Physics I Lecture04- Motion in Two Dimensions-I

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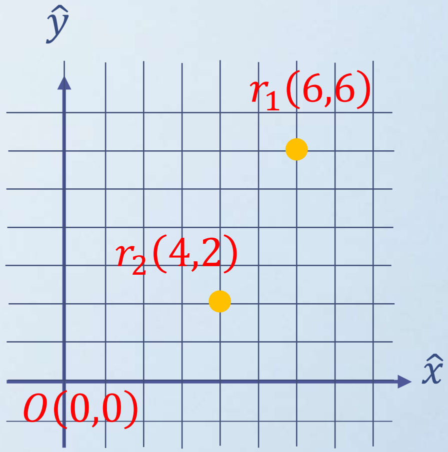
CONTENTS

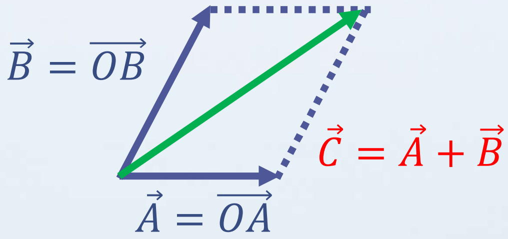
1. VECTORS

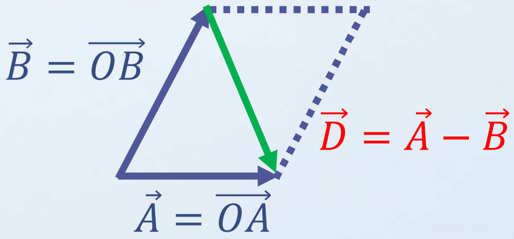
The Vector:

$$\vec{r}_1 = 6\hat{\imath} + 6\hat{\jmath} \quad \vec{r}_2 = 4\hat{\imath} + 2\hat{\jmath}$$



The Addition of The Displacement Vector:





Notation 1: $\overrightarrow{OA} - \overrightarrow{OB} = \overrightarrow{BO} + \overrightarrow{OA} = \overrightarrow{BA}$

Notation 2: $\vec{r}_{AO} - \vec{r}_{BO} = \vec{r}_{AO} + \vec{r}_{OB} = \vec{r}_{AB}$

1. VECTORS

Vector Space:

Zero Vector: $\vec{0}$

Equality: $\vec{\boldsymbol{A}} = \vec{\boldsymbol{B}}$

Addition: $\vec{\boldsymbol{A}} + \vec{\boldsymbol{B}}$

Multiplying by a Scalar: $c\vec{A}$, $0\vec{A} = \vec{0}$

Inverse: $-1 \cdot \vec{A} = -\vec{A}$

Subtraction: $\vec{A} - \vec{B}$

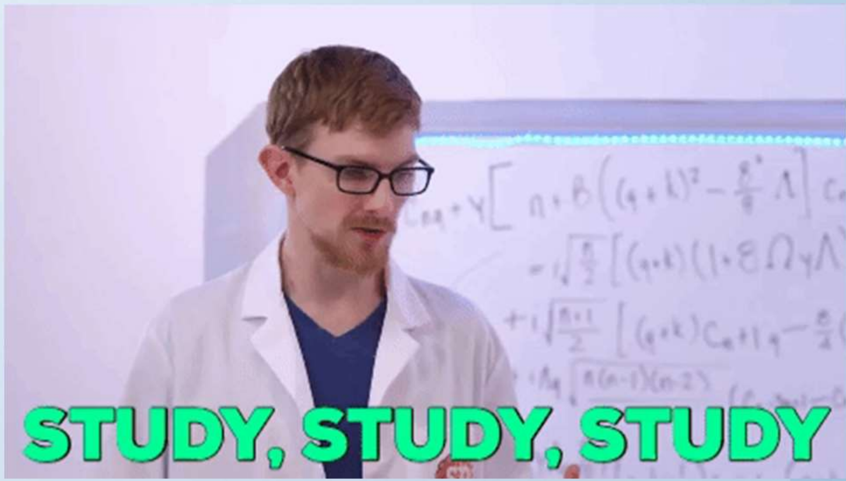
Associative, Distributive, Commutative

Component of a Vector:

$\vec{A} = (A_x, A_y) = A_x\hat{\imath} + A_y\hat{\jmath}$, $|\vec{A}| =?$

Unit Vectors:

$\hat{A} = \vec{A}/|\vec{A}|$



https://giphy.com/gifs/soulpancake-school-3ohBVyLs9CkVV1yZEY

2. POSITION, VELOCITY, AND ACCELERATION

Displacement: $\Delta\vec{r} = \vec{r}_f - \vec{r}_i = (x_f - x_i)\hat{\imath} + (y_f - y_i)\hat{\jmath} + (z_f - z_i)\hat{k}$

Average Speed: $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$

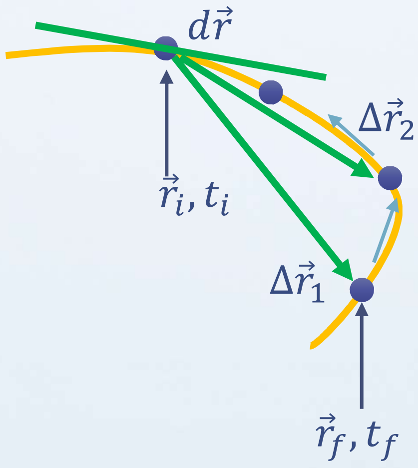
Instantaneous Velocity: $\vec{v} = \lim_{t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k}$

Average Acceleration: $\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$

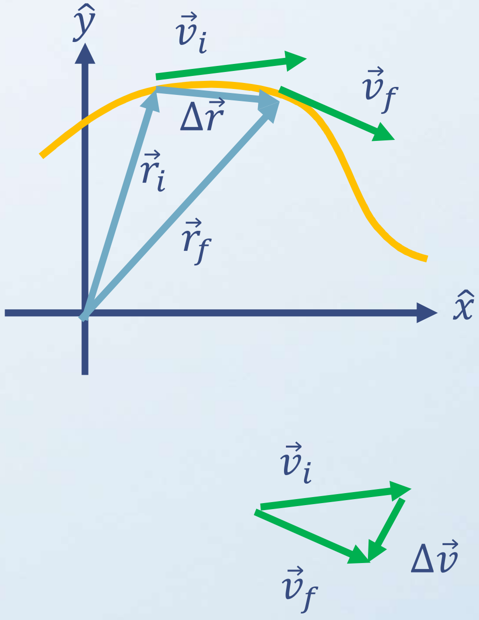
Instantaneous Acceleration: $\vec{a} = \lim_{t\to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$

2. POSITION, VELOCITY, AND ACCELERATION

Trajectory:



Displacement & Velocity Variation



3. 2D MOTION WITH CONSTANT ACCELERATION

Constant Acceleration:

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$$

$$\vec{v} = (v_{0x} + a_x t)\hat{\imath} + (v_{0y} + a_y t)\hat{\jmath} = \vec{v}_0 + \vec{a}t$$

$$\vec{r} = \left(x_0 + v_{0x}t + \frac{a_x t^2}{2}\right)\hat{\imath} + \left(y_0 + v_{0y}t + \frac{a_y t^2}{2}\right)\hat{\jmath}$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

3. 2D MOTION WITH CONSTANT ACCELERATION

Example: A particle move through the origin of an xy coordinate system at t = 0 with initial velocity $\vec{v}_0 = 20\hat{\imath} - 15\hat{\jmath}$ (m/s). The particle moves in the xy plane with an acceleration $\vec{a} = 4\hat{\imath}$ (m/s^2). Determine the components of the velocity as a function of time and the total velocity vector at any time.

initial conditions: $\vec{r}_0 = 0\hat{\imath} + 0\hat{\jmath}$ (m), $\vec{v}_0 = 20\hat{\imath} - 15\hat{\jmath}$ (m/s)

start from the differential equation: $\vec{a} = \frac{d\vec{v}}{dt} = 4\hat{\imath}$

$$\frac{d\vec{v}}{dt} = 4\hat{\imath} \Rightarrow d\vec{v} = 4dt\hat{\imath} \Rightarrow \int_{20\hat{\imath}-15\hat{\jmath}}^{\vec{v}(t)} d\vec{v} = 4\int_0^t dt\,\hat{\imath}$$

$\vec{v}(t) = (20 + 4t)\hat{\imath} - 15\hat{\jmath}$ (m/s)

$$\vec{v}(t) = \frac{d\vec{x}}{dt} = (20 + 4t)\hat{\imath} - 15\hat{\jmath} \Rightarrow \vec{r}(t) = (20t + 2t^2)\hat{\imath} - 15t\hat{\jmath} \text{ (m)}$$

