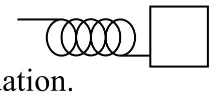
Lecture 07 Energy of a System

Energy approach to describe motion is especially useful <u>when the force acting on the particle is not a constant</u>.



Example: $\vec{F} = m\frac{d^2x}{dt^2} = -kx$ → You need to solve the differential equation.

It would be much easier if you know **the concept of energy** and learn the **energy conservation law**.

$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kx_{max}^2 = \frac{1}{2}mv_{max}^2$$

7.1 Systems and Environments

Understand the system with its environment (system boundary).

A valid system may

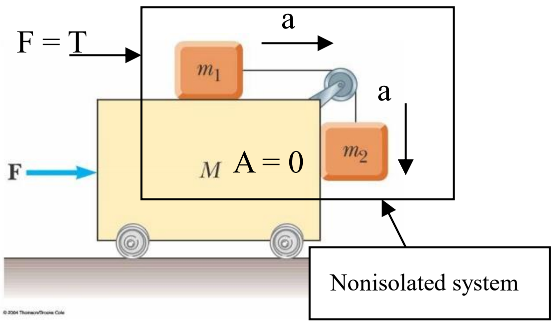
1. be a single object or particle
2. be a collection of objects or particles
3. be a region of space (filling with gas or liquid, such as the interior of an automobile engine)
4. vary in size and shape (rubber ball)

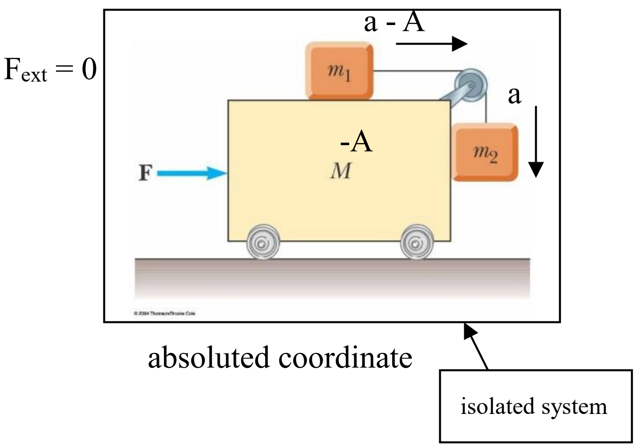
F = 0

$m_2g - T = m_2a$, $T = m_1(a - A)$, $T = MA$

for m2: $a = \frac{(M + m_1)m_2g}{Mm_1 + m_1m_2 + m_2M}$

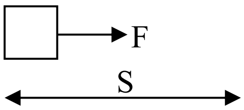
for M: $A = \frac{m_1m_2g}{Mm_1 + m_1m_2 + m_2M}$



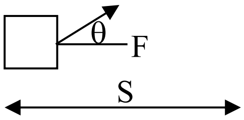


7.2 Work Done by a Constant Force

$W = FS$



$W = (F\cos\theta)S$



7.3 The Scalar Product of Two Vectors

$\vec{A}\cdot\vec{B} = AB\cos\theta$

Commutative: $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

Distributive (associative): $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$

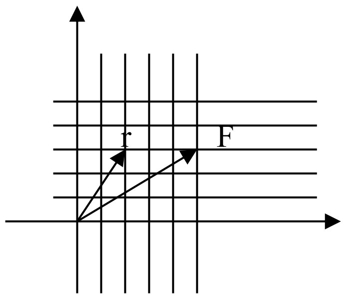
$\hat{i}\cdot\hat{i}=1,\_\hat{j}\cdot\hat{j}=1,\_\hat{k}\cdot\hat{k}=1,\_\hat{i}\cdot\hat{j}=0,\_\hat{i}\cdot\hat{k}=0,\_\hat{j}\cdot\hat{k}=0$

$\vec{A}\cdot\vec{B} = A_x B_x \hat{i}\cdot\hat{i} + A_y B_y \hat{j}\cdot\hat{j} + A_z B_z \hat{k}\cdot\hat{k}$

Example: The vector $\vec{A}$ and $\vec{B}$ are given by $\vec{A}=2\hat{i}+3\hat{j}$ and $\vec{B}=-\hat{i}+2\hat{j}$. (a) Determine the scalar product $\vec{A}\cdot\vec{B}$. (b) Find the angle θ between A and B.

A force that turn your direction of motion does not transfer kinetic energy to you. → Separate them to motional and rotational energy.

