Physics I Lecture01-Mathematics-Part I

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1. QUADRATIC EQUATION

Example: When a driver spots a running motorcycle moving with a constant speed (10 m/s) a distance of 5 m ahead of his car, he starts to accelerate at a 6 m/s^2. When will the driver catch up with the motorcycle?

$$S_M(t) = 5 + 10t$$

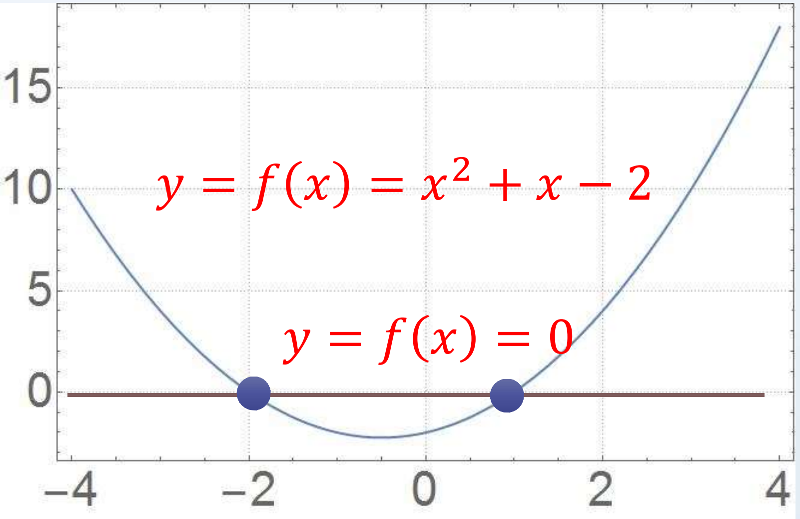
$$S_C(t) = \frac{6}{2}t^2$$

Assume that the driver catch up with the motorcycle at $t = t_1$

$$S_M(t_1) = S_C(t_1) \rightarrow 5 + 10t_1 = 3t_1^2$$

$$3t_1^2 - 10t_1 - 5 = 0 \rightarrow t_1 = \frac{10 \pm \sqrt{10^2 + 4 \times 5 \times 3}}{2 \times 3} = -0.44 \text{ or } 3.8 \text{ (s)}$$

1. QUADRATIC EQUATION



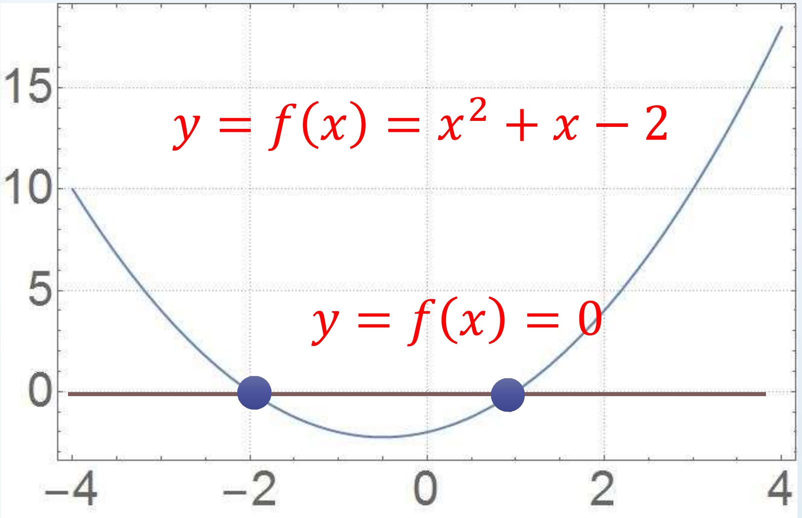
$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

1. QUADRATIC EQUATION



$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{(2a)^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{(2a)^2}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. LINEAR EQUATIONS

$$
\begin{array}{lll}
- & a_1 x + b_1 y = c_1 & \times a_2 \\
+ & a_2 x + b_2 y = c_2 & \times a_1 \\
\hline
\end{array}
$$

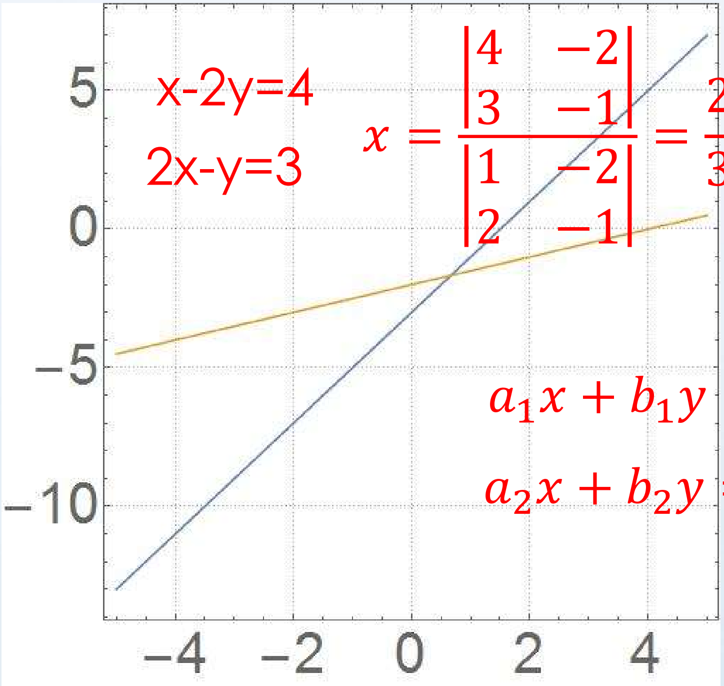
$$(a_1 b_2 - a_2 b_1) y = a_1 c_2 - a_2 c_1$$

$$y = \frac{a_1 c_2 - a_2 c_1}{a_1 b_2 - a_2 b_1}$$

$$a_1 b_2 - a_2 b_1 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

2. LINEAR EQUATIONS

Cramer's Rule



$$x = \frac{\begin{vmatrix} 4 & -2 \\ 3 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & -2 \\ 2 & -1 \end{vmatrix}} = \frac{2}{3}$$