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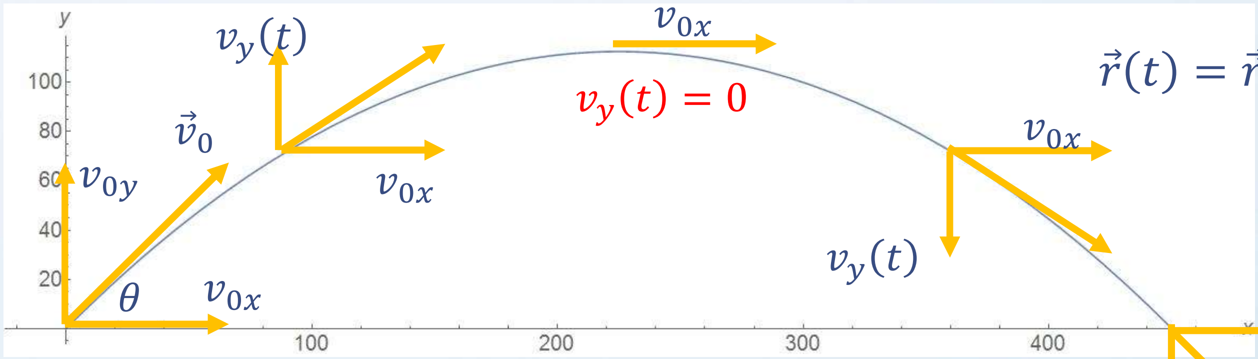
4. PROJECTILE MOTION

Trajectory of Projectile Motion:



$$\vec{v} = \vec{v}_0 - \hat{j}gt \qquad \frac{d\vec{r}}{dt} = \hat{\imath}v_0\cos(\theta) + \hat{j}(v_0\sin(\theta) - gt)$$

$$\vec{r}(t) = \vec{r}_0 + \hat{\imath}v_0\cos(\theta)\,t + \hat{j}\left(v_0\sin(\theta)\,t - \frac{1}{2}gt^2\right)$$

$$x(t) = v_0\cos(\theta)\,t \to t = \frac{x}{v_0\cos(\theta)}$$

$$y(t) = v_0\sin(\theta)\,t - \frac{1}{2}gt^2$$

$$y = \frac{v_0\sin(\theta)}{v_0\cos(\theta)}x - \frac{gx^2}{2v_0^2(\cos(\theta))^2}$$

$$y = H - \frac{g}{2v_0^2\cos^2(\theta)}\left(x - \frac{R}{2}\right)^2$$

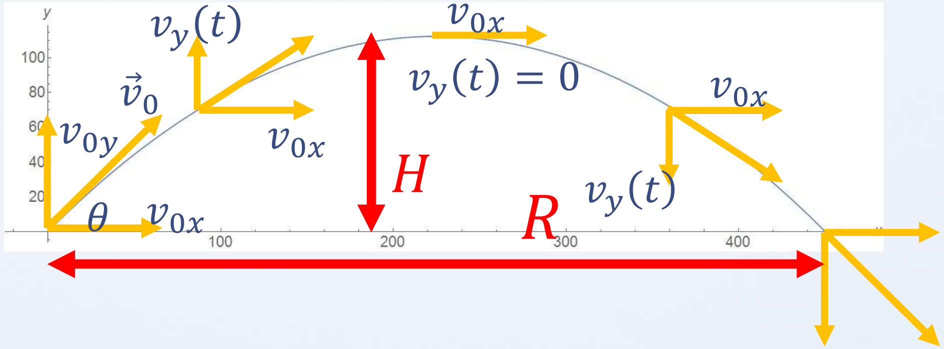
$$\tan\theta = \frac{v_{0y}}{v_{0x}}$$

$\vec{v}_0 = v_{0x}\hat{\imath} + v_{0y}\hat{\jmath} = v_0\cos\theta\,\hat{\imath} + v_0\sin\theta\,\hat{\jmath}$ at $t = 0$

$\vec{r}_0 = 0\hat{\imath} + 0\hat{\jmath}$ at t = 0

$$\frac{d\vec{v}}{dt} = -g\hat{\jmath} \to d\vec{v} = -\hat{\jmath}g\,dt \qquad \int_{\vec{v}_0}^{\vec{v}(t)} d\vec{v} = -\hat{\jmath}g\int_0^t dt$$

