Chapter 24 Electric Potential

conservative forces -> potential energy - What is a conservative force?

Electric potential (V = U / q): the potential energy (U) per unit charge (q) is a function of the position in space

Goal:

1. establish the relationship between the electric field and electric potential
2. calculate the electric potential of various continuous charge distribution
3. use the electric potential to determine the electric field

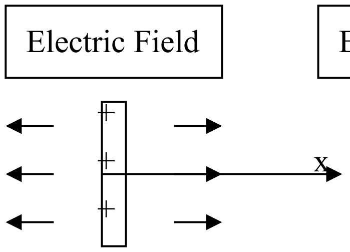
24.1 Electric Potential and Potential Difference

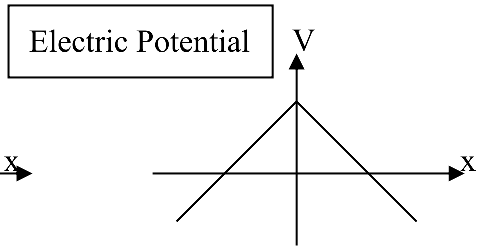
$$dU = -\vec{F} \cdot d\vec{l}$$

$$\Delta U = -\int_A^B \vec{F} \cdot d\vec{l} = -q_0 \int_A^B \vec{E} \cdot d\vec{l} \quad \text{(potential energy difference)}$$

$$V \equiv \frac{U}{q_0}, \quad \Delta V = V_B - V_A = \frac{\Delta U}{q_0} = -\int_A^B \vec{E} \cdot d\vec{l} \quad \text{(electric potential difference)}$$

Continuity of V





$$dV = -\vec{E} \cdot d\vec{l}$$

The potential function is continuous everywhere.

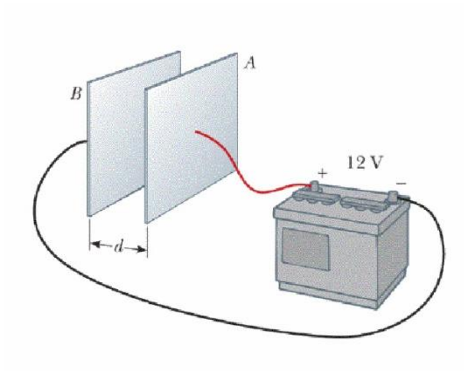
Units

$1\_Volt = 1\_V = 1\_J/C$, $1\_N/C = 1\_V/m$

$1\_eV = 1.602 \cdot 10^{-19} J$

300 K = ? eV, visible light: ? eV

24.2 Potential difference in a uniform electric field

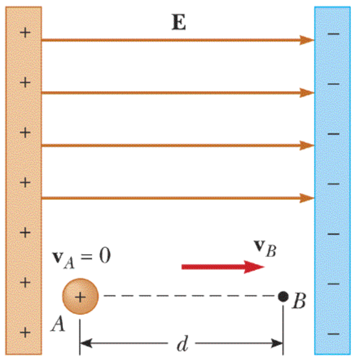


Serway/Jewett; Principles of Physics, 3/e
Figure 20.4

$$V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{l} = -Ed\,,\quad \Delta U = q\Delta V = -qEd$$

What is equipotential surface??

Example: The electric field between two parallel plates of opposite charge



Example: Motion of a proton in a uniform electric field E = 8.0 x 10^4 V/m, d = 0.5 m (a) Find the change in **electric potential** between the points A and B. (b) Find the change in **potential energy**.

24.3 Electric Potential and Potential Energy Due to Point Charges

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r}$$

$$V_B - V_A = -\int_{rA}^{rB}\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}\cdot\hat{r}dr = \frac{q}{4\pi\varepsilon_0 r_B} - \frac{q}{4\pi\varepsilon_0 r_A}$$

let $r_B = r$ and $r_A = \infty$ --> $V = \dfrac{q}{4\pi\varepsilon_0 r}$ and $U = q_0 V = \dfrac{q_0 q}{4\pi\varepsilon_0 r}$

Example: Potential Energy of a Hydrogen Atom

(a) What is the electric potential at a distance r = 0.529 X 10^{-10} m from a proton?