$$a_1 x + b_1 y = c_1$$

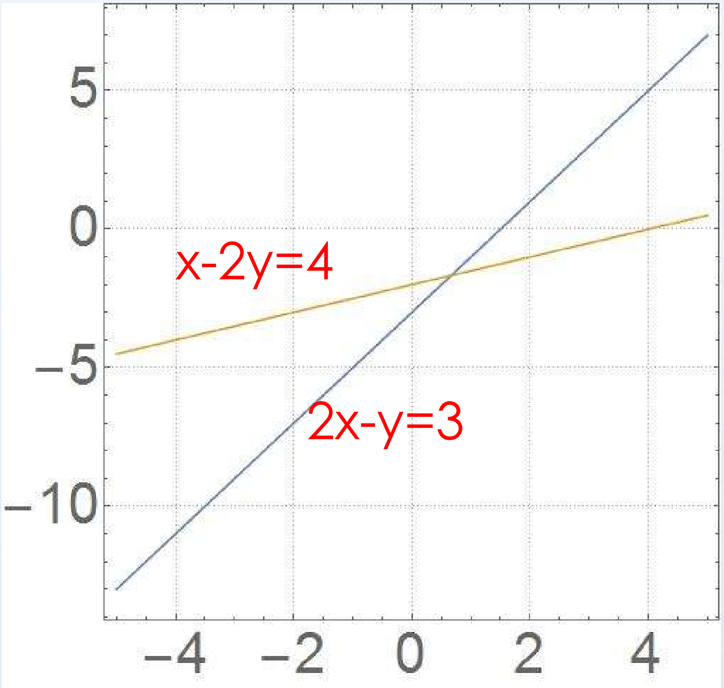
$$a_2 x + b_2 y = c_2$$

$$a_1 c_2 - a_2 c_1 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

$$y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

2. LINEAR EQUATIONS

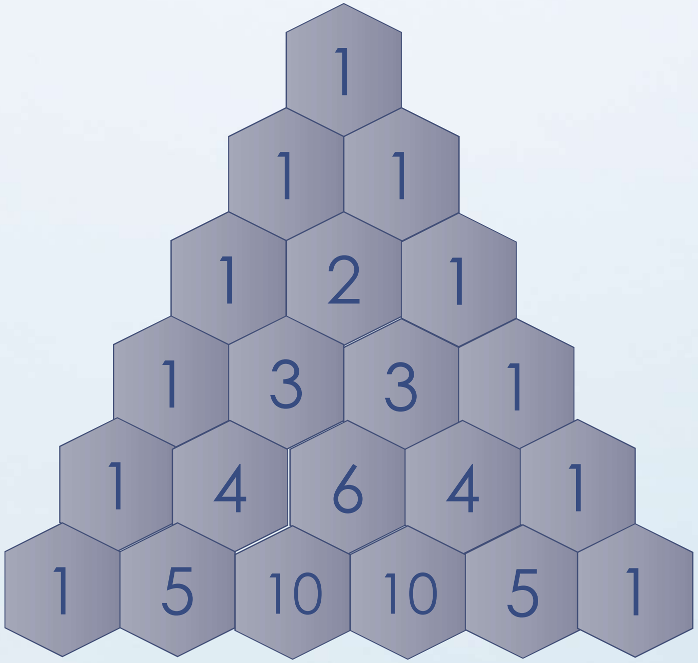


$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

3. BINOMIAL SERIES

Pascal's triangle:



$$(1+x)^2 = (1+x)(1+x)$$

$$(1+x)^2 = 1 + 2x + x^2$$

$$(1+x)^2 = C_0^2 + C_1^2 x + C_2^2 x^2$$

$$(1+x)^n = C_0^n + C_1^n x + C_2^n x^2 + \cdots + C_n^n x^n$$

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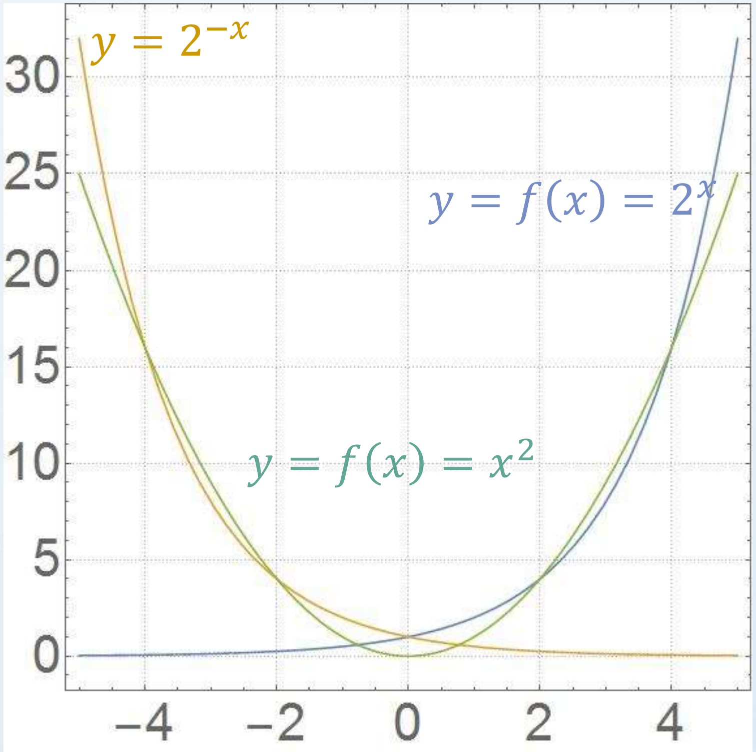
4. POWER, EXPONENT