4. PROJECTILE MOTION



Maximum Height & Horizontal Distance

$$\frac{T}{2} = \frac{v_0 \sin\theta}{g} \quad \Rightarrow \quad H = \frac{g}{2}\left(\frac{T}{2}\right)^2 = \frac{v_0^2 \sin^2\theta}{2g}$$

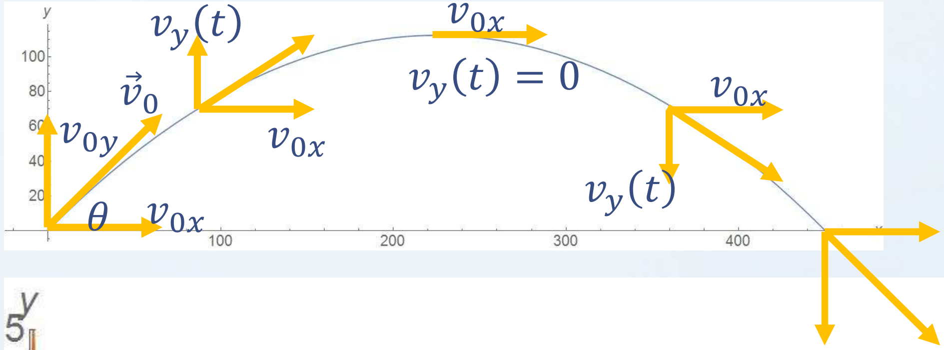
H is maximum at $\theta = \frac{\pi}{2}$

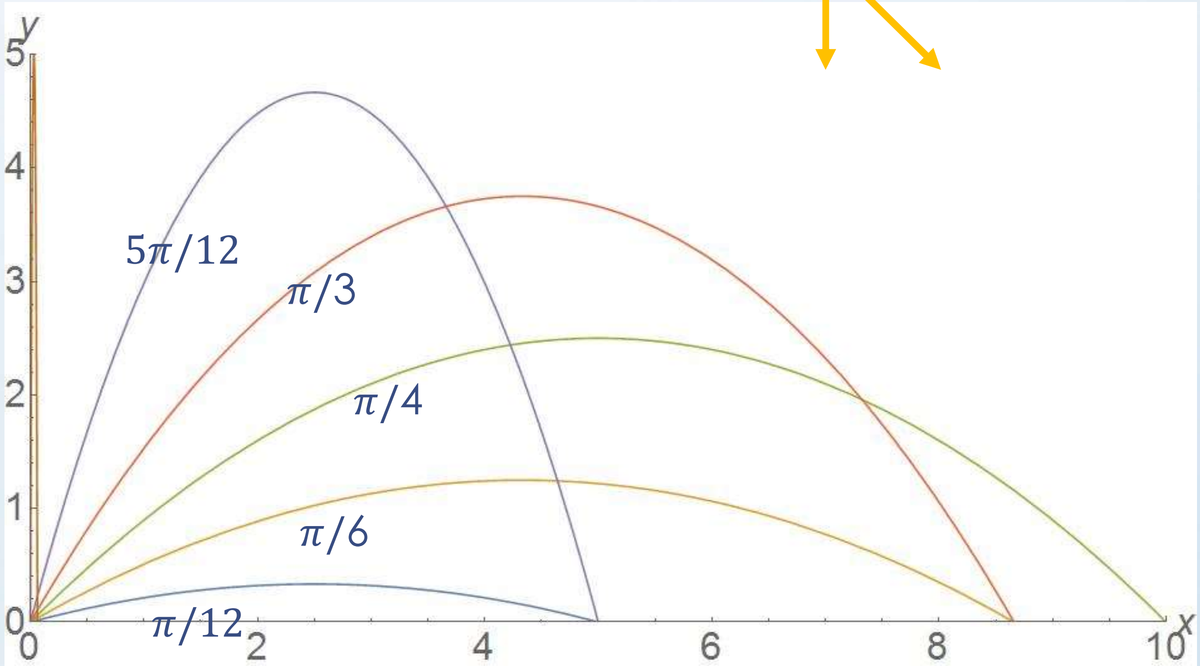
$$R = v_0 \cos\theta \, T = \frac{v_0^2 2 \sin\theta \cos\theta}{g} = \frac{v_0^2 \sin(2\theta)}{g}$$

R is maximum at $\theta = \frac{\pi}{4}$

https://giphy.com/search/cut-off

4. PROJECTILE MOTION





4. PROJECTILE MOTION

Example: A long-jumper leaves the ground at an angle of $\pi/6$ rad above the horizontal and at a speed of 9.8 m/s. How far does he jump in the horizontal direction? What is the maximum height reached?