Example: A particle moving in the xy plane undergoes a displacement $\Delta\vec{r}=(2.0\hat{i}+3.0\hat{j})$ m as a constant force $\vec{F}=(5.0\hat{i}+2.0\hat{j})$ N acts on the particle.

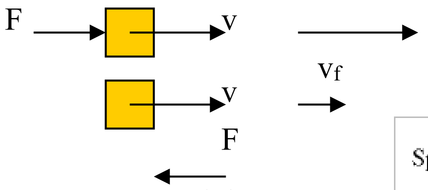


Example: Using the definition of the scalar product, find the angles between (a) $\vec{A}=3\hat{i}-2\hat{j}$ and $\vec{B}=4\hat{i}-4\hat{j}$; (b) $\vec{A}=-2\hat{i}+4\hat{j}$ and $\vec{B}=3\hat{i}-4\hat{j}+2\hat{k}$; (c) $\vec{A}=\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{B}=3\hat{j}+4\hat{k}$.

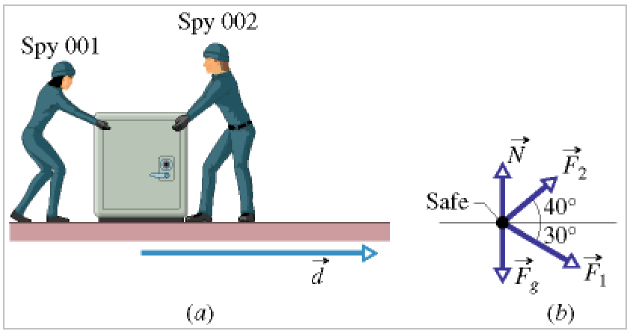
Motion in One Dimension with Constant Forces

work: $W=\vec{F}\cdot\vec{S}$

Positive or Negative – What's difference?



Kinetic Energy = Inertial Energy



Unit of Work: N·m = J

Work is an energy transfer.

Example: Figure shows two industrial spies sliding an initially stationary 225 kg floor safe a displacement $\vec{d}$ of magnitude 8.50 m, straight toward their truck. The push $\vec{F}_1$ of Spy 001 is 12.0 N, directed at an angle of 30° downward from the horizontal; the pull $\vec{F}_2$ of Spy 002 is 10.0 N, directed at 40° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.

$\sin 30^o = 0.5$, $\cos 30^o = 0.87$, $\sin 40^o = 0.64$, $\cos 40^o = 0.77$

$\vec{F}_1 = 12 \cdot \cos 30^o \hat{i} - 12 \cdot \sin 30^o \hat{j}$, $\vec{F}_2 = 10 \cdot \cos 40^o \hat{i} + 10 \cdot \sin 40^o \hat{j}$

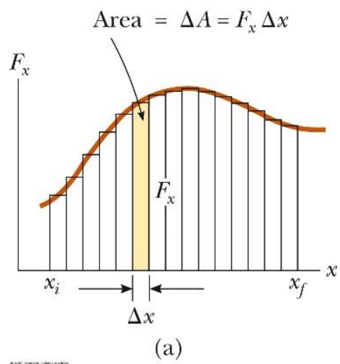
$\vec{F} = \vec{F}_1 + \vec{F}_2 = 10.4\hat{i} - 6\hat{j} + 7.7\hat{i} + 6.4\hat{j} = 18.1\hat{i} + 0.4\hat{j}$

$\vec{F} \cdot \vec{S} = (18.1 \cdot \hat{i} + 0.4 \cdot \hat{j}) \cdot (8.5 \cdot \hat{i}) = 153.9$

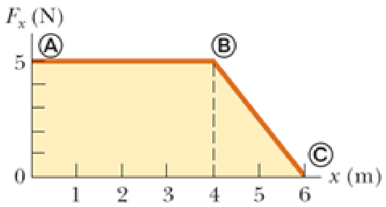
7.4 Work Done by a Varying Force

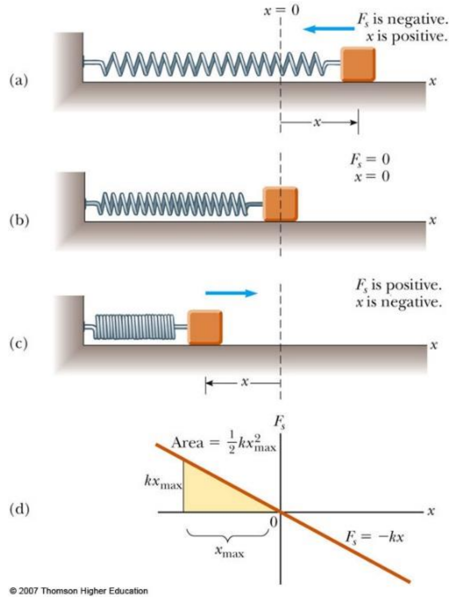
$$W \approx F_x \Delta x \rightarrow W \approx \sum_{xi}^{xf} F_x \Delta x \rightarrow \lim_{\Delta x \to 0} \sum_{xi}^{xf} F_x \Delta x = \int_{xi}^{xf} F_x dx$$