$$a^m = a^{m-n+n} \quad \Rightarrow a^x \text{ such as } 2^x, 2^{-x}$$

$$a^m = a^{m-n} \times a^n$$

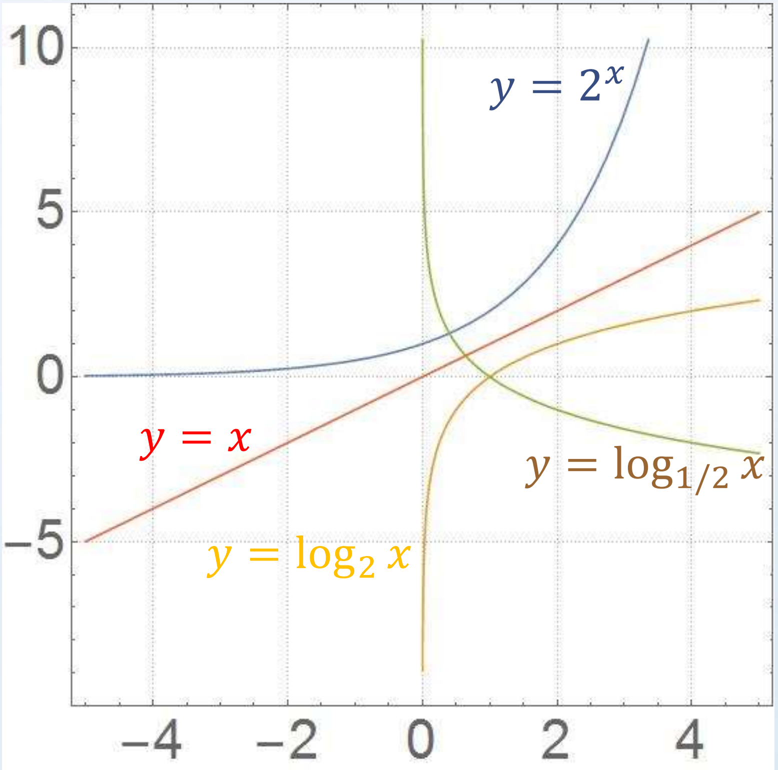
$$\frac{a^m}{a^n} = a^{m-n}$$

$$m = 0, \frac{1}{a^n} = a^{-n}$$

$$A \times A = a = a^{\frac{1}{2}+\frac{1}{2}}$$

$$A = a^{\frac{1}{2}} = \sqrt{a}$$

5. LOGARITHMS



$$y = a^x$$

$$x = log_a(y)$$

$$100 = 10^2$$

$$log_{10}(100) = 2$$

$$log_{10}(1) = 0$$

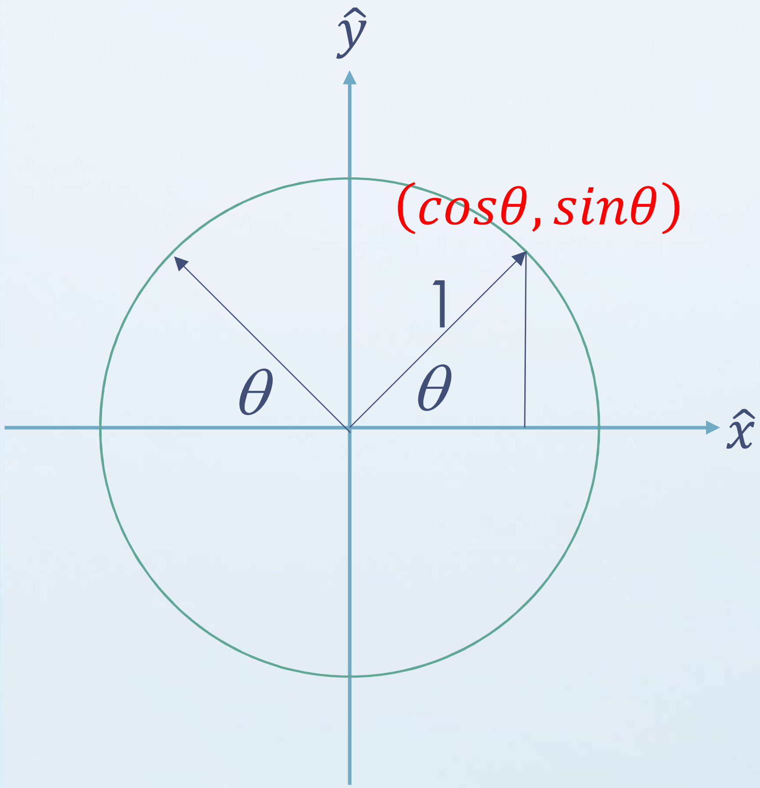
$$y_1 = a^x, y_2 = a^y$$

$$y_1 y_2 = a^x a^y = a^{x+y}$$

$$log(y_1 y_2) = x + y$$
$$= log(y_1) + log(y_2)$$

$$e = 2.718281828459045 \qquad \log_e y = \ln(y)$$

6. TRIGONAL FUNCTIONS



addition formula

$$sin(A \pm B)$$
$$= sinAcosB \pm cosAsinB$$

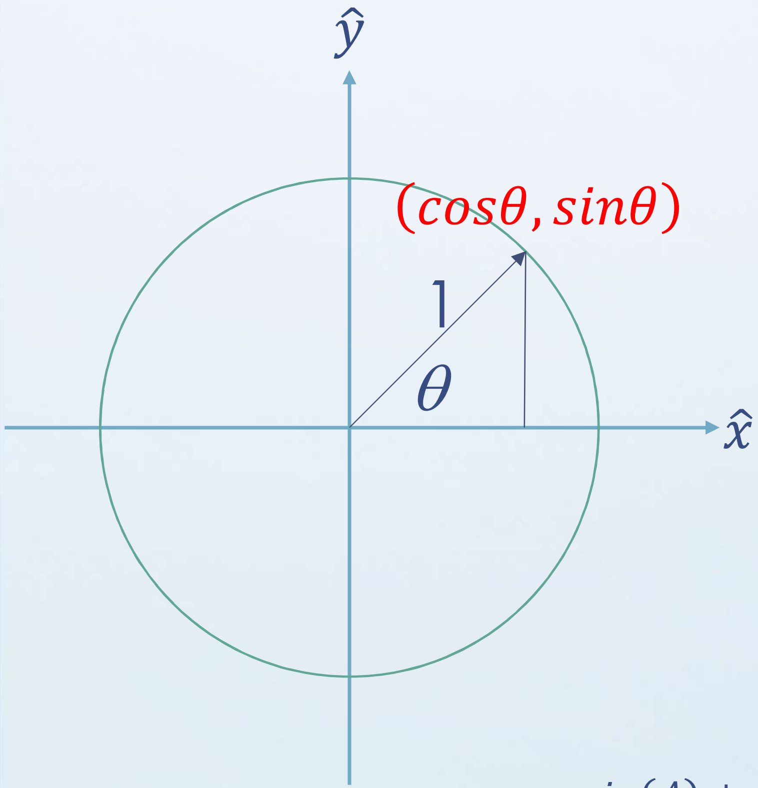
$$\cos(A \pm B)$$
$$= cosAcosB \mp sinAsinB$$

$$sin(\pi - \theta) = sin\theta$$

$$cos(\pi - \theta) = -cos\theta$$

$$sin\left(\frac{\pi}{2} - \theta\right) = cos\theta$$

6. TRIGONAL FUNCTIONS



$$sin(A) = sin\left(\frac{A+B}{2} + \frac{A-B}{2}\right)$$

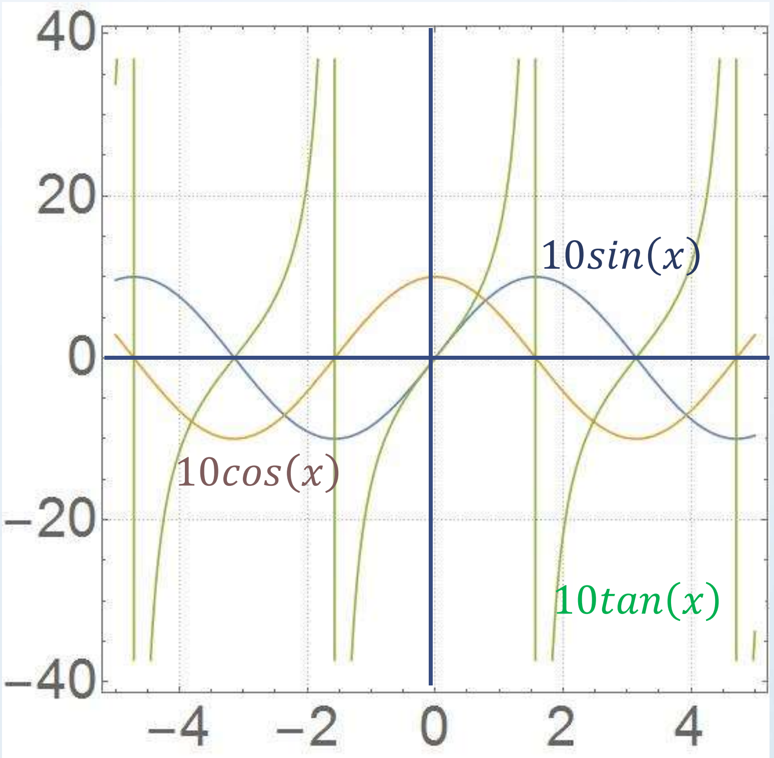
$$sin(A) = sin\left(\frac{A+B}{2}\right)cos\left(\frac{A-B}{2}\right) + cos\left(\frac{A+B}{2}\right)sin\left(\frac{A-B}{2}\right)$$