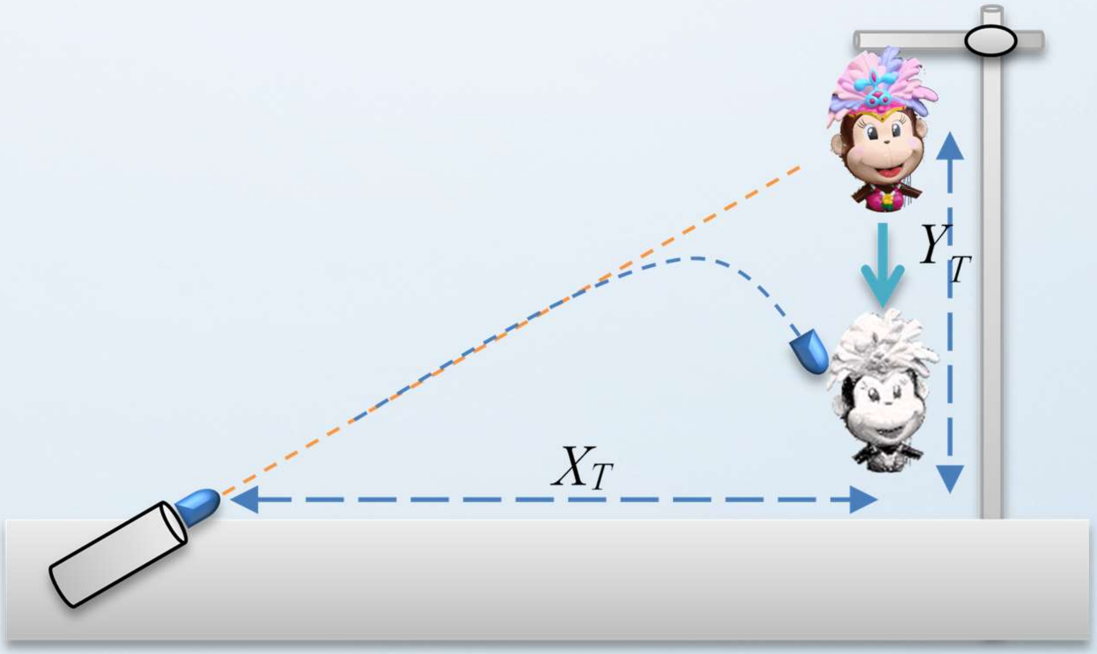
$$\frac{T}{2} = \frac{9.8\sin(\pi/6)}{9.8} = 0.50 \quad \text{(s)}$$

$$R = v_0 \cos(\pi/6)\, T = 9.8 \times \frac{\sqrt{3}}{2} \times 1 = 8.5 \quad \text{(m)}$$

$$H = \frac{g}{2}\left(\frac{T}{2}\right)^2 = 4.9(0.50)^2 = 1.2 \quad \text{(m)}$$

4. PROJECTILE MOTION

Example: A projectile is fired at a target T in a way that the projectile leaves the gun at the same time the target is dropped from rest. Show that if the gun is initially aimed at the stationary target, the projectile hits the target.



4. PROJECTILE MOTION

Example: A projectile is fired at a target T in a way that the projectile leaves the gun at the same time the target is dropped from rest. Show that if the gun is initially aimed at the stationary target, the projectile hits the target.

Assume the aiming angle of θ, the initial speed of v

Traveling time: $X_T / v\cos\theta$

Hitting condition:

$$\frac{g}{2}\left(\frac{X_T}{v\cos\theta}\right)^2 + v\sin\theta\left(\frac{X_T}{v\cos\theta}\right) - \frac{g}{2}\left(\frac{X_T}{v\cos\theta}\right)^2 = Y_T$$