$$W = \int_{xi}^{xf} F(x) \cdot dx = \int_{ri}^{rf} \vec{F}(\vec{r}) \cdot d\vec{r}$$



Example: Calculating Total Work Done from a Graph





Work Done by a Spring

$F_x = -kx$

Sprint on block: $W = \int_{xi}^{xf} -kx dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$

Block on sprint: $W_{app} = \int_{xi}^{xf} -kx dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$

Example: $\vec{F} = 3x^2\hat{i} + 4\hat{j}$ N, with x in meters, acts on a particle, changing only the kinetic energy of the particle. How much work is done on the particle as it moves from coordinates (2 m, 3 m) to (3 m, 0 m)? Does the speed of the particle increase, decrease, or remain the same?

$\vec{r}_i = (2,3)$, $\vec{r}_f = (3,0)$, $d\vec{r} = \hat{i}dx + \hat{j}dy$, $dW = \vec{F} \cdot d\vec{r} = (3x^2\hat{i} + 4\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$

$$W = \int_{(2,3)}^{(3,0)} 3x^2 dx + 4dy = \int_2^3 d\left[x^3\right] + \int_3^0 d\left[4y\right] = 7$$

The speed of the particle increase: $\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + 7$

Example: An archer pulls her bowstring back 0.400 m by exerting a force that increases uniformly from zero to 230 N. (a) What is the equivalent spring constant of the bow? (b) How much work does the archer do in pulling the bow?

(a) $F = -kx$ -> $230 = k \cdot 0.4$, $k = 575$ N/m

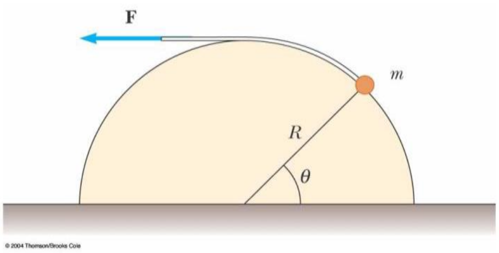
(b) $W = \frac{1}{2}kx^2 = \frac{1}{2} * 575 * 0.4^2 = 46$ J

Example: A small particle of mass m is pulled to the top of a frictionless half-cylinder (of radius R) by a cord that passes over the top of the cylinder. (a) If the particle moves at a constant speed, show that F = mgcosθ. (Note: If the particle moves at constant speed, the component of its acceleration tangent to the cylinder must be zero at all times.)

(b) By directly integrating W = ∫F·dr, find the work done in moving the particle at constant speed from the bottom to the top of the half-cylinder.

Hint (a): Draw the force graph, the force component that do work should be along the circle.

Hint (b): F is along the arc, the distance along the circle is ds = R dθ. Integrate from θ = 0 to θ = θ to get the work done by the string.



7.5 Kinetic Energy and the Work-Kinetic Energy Theorem

The Work-Kinetic Energy Theorem

work – a mechanism for transferring energy into a system

work-kinetic energy theorem: work transfer to kinetic energy in a non-isolated system

$$W = \int F(x)\cdot dx = \int ma\cdot dx = \int m\frac{dv}{dt}\cdot dx\,, \quad \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt}$$

$$W = \int m\frac{dv}{dx}v\cdot dx = \int mv\cdot dv = \int d\left[\frac{1}{2}mv^2\right] = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

kinetic energy $K = \frac{1}{2}mv^2$, work-kinetic energy theory: $W = \Delta K$

Example: A 6.0 kg block initially at rest is pulled to the right along a horizontal, frictionless surface by a constant force of 12 N. Find the speed of the block after it has moved 3.0m.

Method 1:

$$F = 12 = ma = 6\frac{dv}{dt} \rightarrow v^2 = v_0^2 + 2as$$

Method 2:

$$F * s = \frac{1}{2} m v^2$$

Example: Does the Ramp Lesson the Work Required?