$$sin(B) = sin\left(\frac{A+B}{2} - \frac{A-B}{2}\right)$$

$$sin(B) = sin\left(\frac{A+B}{2}\right)cos\left(\frac{A-B}{2}\right) - cos\left(\frac{A+B}{2}\right)sin\left(\frac{A-B}{2}\right)$$

$$sin(A) + sin(B) = 2sin\left(\frac{A+B}{2}\right)cos\left(\frac{A-B}{2}\right)$$

6. TRIGONAL FUNCTIONS

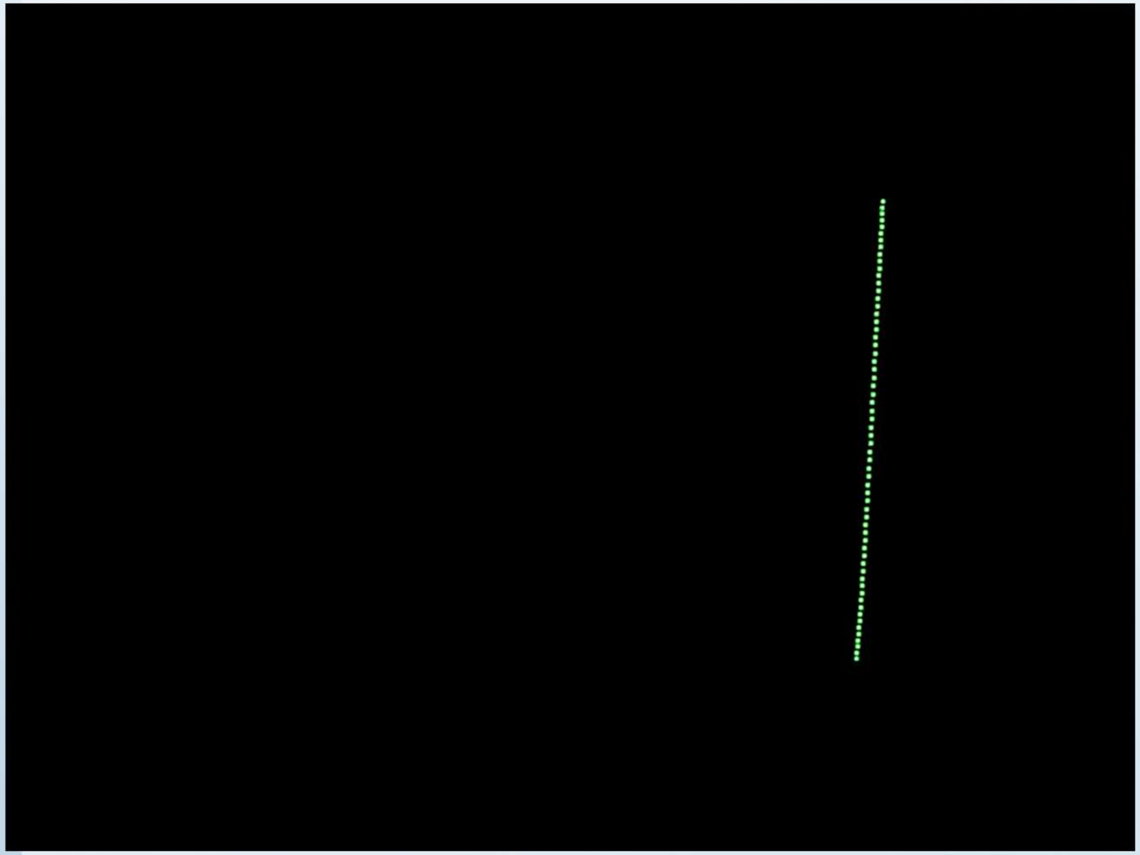


$$y = f(x) = 10sin(x)$$

$$y = f(x) = 10cos(x)$$

$$y = f(x) = 10tan(x)$$

APPLICATION OF SINUSOIDAL FUNCTIONS

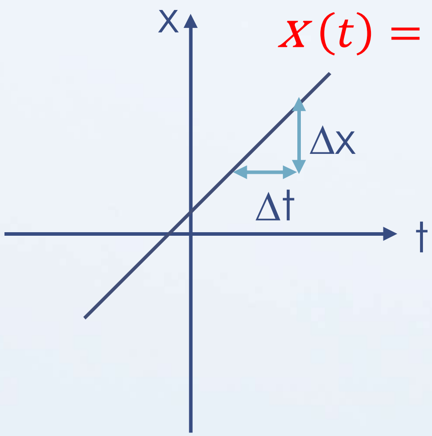


Lissajous curve

$$x(t) = A\sin(Ct + \delta)$$

$$y(t) = B\sin(Dt)$$

7. DIFFERENTIAL CALCULUS – CONSTANT VELOCITY

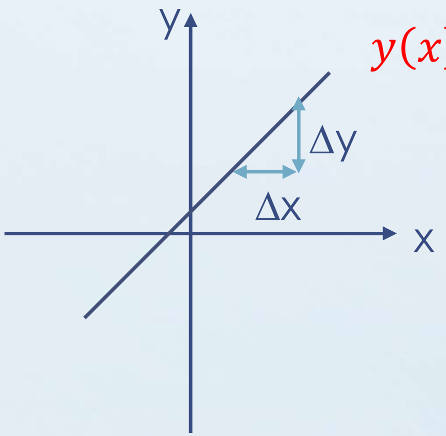


$$x(t) = x_0 + v_0 t$$

constant velocity

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$= \frac{x(t_2) - x(t_1)}{t_2 - t_1} = v_0$$