$$v\sin\theta\left(\frac{X_T}{v\cos\theta}\right) = Y_T$$

The required condition for hitting the target is $\tan\theta = \frac{Y_T}{X_T}$

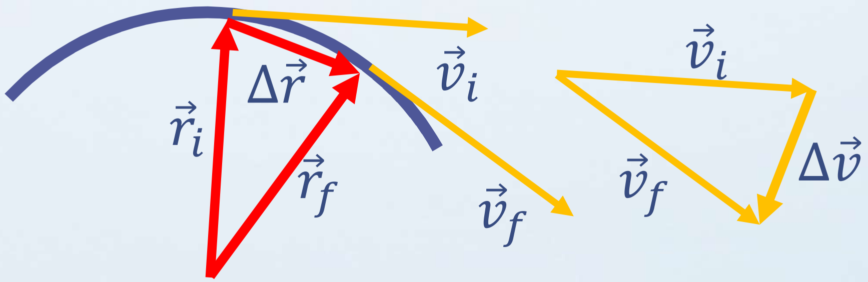
➡ aiming at the target

5. UNIFORM CIRCULAR MOTION

A particle is in uniform circular motion. The radius of its trajectory is r and its speed in the motion is v.

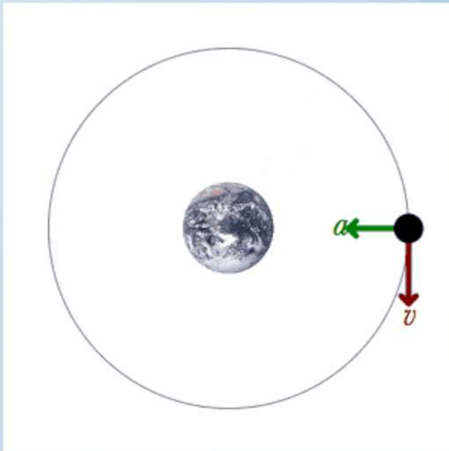
important physical quantities: $T = \frac{2\pi r}{v}$ $\quad \omega = \frac{d\theta}{dt} = 2\pi f = \frac{2\pi}{T} = \frac{v}{r}$ $\quad v = \omega r$

What's the centripetal acceleration a_r?



$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} \qquad a_{avg} = \frac{|\Delta\vec{v}|}{\Delta t} \qquad \frac{|\Delta\vec{v}|}{v} = \frac{|\Delta\vec{r}|}{r} \quad \Rightarrow \quad a_{avg} = \frac{v}{r}\frac{|\Delta\vec{r}|}{\Delta t}$$

$$a = \lim_{\Delta t \to 0} a_{avg} = \lim_{\Delta t \to 0} \frac{v}{r}\frac{|\Delta\vec{r}|}{\Delta t} = \frac{v^2}{r}$$



https://giphy.com/gifs/kinetic-swkRiEwhaKhDW

5. UNIFORM CIRCULAR MOTION

A particle is in uniform circular motion. The radius of its trajectory is r and its speed in the motion is v.

Use polar coordinate, the positional vector

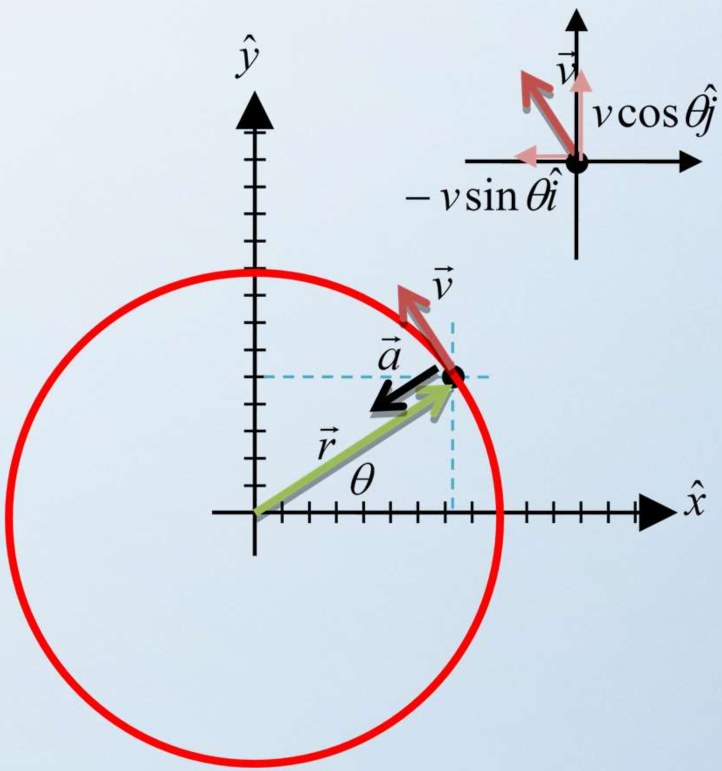
$\vec{r} = r\cos\theta\,\hat{\imath} + r\sin\theta\,\hat{\jmath}$, where $\theta = \omega t$