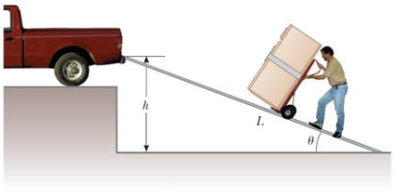


Figure 7.14 (Conceptual Example 7.7) A refrigerator attached to a frictionless, wheeled hand truck is moved up a ramp at constant speed.

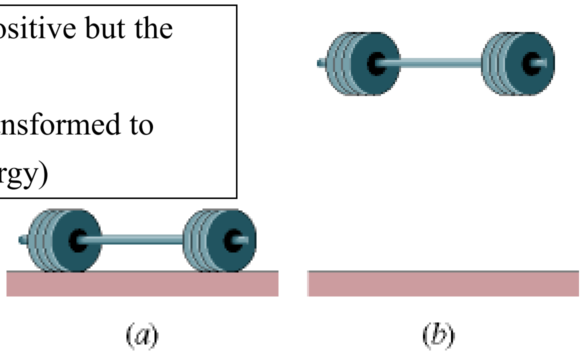
$$F = mg \sin\theta, \quad s = L = \frac{h}{\sin\theta}$$

→ $W = FS = mgh$

7.6 Potential Energy of a System

As the objects undergoes the upward displacement by the applied force $\vec{F}$, you stored energy $W = \vec{F} \cdot \vec{h} = mg\hat{z} \cdot (y_f - y_i)\hat{z}$ in the system.

What happens? In the process work positive but the kinetic energy remains the same.
Ans: work done by external forces is transformed to potential energy (stored in potential energy)



$U = mgy$, $W = \Delta U_g$

Potential energy of the system is increased by work of external force.

Two kinds of typical potential energy: gravitational potential energy, elastic potential energy.

Potential Energy of a Spring

$$\vec{F} = -k\vec{d}$$

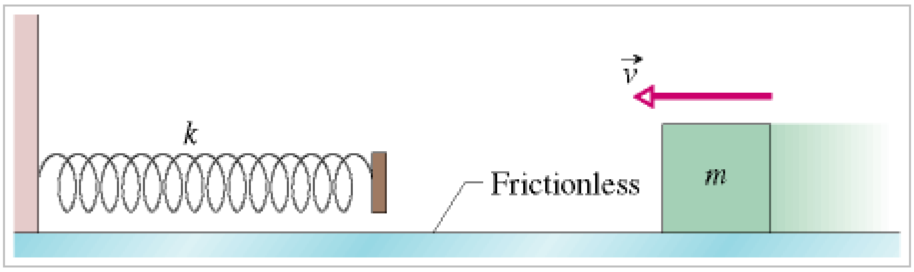
$$F = -kx, \quad dW = F(x)dx, \quad dW = -kx \cdot dx, \quad \int dW = \int -k \cdot d[\frac{1}{2}x^2]$$

$$\int_0^W dW = \int_0^x d\left[-\frac{k}{2}x^2\right] = -\frac{1}{2}kx^2$$

$$F = -kx = m \cdot a = m \cdot \frac{dv}{dt}, \quad m \cdot \frac{dv}{dt} = -kx, \quad m \cdot dv = -kx \cdot dt, \quad m \cdot v \cdot dv = -kx \cdot v \cdot dt,$$

$$mv \cdot dv = -kx\frac{dx}{dt}dt = -kx \cdot dx, \quad d\left[\frac{1}{2}mv^2\right] = d\left[-\frac{1}{2}kx^2\right]$$

Example: Acumin canister of mass m = 0.40 kg slides across a horizontal frictionless counter with speed v = 0.50 m/s. It then runs into and compresses a spring of spring constant k = 750 N/m. When the canister is momentarily stopped by the spring, by what distance d is the spring compressed?



$$E = \frac{1}{2}kx^2 = \frac{1}{2}mv^2, \quad 750 \cdot x^2 = 0.4 \cdot 0.5^2, \quad x = 0.012 \cdot m = 1.2 \cdot cm$$

7.7 Conservative and Nonconservative Forces

Conservative Forces

isolated system -> $W = \Delta E = 0$

1. The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle
2. The work done by a conservative force on a particle moving through any closed path is zero.

Conservative -> the force can be derived from a potential function

Nonconservative forces -> can not find a scalar function – potential -> energy is not conserved

Example: A single constant force $\vec{F} = (3\hat{i} + 5\hat{j})N$ acts on a 4.00-kg particle. (a) Calculate the work done by this force if the particle moves from the origin to the point having the vector position $\vec{r} = (2\hat{i} - 3\hat{j})m$. Does this result depend on the path? Explain. (b) What is the speed of the particle at r if its speed at the origin is 4.00 m/s? (c) What is the change in its potential energy?