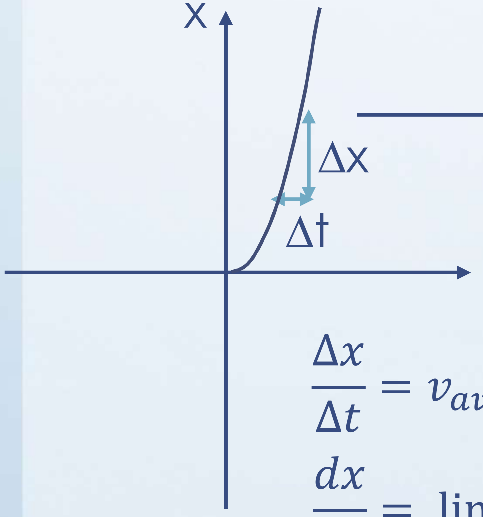


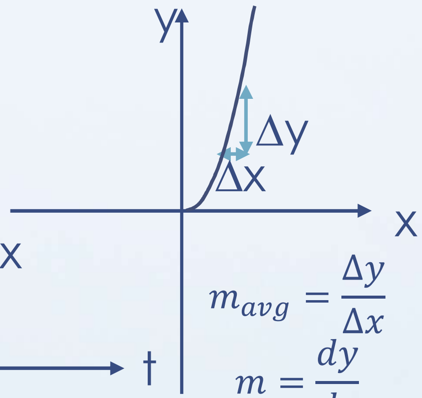
$$y(x) = f(x) = ax + b$$

$$m = \frac{\Delta y}{\Delta x} = \frac{y(x_2) - y(x_1)}{x_2 - x_1}$$

7. DIFFERENTIAL CALCULUS – CONSTANT ACCELERATION

constant acceleration – freely falling object



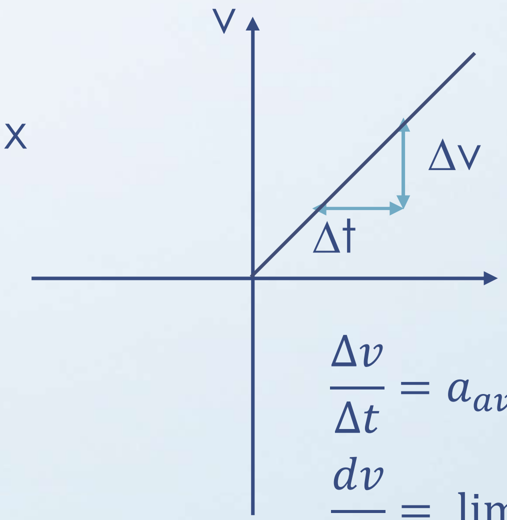


$$m_{avg} = \frac{\Delta y}{\Delta x}$$

$$m = \frac{dy}{dx}$$

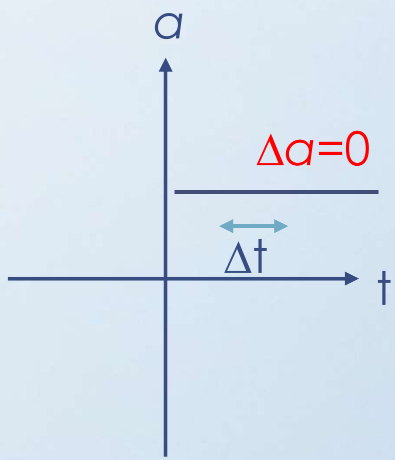
$$\frac{\Delta x}{\Delta t} = v_{avg}$$

$$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = v$$



$$\frac{\Delta v}{\Delta t} = a_{avg}$$

$$\frac{dv}{dt} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = a$$



Differential Calculation: $\frac{dx(t)}{dt} = \lim_{\Delta t \to 0} \frac{\Delta x(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t)-x(t)}{t+\Delta t - t}$

7. DIFFERENTIAL CALCULUS – DEFINITION OF DIFFERENTIATION

Differential Calculation: $\frac{dx(t)}{dt} = \lim_{\Delta t \to 0} \frac{\Delta x(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t)-x(t)}{t+\Delta t-t}$

For example, $x(t) = t^2$,

$$\frac{dx(t)}{dt} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{(t+\Delta t)^2 - t^2}{\Delta t} = 2t$$

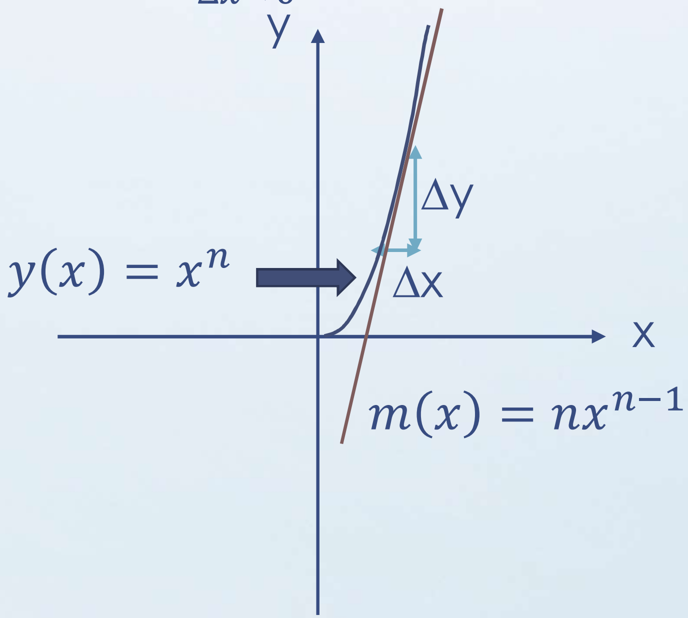
$$\frac{d(x^n)}{dx} = nx^{n-1} \qquad \frac{d(\sin(x))}{dx} = \cos(x) \qquad \frac{d(\cos(x))}{dx} = -\sin(x)$$

$$\frac{d(e^x)}{dx} = e^x \qquad \frac{d(\ln(x))}{dx} = \frac{1}{x}$$

7. DIFFERENTIAL CALCULUS – DEFINITION OF DIFFERENTIATION

$$\frac{d}{dx}(x^n) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^n - x^n}{\Delta x} = \lim_{\Delta x \to 0} \frac{x^n + C_1^n x^{n-1}\Delta x + C_2^n x^{n-2}(\Delta x)^2 + \cdots - x^n}{\Delta x}$$

$$= \lim_{\Delta x \to 0}\left(C_1^n x^{n-1} + C_2^n x^{n-2}\Delta x + \cdots\right) = n x^{n-1}$$



$$d(x^n) = m(x)dx = n x^{n-1} dx$$

$$dy = \frac{dy}{dx}dx = m(x)dx$$

7. DIFFERENTIAL CALCULUS – CHAIN RULE

rules of differentiation

$$\frac{d}{dx}[f(x)g(x)] = \frac{df(x)}{dx}g(x) + f(x)\frac{dg(x)}{dx}$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{d}{dx}[f(x)g^{-1}(x)] = \frac{df(x)}{dx}g^{-1}(x) + f(x)\frac{dg^{-1}(x)}{dx}$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{df(x)}{dx}g^{-1}(x) + f(x)\left(-g^{-2}(x)\frac{dg(x)}{dx}\right)$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\left(\frac{df(x)}{dx}g(x) - f(x)\frac{dg(x)}{dx}\right)}{g^2(x)}$$

$$\frac{d}{dx}[f(g(x))] = \frac{df(g)}{dg}\frac{dg(x)}{dx}$$

chain rule

7. DIFFERENTIAL CALCULUS – CHAIN RULE

All calculations are based on the knowledge you already learned!