The velocity vector is derived by differentiation

$$\vec{v} = \frac{d\vec{r}}{dt} = -v\sin\theta\,\hat{\imath} + v\cos\theta\,\hat{\jmath}$$

Differentiate it again

$$\vec{a} = -\frac{v^2}{r}(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$$



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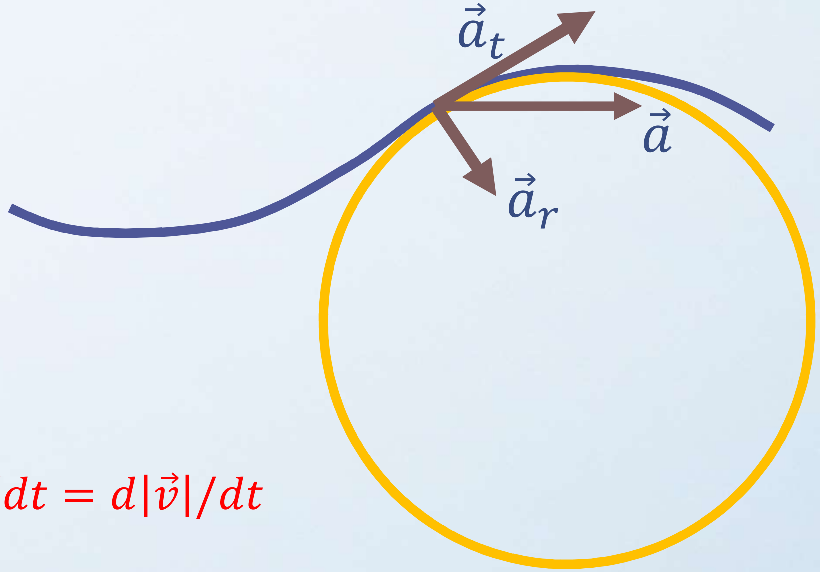
4. Projectile Motion
5. Uniform Circular Motion
6. Tangential and Radial Acceleration
7. Relative Velocity

6. TANGENTIAL AND RADIAL ACCELERATION



The change in speed: $a_t = dv/dt = d|\vec{v}|/dt$

The change in direction: $a_r = v^2/r$

The total acceleration: $a = \sqrt{a_t^2 + a_r^2}$

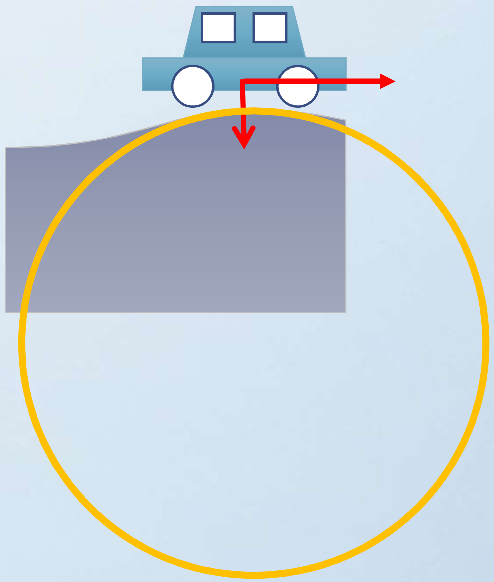
6. TANGENTIAL AND RADIAL ACCELERATION

Example: A car exhibits a constant acceleration of 0.300 m/s^2 parallel to the roadway. The car passes over a rise in the roadway such that the top of the rise is shaped like a circle of radius 500.0 m. At the moment the car is at the top of the rise, its velocity vector is horizontal and has a magnitude of 6.00 m/s. What is the direction and the magnitude of the total acceleration vector for the car at this moment?