(b) What is the electric potential energy of the electron and the proton at this separation?

(a) $V = \dfrac{ke}{r} = \dfrac{(9\times10^9)(1.602\times10^{-19})}{0.529\times10^{-10}} = 27.2\_V$

(b) $U = -eV = -27.2\_eV$

Example: Potential Energy of Nuclear-Fission Products

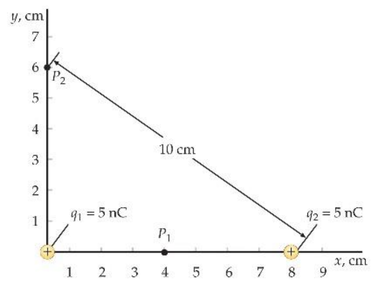
In nuclear fission, a uranium-235 nucleus captures a neutron and splits apart into two lighter nuclei. Sometimes the two fission products are a barium nucleus (charge 56e) and a krypton nucleus (charge 36e). Assume that immediately after the split these nuclei are positive point charges separated by $r = 14.6 \times 10^{-15}\_m$. Calculate the potential energy of this two-charge system in electron volts.

$$U = \frac{(9\times10^{9})(56)(1.602\times10^{-19})(36)(1.602\times10^{-19})}{14.6\times10^{-15}} J = \frac{(9\times10^{9})(56)(1.602\times10^{-19})(36)}{14.6\times10^{-15}} eV$$

$$= 199\_MeV$$

$$V = \sum \frac{q_i}{4\pi\varepsilon_0 r_i}$$ -> easier for calculation without consideration of vector addition

Example: Potential Due to Two Point Charges



P1: $V = \frac{k(5nC)}{0.04} + \frac{k(5nC)}{0.04}$

P2: $V = \frac{k(5nC)}{0.10} + \frac{k(5nC)}{0.06}$

Example: A point charge q_1 is at the origin, and a second point charge q_2 is on the x-axis at $x = a$. Find the potential everywhere on the x-axis.

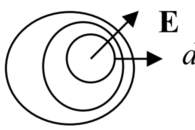
$$V = \frac{kq_1}{|x|} + \frac{kq_2}{|x-a|}$$

24.4 Obtaining the Value of the Electronic Field from the Electric Potential

$$dU = -\vec{F}\cdot d\vec{l} \quad \text{-->} \quad dV = -\vec{E}\cdot d\vec{l} \quad \text{-->} \quad E = -\frac{dV}{dl}$$

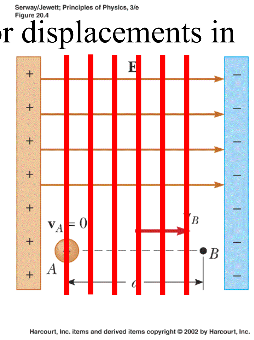
The electric field points in the direction in which the potential decrease most rapidly. (1D? 2D? 3D?)

$$dV = -\vec{E}\cdot d\vec{l} = -Edl\cos\theta = -E_t dl$$



$$E_t = -dV/dl$$

If the potential V depends only on x, there will be no change in V for displacements in the y and z direction.



$$dV(x) = -\vec{E}\cdot d\vec{l} = -\vec{E}\cdot(\hat{i}dx) = -(E_x\hat{i} + E_y\hat{j} + E_z\hat{k})\cdot(\hat{i}dx) = -E_x dx$$

--> $E_x = -\dfrac{dV}{dx}$

If displacements perpendicular to the radial direction give no change in V,

$$dV = -\vec{E}\cdot d\vec{l} = -\vec{E}\cdot(\hat{r}dr) = -E_r dr$$

--> $E_r = -\dfrac{dV}{dr}$

Example: Find the electric field for the electric potential function V given by $V = 100 - 25x\_(V)$.