$$\frac{d(\sin(x))}{dx} = \cos(x) \;\; \& \;\; \frac{dx^3}{dx} = 3x^2 \rightarrow \frac{d\sin(x^3)}{dx} = ?$$

Let $f(g) = \sin(g)\,, g(x) = x^3$ $\quad \frac{df(g(x))}{dx} = \frac{df(g)}{dg}\frac{dg(x)}{dx}$

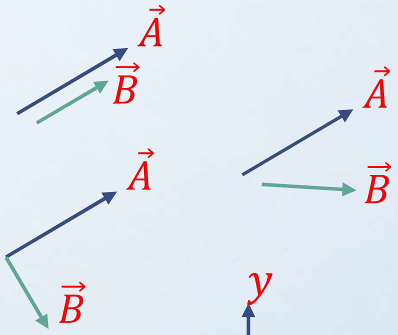
$$\frac{d\sin(x^3)}{dx} = \frac{d\sin(x^3)}{d(x^3)}\frac{dx^3}{dx} = \cos(x^3)\,3x^2$$

$$\frac{d\left(\cos(x^2+2x)+x^3\right)}{dx} = \left(\frac{d}{dx}\right)\left(\cos(x^2+2x)+x^3\right) = \frac{d\cos(x^2+2x)}{dx} + \frac{dx^3}{dx}$$

$$= \frac{d\cos(x^2+2x)}{d(x^2+2x)}\frac{d(x^2+2x)}{dx} + 3x^2 = -\sin(x^2+2x)\,(2x+2) + 3x^2$$

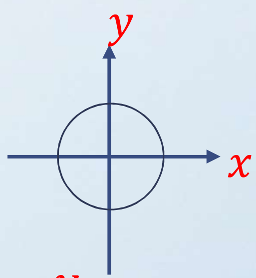
8. PARTIAL DIFFERENTIAL CALCULUS – ORTHOGONAL COORDINATES

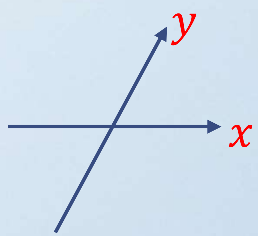
Vector in space to demonstrate the concept



Linearly Dependent: $a\vec{A} + b\vec{B} = 0,\ a \neq 0,\ b \neq 0$
Linearly Independent: $a\vec{A} + b\vec{B} = 0,\ a = 0,\ b = 0$
Orthogonal: $\vec{A} \cdot \vec{B} = 0$

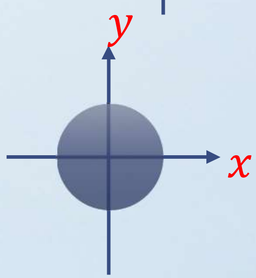
Functions are similar, but not exactly the same





Dependent: x, y, $y = y(x)$ or $x = x(y)$
Independent: no function relation between x and y

Partial Differentiation of $f(x, y)$: $\frac{\partial f(x,y)}{\partial x}$, take y as a const



Complete Differentiation of $f(x, y)$:

$$\frac{df(x,y)}{dx} = \frac{\partial f(x,y)}{\partial x} + \frac{\partial f(x,y)}{\partial y}\frac{dy}{dx}, y = y(x)$$

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9. TAYLOR EXPANSION

Any function $f(x)$ can be expanded at $x = a$ $\quad f(x) = C_0 + C_1(x-a) + C_2(x-a)^2 + \cdots$

Taylor expansion of $f(x)$ at $x = a$

$$f(x) = f(a) + \frac{df(x)}{dx}\bigg]_{x=a} \frac{(x-a)}{1!} + \frac{d^2 f(x)}{dx^2}\bigg]_{x=a} \frac{(x-a)^2}{2!} + \cdots$$

Taylor expansion of $f(x)$ at $x = 0$ $\quad f(x) = C_0 + C_1 x + C_2 x^2 + \cdots$

$$f(x) = f(0) + f'(0)\frac{x}{1!} + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} + \ldots.$$

For example, $f(x) = x^2 + 2x$, $f(0) = 0$, $f'(0) = 2$, $f''(0) = 2$

$f'''(0) = f''''(0) = \cdots = 0$

Taylor expansion gives $f(x) = 0 + \frac{2}{1!}x + \frac{2}{2!}x^2$

9. TAYLOR EXPANSION

Series of e $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$

Series of e^x $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

Differentiation of e^x $\frac{de^x}{dx} = 0 + \frac{1}{1!} + \frac{2x^1}{2!} + \frac{3x^2}{3!} + \cdots$

$$\frac{de^x}{dx} = 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \cdots = e^x$$

$\frac{d\ln x}{dx} = ?$ Let $y = \ln x \rightarrow x = e^y$ $\frac{dx}{dx} = \frac{de^y}{dx} = \frac{de^y}{dy}\frac{dy}{dx} = e^y\frac{dy}{dx}$

$1 = e^y\frac{dy}{dx}$ ➡ $\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$ ➡ $\frac{d\ln x}{dx} = \frac{1}{x}$

9. TAYLOR EXPANSION

Differentiation of $\sin(x)$:

$$\frac{d\sin(x)}{dx} = \lim_{\Delta x \to 0} \frac{\sin(x+\Delta x) - \sin(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sin(x)\cos(\Delta x) + \cos(x)\sin(\Delta x) - \sin(x)}{\Delta x} = \cos(x)$$

Differentiation of $cos(x)$:

$$\frac{d\cos(x)}{dx} = \lim_{\Delta x \to 0} \frac{\cos(x+\Delta x) - \cos(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cos(x)\cos(\Delta x) - \sin(x)\sin(\Delta x) - \cos(x)}{\Delta x} = -\sin(x)$$

Taylor expansion:

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

9. TAYLOR EXPANSION

$$\sin(x) = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \cdots = \sin(0) + \frac{\cos(0)}{1!}x + \frac{-\sin(0)}{2!}x^2 + \frac{-\cos(0)}{3!}x^3 + \cdots$$

$$\sin(x) = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \pm \cdots$$

$$\cos(x) = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \cdots = \cos(0) + \frac{-\sin(0)}{1!}x + \frac{-\cos(0)}{2!}x^2 + \frac{\sin(0)}{3!}x^3 + \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \pm \cdots$$

Imaginary number: $i \equiv \sqrt{-1}, or\ i^2 = -1$

Notation:

$$e^{ix} = 1 + \frac{(ix)}{1!} + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \cdots$$

$$e^{ix} = \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots\right) + i\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right)$$

