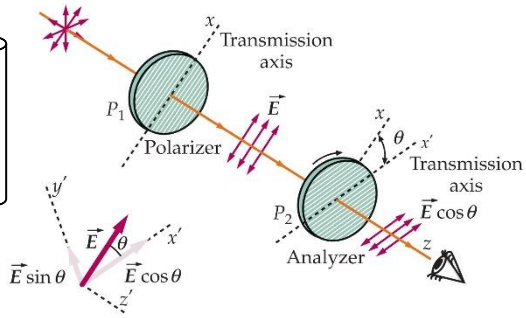
1. plane polarized or linearly polarized EM waves
2. elliptically polarized EM waves

## Polarization by Selective Absorption

Polaroid

perpendicular to the chain: pass




$I \propto (E\cos\theta)^2$

→ $I = I_0 \cos^2\theta$ (Law of Malus)

Specific Quantum Feature:



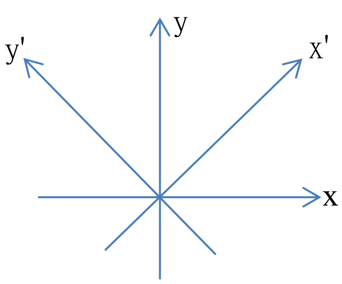
Once the x'-filter intervenes and selects the x'-polarized beam, it is immaterial whether the beam was previously x-polarized.

Correspondence between the SG experiment and light polarization filter:

$S_z\pm$ atoms <-> x-, y-polarized light

$S_x\pm$ atoms <-> x'-,y'-polarized light

Electric field of light wave:



$$\hat{x}' E_0 \cos(kz - \omega t) = \hat{x}\frac{E_0}{\sqrt{2}}\cos(kz - \omega t) + \hat{y}\frac{E_0}{\sqrt{2}}\cos(kz - \omega t)$$

$$\hat{y}' E_0 \cos(kz - \omega t) = -\hat{x}\frac{E_0}{\sqrt{2}}\cos(kz - \omega t) + \hat{y}\frac{E_0}{\sqrt{2}}\cos(kz - \omega t)$$

We might be able to represent the spin state of a silver atom by

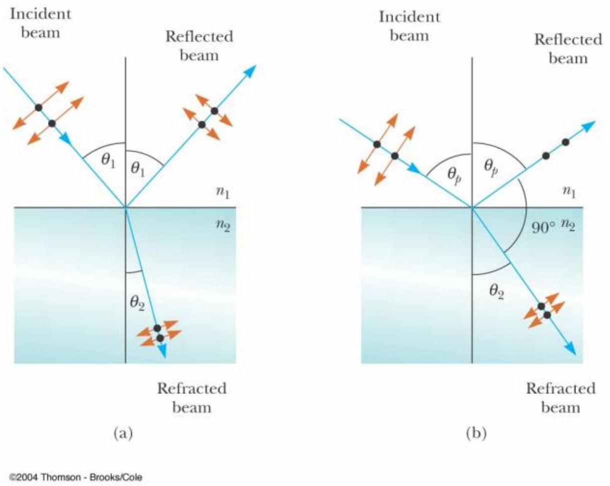
$$|S_x;+\rangle \overset{?}{=} \frac{1}{\sqrt{2}}|S_z;+\rangle + \frac{1}{\sqrt{2}}|S_z;-\rangle$$

$$|S_x;-\rangle \overset{?}{=} -\frac{1}{\sqrt{2}}|S_z;+\rangle + \frac{1}{\sqrt{2}}|S_z;-\rangle$$

Symmetry arguments tell that the $S_y\pm$ states are similar to $S_x\pm$ states, how can we write it decomposed into $S_z\pm$ states?

# Polarization by Reflection:

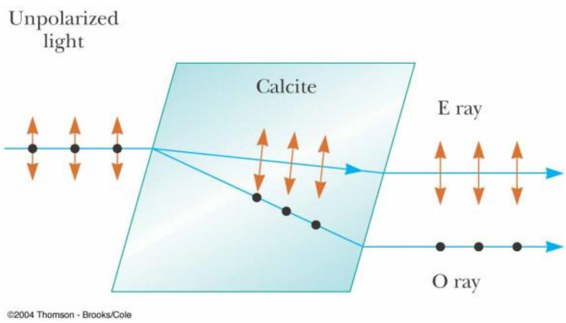
The polarization of the reflected light depends on the angle of incidence. If the angle of incidence is $0^o$, the reflected beam is unpolarized. For other angles, the reflected light is polarized to some extent. For the particular case of Brewster's condition, the reflected light is completely polarized.

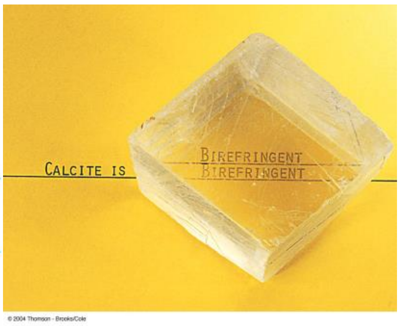



Polarizing angle $\theta_p$

$$\frac{n_2}{n_2} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin\theta_p}{\sin(90-\theta_p)} = \tan\theta_p$$ **Brewster's Law (David Brewster 1781-1868)**

## Polarization by Double Refraction




## Polarization by Scattering