Example: Potential Due to An Electric Dipole

An electric dipole consists of a positive charge $+q$ on the x-axis at $x = a\hat{i}$ and a negative charge $-q$ on the x-axis at $x = -a\hat{i}$. Find the potential on the x-axis for $x >> a$ in terms of the **electric dipole moment** $p = 2qa$.

$x > a$ --> $V = \dfrac{kq}{x-a} + \dfrac{k(-q)}{x+a} = \dfrac{2kqa}{x^2 - a^2}$

$x >> a$ --> $V \approx \dfrac{2kqa}{x^2} = \dfrac{kp}{x^2}$

General Relation Between $\vec{E}$ and V

$$E_x = -\frac{dV}{dx} \quad \& \quad E_y = -\frac{dV}{dy} \quad \& \quad E_z = -\frac{dV}{dz}$$

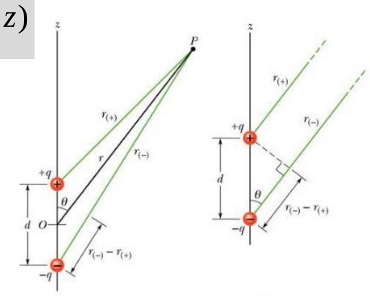
$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k} = -\frac{dV}{dx}\hat{i} - \frac{dV}{dy}\hat{j} - \frac{dV}{dz}\hat{k} = -\left(\hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{d}{dz}\right)V \equiv -\vec{\nabla}V$$

Obtaining electric field from electric potential

$$\vec{E} = -\vec{\nabla}V = -\hat{i}\frac{\partial}{\partial x}V(x,y,z) - \hat{j}\frac{\partial}{\partial y}V(x,y,z) - \hat{k}\frac{\partial}{\partial z}V(x,y,z)$$

$$\vec{E} = -\vec{\nabla}V = -\hat{r}\frac{\partial}{\partial r}V(r,\theta,\phi) - \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta}V(r,\theta,\phi) - \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}V(r,\theta,\phi)$$

$$\vec{E} = -\vec{\nabla}V = -\hat{r}\frac{\partial}{\partial r}V(r,\theta,z) - \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta}V(r,\theta,z) - \hat{z}\frac{\partial}{\partial z}V(r,\theta,z)$$



Potential due to an electric dipole

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{|\vec{r}-\vec{r}_1|} + \frac{1}{4\pi\varepsilon_0}\frac{-q}{|\vec{r}+\vec{r}_1|}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{r^2+r_1^{\,2}-2rr_1\cos\theta}} - \frac{1}{\sqrt{r^2+r_1^{\,2}+2rr_1\cos\theta}}\right)$$

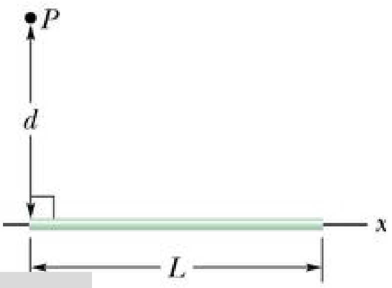
$$= \frac{q}{4\pi\varepsilon_0}\frac{1}{r}\left(1-\frac{1}{2}\left(-2\frac{r_1}{r}\cos\theta\right)-\left(1-\frac{1}{2}2\frac{r_1}{r}\cos\theta\right)\right) = \frac{q}{4\pi\varepsilon_0 r}\frac{2r_1\cos\theta}{r} = \frac{qd\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$\vec{E} = \hat{r}\left(-\frac{\partial}{\partial r}\right)V + \hat{\theta}\left(-\frac{\partial}{r\partial\theta}\right)V = ?$$

24.5 Electric Potential Due to Continuous Charge Distributions

$$dV = \frac{kdq}{r} \rightarrow V = \int\frac{kdq}{r} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

V Due to an Infinite Line Charge:



(a)

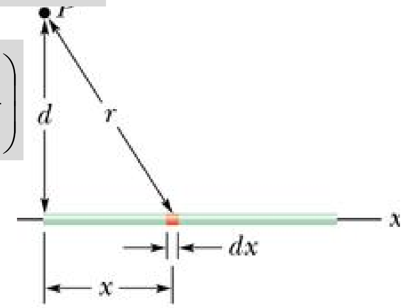
METHOD 1:

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda dx}{\sqrt{d^2+x^2}}$$

let $x = d\tan\theta$

$$V = \frac{\lambda}{4\pi\varepsilon_0}\int_{\tan^{-1}\frac{0}{d}}^{\tan^{-1}\frac{L}{d}}\sec\theta d\theta = \frac{\lambda}{4\pi\varepsilon_0}\int_{\tan^{-1}\frac{0}{d}}^{\tan^{-1}\frac{L}{d}}\frac{1}{2}\left(\frac{1}{1-\sin\theta}+\frac{1}{1+\sin\theta}\right)d\sin\theta$$

$$V = \frac{\lambda}{4\pi\varepsilon_0}\frac{1}{2}\left[\ln\frac{1+\sin\theta}{1-\sin\theta}\right]_0^{\tan^{-1}\frac{L}{d}} = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{L+\sqrt{L^2+d^2}}{d}\right)$$

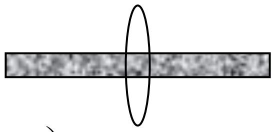


(b)

METHOD 2:

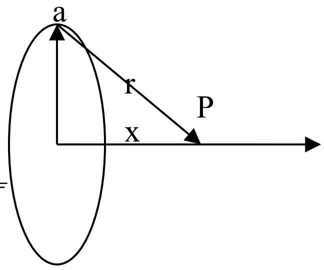
Obtain E by applying Gauss's law:

$$\vec{E} = \frac{4\pi k\lambda L}{2\pi r L}\hat{r} = \frac{2k\lambda}{r}\hat{r}$$



$$V_P - V_{ref} = -\int_{r\_ref}^{r\_P} \frac{2k\lambda}{r}dr = 2k\lambda \ln\left(\frac{R_P}{R_{ref}}\right)$$

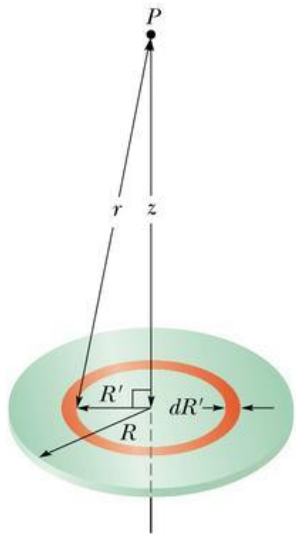
V on The Axis of a Charged Ring



$$V = \int \frac{kdq}{r} = k\int \frac{\lambda ds}{\sqrt{x^2+a^2}} = k\int_0^{2\pi} \frac{\lambda a d\theta}{\sqrt{x^2+a^2}} = \frac{k2\pi a\lambda}{\sqrt{x^2+a^2}} = \frac{kQ}{\sqrt{x^2+a^2}}$$

$$E_x = -dV/dx = ?$$

V on The Axis of a Uniformly Charged Disk:



$$V = \int \frac{kdq}{r} = \int \frac{k\sigma 2\pi r dr}{\sqrt{z^2+r^2}} = \int_0^R \frac{k\sigma 2\pi r dr}{\sqrt{z^2+r^2}} = 2\pi k\sigma\left(\sqrt{z^2+r^2}\right)_0^R$$

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^R dr \int_0^{2\pi} r d\theta \frac{\sigma}{\sqrt{z^2+r^2}}$$

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - z\right)$$

for $z >> R$ -->

$$V(z) = 2\pi k\sigma\left(z\left(1+\frac{R^2}{z^2}\right)^{1/2} - z\right) = 2\pi k\sigma\left(z\left(1+\frac{1}{2}\frac{R^2}{z^2}\right) - z\right) = \frac{k\pi R^2\sigma}{z} = \frac{kQ}{z}$$

Example: Find the electric field along z direction.

$$E_z = -\frac{\partial}{\partial z}V = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

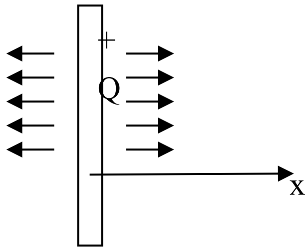
24.6 Electric potential Due to a Charged Conductor

$\vec{E} = 0$ inside a conductor -> $V = const$

The conductor is a three dimensional equipotential surface.