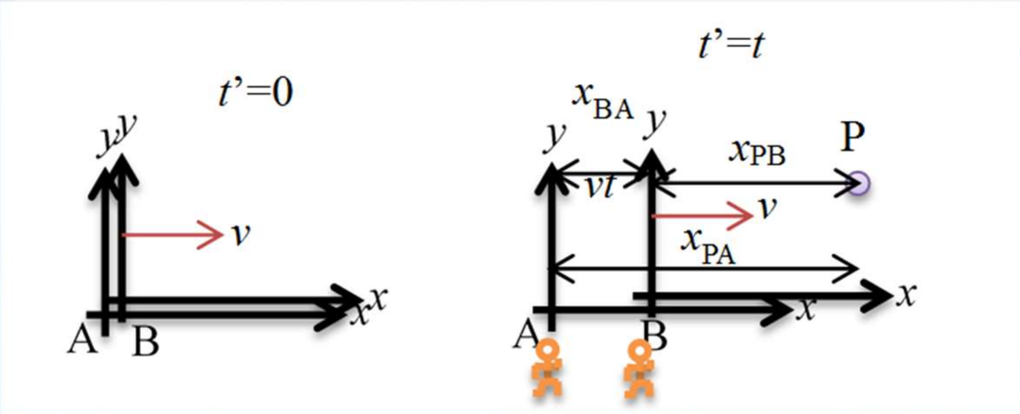
$$\vec{a}_t = 0.300\hat{\imath}$$

$$\vec{a}_r = -\frac{(6.00)^2}{500.0}\hat{\jmath} = -0.0720\hat{\jmath}$$

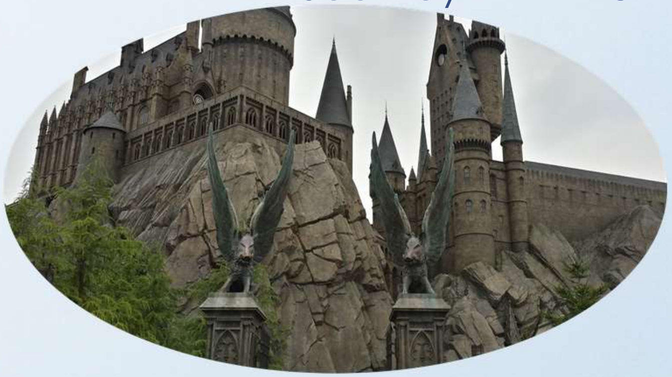
$$\vec{a} = 0.300\hat{\imath} - 0.0720\hat{\jmath} \rightarrow a = \sqrt{0.3^2 + 0.072^2}$$



7. RELATIVE VELOCITY



Harry Potter and the Forbidden Journey™ in 4K3D



$$x_{PA} = x_{PB} + x_{BA} \quad \Rightarrow \quad x_{PA} = x_{PB} + vt$$

$$\left(\frac{d}{dt}\right) x_{PA} = \left(\frac{d}{dt}\right) x_{PB} + \left(\frac{d}{dt}\right) vt \quad \Rightarrow \quad v_{PA} = v_{PB} + v_{BA}$$

7. RELATIVE VELOCITY

Example: Barbara's velocity relative to Alex is a constant v_{BA} = 50.0 km/h and car P is moving in the negative direction of the x axis. If Alex measures a constant velocity v_{PA} = -80.0 km/h for car P, what velocity will Barbara measure?

Use index to solve the problem, ask $v_{pB} =?$

$$v_{pB} = v_{pA} + v_{AB}$$

$$v_{pA} = -80.0 \text{ (km/h)}, \ v_{AB} = -v_{BA} = -50.0 \text{ (km/h)}$$

$$v_{pB} = v_{pA} + v_{AB} = -130 \text{ (km/h)}$$

7. RELATIVE VELOCITY

Example: A plane moves due east (directly toward the east) while the pilot points the plane somewhat south of east, toward a steady wind that blows to the northeast. The plane has velocity V_{PW} relative to the wind, with an airspeed (speed relative to the wind) of 215 km/h, directed at angle θ south of east. The wind has velocity V_{WG} relative to the ground, with a speed of 65.0 km/h, directed 30.0° east of north. What is the magnitude of the velocity V_{PG} of the plane relative to the ground, and what is θ?

$$\vec{v}_{WG} = 65.0\sin(\pi/6)\,\hat{\imath} + 65.0\cos(\pi/6)\,\hat{\jmath}$$

$$\vec{v}_{PW} = 215\cos\theta\,\hat{\imath} - 215\sin\theta\,\hat{\jmath}$$

$$\vec{v}_{PG} = \vec{v}_{PW} + \vec{v}_{WG} = (32.5 + 215\cos\theta)\hat{\imath} + (56.3 - 215\sin\theta)\hat{\jmath}$$

v_{PG} is in the due east direction

$$56.3 - 215\sin\theta = 0 \;\Rightarrow\; \theta \cong 15.2^o$$

$$\vec{v}_{PG} = (32.5 + 215\cos\theta)\hat{\imath} = 240\hat{\imath}\ \text{(km/h)}$$

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