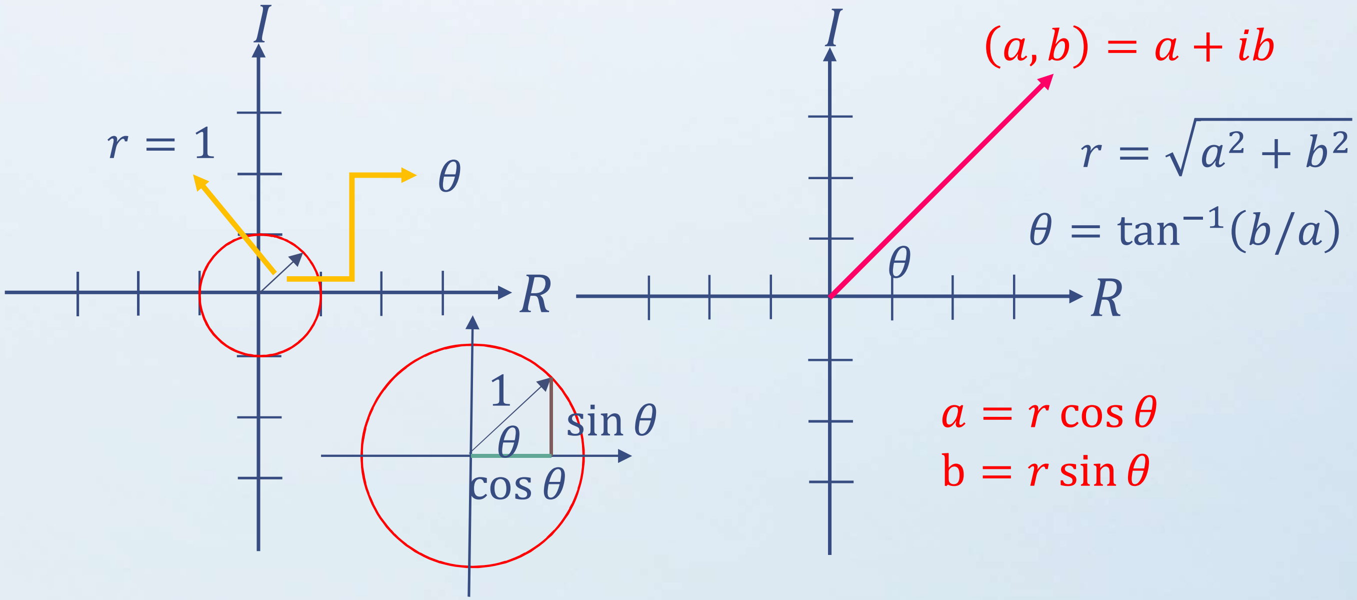
$$e^{ix} = \cos x + i \sin x$$

10. COMPLEX NUMBERS

Embody the notation of complex number $e^{ix} = \cos x + i \sin x$

Present in the polar coordinate $e^{i\theta} = \cos\theta + i\sin\theta$

$re^{i\theta} = r\cos\theta + ir\sin\theta$



10. COMPLEX NUMBERS (APPLICATION)

Trigonal functions – addition formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{i\alpha} = \cos\alpha + i\sin\alpha \qquad e^{i\beta} = \cos\beta + i\sin\beta$$

$$e^{i(\alpha+\beta)} = e^{i\alpha}e^{i\beta} \Rightarrow$$

$$\cos(\alpha+\beta) + i\sin(\alpha+\beta) = (\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta)$$

$$\cos(\alpha+\beta) + i\sin(\alpha+\beta) = (\cos\alpha\cos\beta - \sin\alpha\sin\beta) + i(\sin\alpha\cos\beta + \sin\beta\cos\alpha)$$

$$\Rightarrow \begin{array}{l} \sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha \\ \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \end{array}$$

Trigonal functions – double angle formula

$$e^{i(2\theta)} = e^{i\theta}e^{i\theta} \Rightarrow \begin{array}{l} \sin(2\theta) = 2\sin\theta\cos\theta \\ \cos(2\theta) = \cos^2\theta - \sin^2\theta \end{array}$$

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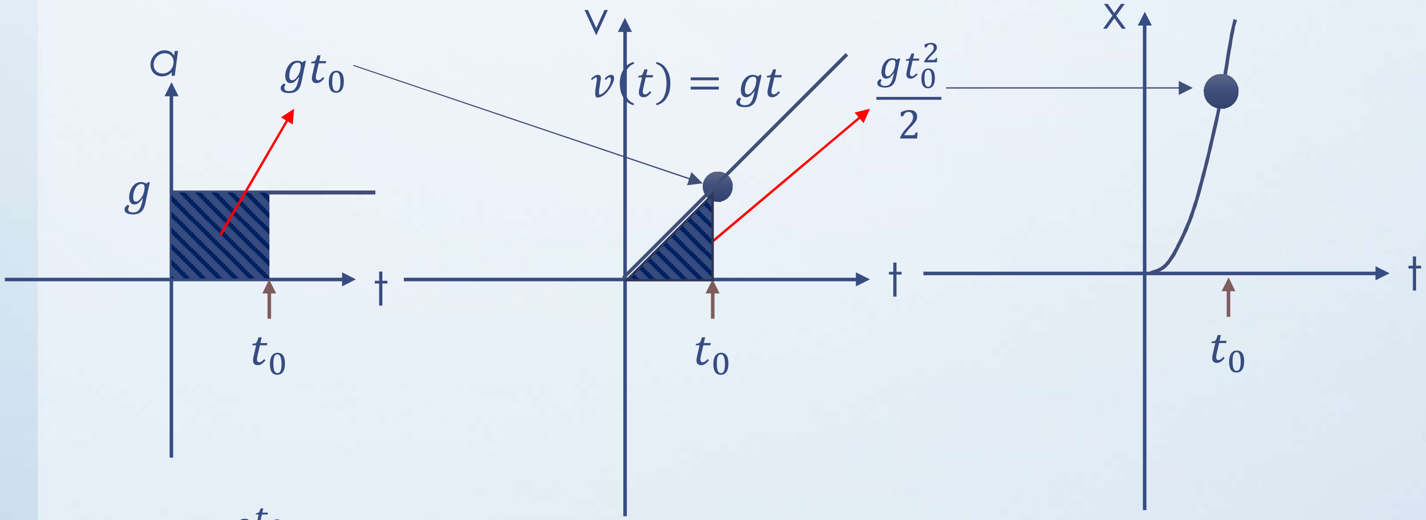
11. INTEGRATION

constant acceleration g – freely falling object



$$v(t_0) = \int_0^{t_0} g dt = [gt]_{t=0}^{t=t_0} = gt_0 - g \cdot 0 = gt_0 \Rightarrow v(t) = gt$$

$$x(t_0) = \int_0^{t_0} gt dt = \left[\frac{gt^2}{2}\right]_{t=0}^{t=t_0} = \frac{gt_0^2}{2} - \frac{g \cdot 0^2}{2} = \frac{gt_0^2}{2} \Rightarrow x(t) = gt^2/2$$

https://giphy.com/gifs/QhaDJN80P3YqI

11. INTEGRATION

Differentiation

$f(x) = x^n \quad \frac{df(x)}{dx} = nx^{n-1}$ $\qquad f(x) = e^x \quad \frac{df(x)}{dx} = e^x$