(a) constant force → $W = \vec{F} \cdot \Delta\vec{r} = 6 - 15 = -9$ J, does not depend on the path

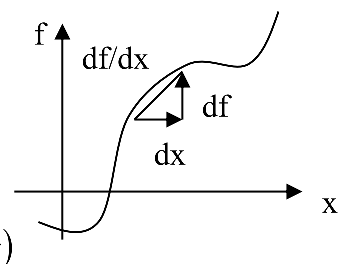
(b) $\Delta K = -9 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

(c) ??

7.8 Relationship Between Conservative Forces and Potential Energy

Differentiation <-> Integration

$df = \frac{df}{dx}dx$, df : infinitesimal displacement of f, $\frac{df}{dx}$: variation of f with respect to x, dx: infinitesimal displacement of x



$\Delta U(x) = -W = -F(x)\Delta x$ → $F(x) = -\frac{\Delta U(x)}{\Delta x} = -\frac{dU(x)}{dx}$

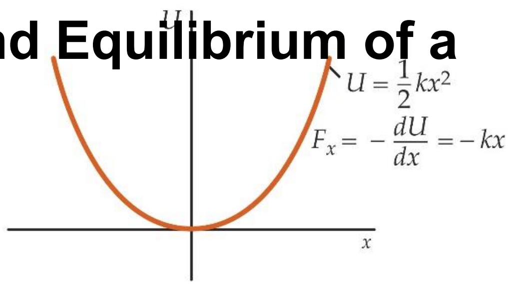
$F_x(x) = -\frac{dU(x)}{dx}$ Extending to two dimensions, if the potential energy is $U(x, y)$, the forces in x and y directions are $F_x = -\frac{\partial U(x,y)}{\partial x}$ and $F_y = -\frac{\partial U(x,y)}{\partial y}$.

$U = \frac{1}{2}kx^2$ → F=?

$U = -\frac{GMm}{r}$ → F=?

$U = \frac{kq_1q_2}{r}$ → F=?

7.9 Energy Diagrams and Equilibrium of a System

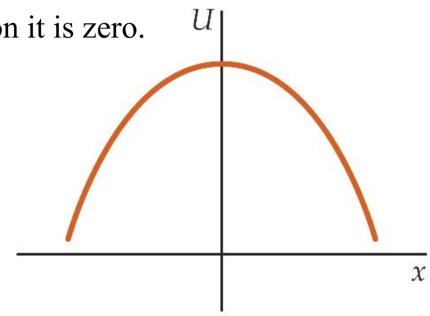


For a general conservative force,

$dU = -F_x dx$ → $F_x = -\frac{dU}{dx}$

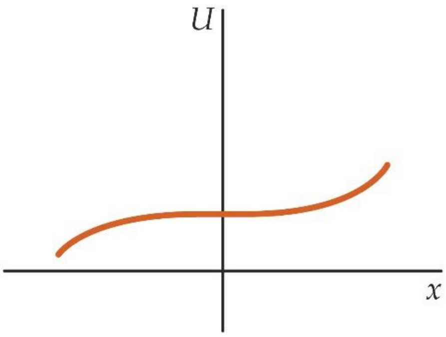
for a blocksprint system: $U = \frac{1}{2}kx^2$, $F_x = -kx$

A particle is in equilibrium if the net force acting on it is zero.



In stable equilibrium, a small displacement in any direction results in a restoring force that accelerates the particle back toward its equilibrium.

In unstable equilibrium, a small displacement results in a force that accelerates the particle away from its equilibrium position.



In neutral equilibrium, a small displacement results in zero force and the particle remains in equilibrium.

Example: In the region –a < x < a the force on a particle is represented by the potential energy function $U = -b\left(\frac{1}{a+x} + \frac{1}{a-x}\right)$, where a and b are positive constants. (a) Find the force. (b) At what value of x is the force zero? (c) stable or unstable?

(a) $F = -b\left(\frac{1}{(x+a)^2} - \frac{1}{(x-a)^2}\right)$

(b) x = 0

(c) unstable

Lenard-Jones potential energy

$$U(x) = 4\varepsilon\left[\left(\frac{\sigma}{x}\right)^{12} - \left(\frac{\sigma}{x}\right)^{6}\right]$$

$F = ?$