The potential V has the same value everywhere on an equipotential surface.

V Due to an Infinite Plane of Charge



METHOD 2:

Obtain E by applying Gauss's law:

$$E = 2\pi k\sigma$$

$$x > 0 \quad \text{-->} \quad V = -\int_{reff}^{Vp} \vec{E}\cdot d\vec{l} = -\int_{reff}^{Vp} 2\pi k\sigma dx = -2\pi k\sigma x + V_0$$

$$x < 0 \quad \text{-->} \quad V = -\int_{reff}^{Vp} \vec{E}\cdot d\vec{l} = \int_{reff}^{Vp} 2\pi k\sigma dx = 2\pi k\sigma x + V_0$$

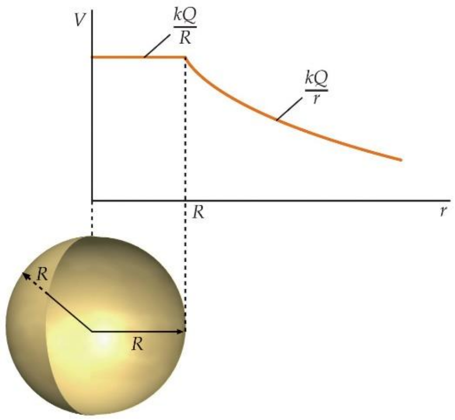
V Inside and Outside a Spherical Shell of Charge

METHOD 2:

Obtain E by applying Gauss's law:

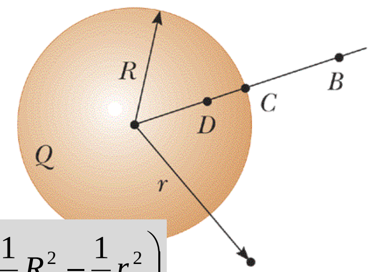
$$r >= R, \quad V = \frac{kQ}{r}$$

$$r < R, \quad V = \frac{kQ}{R}$$



V for a Uniformly Charged Sphere

Serway/Jewett; Principles of Physics, 3/e
Figure 20.12a



(a)

(a) r > R

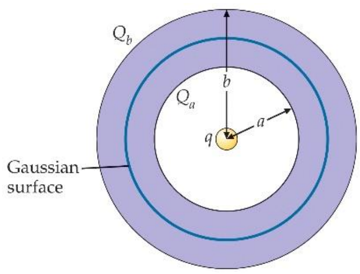
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}, \quad E = -\int_{\infty}^{r} \vec{E}\cdot\hat{r}dr = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

(b) r < R

$$E = \frac{Q}{4\pi\varepsilon_0 R^3} r\hat{r}, \quad V(r) - V(R) = -\int_{R}^{r} E\cdot\hat{r}dr = \frac{Q}{4\pi\varepsilon_0 R^3}\left(\frac{1}{2}R^2 - \frac{1}{2}r^2\right)$$

$$V(r) = \frac{Q}{4\pi\varepsilon_0 R^3}\left(\frac{3}{2}R^2 - \frac{1}{2}r^2\right)$$

Example: A hollow uncharged spherical conducting shell has an inner radius a and an outer radius b. A positive charge q is in the cavity, at the center of the sphere. (a) Find the charge on each surface of the conductor. (b) Find the potential.