$f(x) = \sin x \quad \frac{df(x)}{dx} = \cos x$ $\qquad f(x) = \ln x \quad \frac{df(x)}{dx} = \frac{1}{x}$

Integration

$f(x) = x^n, \int f(x)dx = \frac{x^{n+1}}{n+1} + c$ $\qquad f(x) = e^x, \int f(x)dx = e^x + c$

$f(x) = \cos x, \int f(x)dx = \sin x + c$ $\qquad f(x) = \frac{1}{x} \quad \int f(x)dx = \ln x + c$

11. INTEGRATION – CHAIN RULE & CHANGE OF VARIABLES

$$\frac{d(\sin(x^2+1))}{dx} = \frac{d(\sin(x^2+1))}{d(x^2+1)} \frac{d(x^2+1)}{dx} = \cos(x^2+1) \times 2x$$

$$\int_0^1 2x\cos(x^2+1)\,dx = [\sin(x^2+1)]_{x=0}^{x=1}, \text{how?}$$

$$\int_0^1 2x\cos(x^2+1)\,dx = \int_0^1 \cos(x^2+1)\,d(x^2) = \int_0^1 \cos(x^2+1)\,d(x^2+1)$$

$$= \int_0^1 d(\sin(x^2+1)) = [\sin(x^2+1)]_{x=0}^{x=1}$$

$$\int_0^{\pi/4} \frac{1}{\cos(x)}dx = \int_0^{\pi/4} \frac{\cos(x)}{\cos^2(x)}dx = \int_0^{\pi/4} \frac{1}{\cos^2(x)}d(\sin(x)) = \int_0^{\pi/4} \frac{1}{1-\sin^2(x)}d(\sin(x))$$

$$= \frac{1}{2}\int_0^{\pi/4}\left(\frac{1}{1-\sin x} + \frac{1}{1+\sin x}\right)d(\sin(x)) = \frac{1}{2}\int_0^{\pi/4}\frac{1}{1+\sin x}d(1+\sin(x)) - \frac{1}{2}\int_0^{\pi/4}\frac{1}{1-\sin x}d(1-\sin(x))$$

$$= \frac{1}{2}[\ln((1+\sin x)/(1-\sin x))]_{x=0}^{x=\pi/4}$$

12. 1ST ORDER DIFFERENTIAL EQUATION

Some problems like population growth

Its growth rate is proportional to the population at that time

Auume the population P as a function of time t is expressed as $P(t)$

Its growth rate (or variation) is $dP(t)/dt$

If the growth rate is propoetional to the population with a const k

That can be expressed as $\frac{dP(t)}{dt} = kP(t)$ We obtain the 1st order DE

To solve the problem, we need to have one condition like $P(t=0) = P_0$

Separation of variable $\frac{dP(t)}{P(t)} = kdt$

$$\int \frac{dP}{P} = \int kdt \quad \int_{P_0}^{P(t')} \frac{dP}{P} = \int_0^{t'} kdt \quad [\ln P]_{P_0}^{P(t')} = [kt]_0^{t'}$$

$$\ln(P(t')) - \ln P_0 = kt' - 0 \quad \ln\left(\frac{P(t')}{P_0}\right) = kt' \quad \frac{P(t')}{P_0} = e^{kt'} \quad P(t') = P_0 e^{kt'}$$

12. 1ST ORDER DIFFERENTIAL EQUATION

Check that $P(t) = P_0 e^{kt}$ is the solution of the 1st order DE $\frac{dP}{dt} = kP$

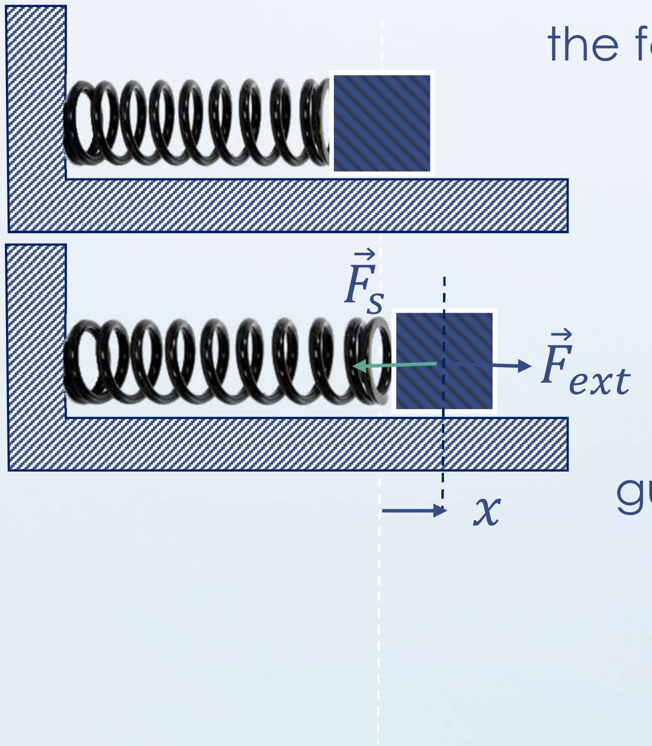
Put the solution into the DE, check if the equation is satisfied or not

$$\frac{dP(t)}{dt} = \frac{d\left(P_0 e^{kt}\right)}{dt} = \frac{P_0 d\left(e^{kt}\right)}{dt} = P_0 \frac{d\left(e^{kt}\right)}{d(kt)} \frac{d(kt)}{dt} = kP_0 e^{kt} = kP(t)$$

The equation is satisfied by using the function $P(t)$

Thus the function $P(t) = P_0 e^{kt}$ is the solution of the DE $\frac{dP}{dt} = kP$

13. 2ND ORDER DIFFERENTIAL EQUATION



Derive the 2nd order DE from
the force equation of the spring system

$$F = -kx$$
$$F = ma$$
$$\Rightarrow -kx = ma$$

$$ma + kx = 0$$

$$\Rightarrow m\frac{d^2x}{dt^2} + kx = 0$$

guess solution $x(t) = A\sin(Bt)$ or $x(t) = A\cos(Bt)$

$$-mB^2A\sin(Bt) + kA\sin(Bt) = 0$$

$$(k - mB^2)A\sin(Bt) = 0$$

$$B = \sqrt{\frac{k}{m}} \qquad x(t) = A\sin\left(\sqrt{k/m}t\right)$$

13. 2ND ORDER DIFFERENTIAL EQUATION

$$m\frac{d^2x}{dt^2} + kx = 0$$

guess solution $x(t) = Ae^{Bt}$, put the guess solution into the DE

$$mB^2Ae^{Bt} + kAe^{Bt} = 0$$

$$(k + mB^2)Ae^{Bt} = 0 \qquad B = i\sqrt{k/m} \quad B = -i\sqrt{k/m}$$

$$x(t) = A_1e^{i\sqrt{k/mt}} + A_2e^{i\sqrt{k/mt}} = C_1 \sin\left(\sqrt{k/mt}\right) + C_2 \cos\left(\sqrt{k/mt}\right)$$

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