$Q_a = -q$, $Q_b = +q$

$r \ge b$, $V = \dfrac{kq}{r}$

$b \ge r \ge a$, $V = \dfrac{kq}{b}$

$a \ge r$, $V = \dfrac{kq}{r} - \dfrac{kq}{a} + \dfrac{kq}{b}$

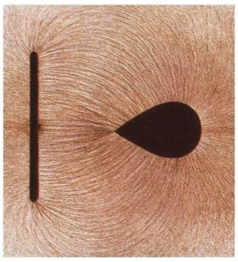
Example: The two spheres are separated by a distance much greater than R_1 and R_2. Find the charges Q_1 and Q_2 on the two spheres if the total charge is Q. Find the ratio of the magnitudes of the electric fields at the surfaces of the spheres.

(1) $Q_1 + Q_2 = Q$

$\dfrac{kQ_1}{R_1} = V_1 = V_2 = \dfrac{kQ_2}{R_2}$ --> $Q_1 = \dfrac{R_1}{R_1 + R_2} Q$

(2) $\dfrac{E_1}{E_2} = \dfrac{\dfrac{kQ_1}{R_1^2}}{\dfrac{kQ_2}{R_2^2}} = \dfrac{R_2}{R_1}$

A charge is placed on a conductor of nonspherical shape.

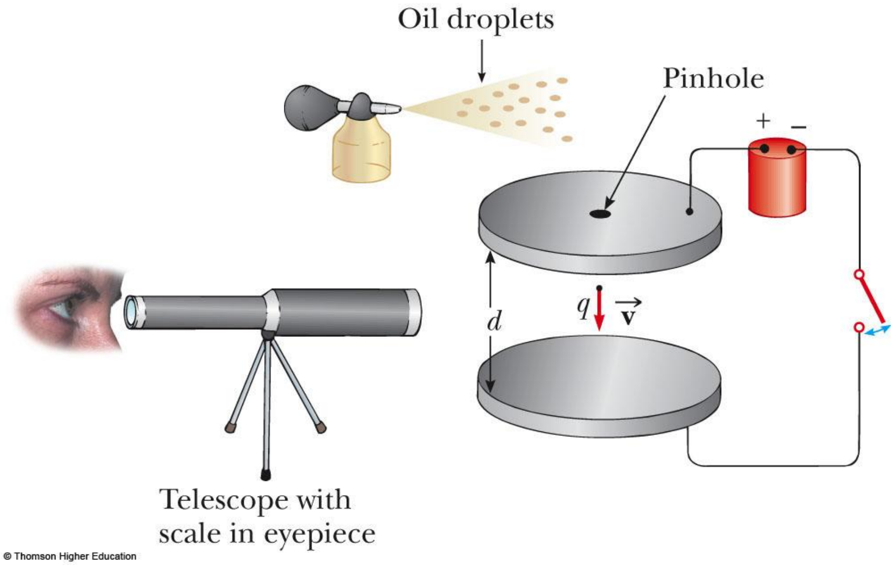


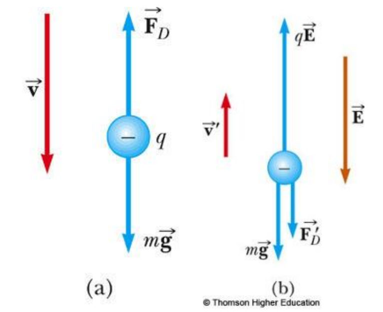
$V = \dfrac{kQ}{R} = \dfrac{k4\pi R^2 \sigma}{R} = 4\pi k \sigma R$

$\sigma = \dfrac{V}{4\pi k R}$

small R --> Large V

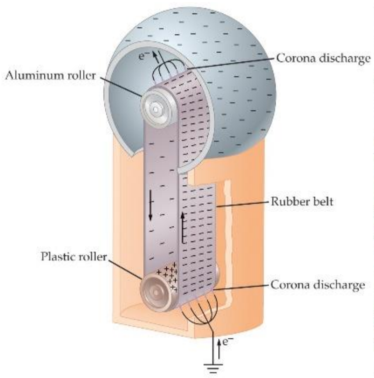
24.7 The Millikan Oil-Drop Experiment





24.8 Applications of Electrostatics

The Van de Graaff Generator



The Electrostatic Precipitator

Xerography and Laser Printers